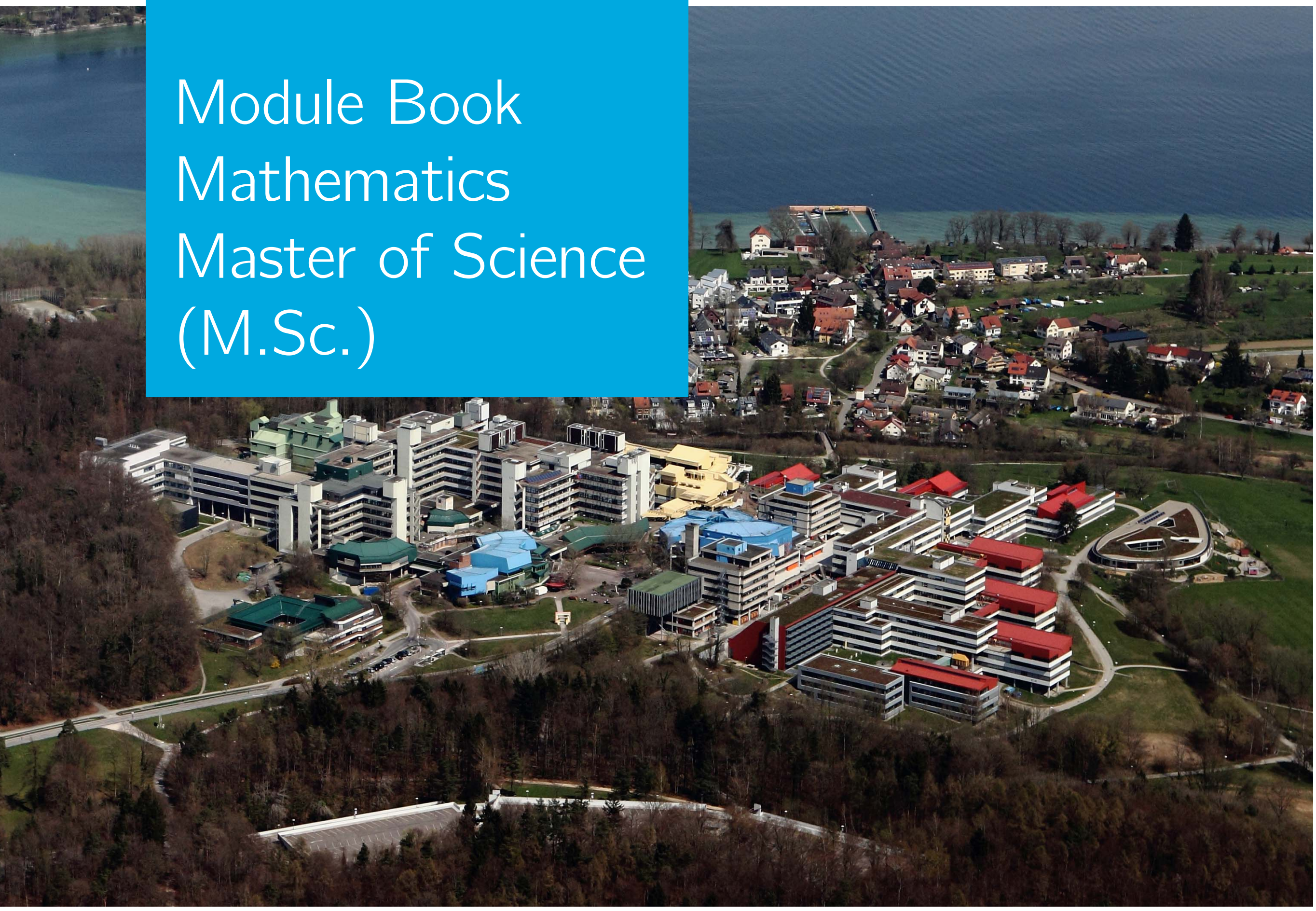
Universität Konstanz

Module Book Mathematics Master of Science (M.Sc.)



Department of Mathematics and Statistics

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1 Qualification Goals

The mathematics degree programme is a scientific education that provides the foundation for a later career in a wide range of branches within the economy, industry or research. The main focus of this education is the study of mathematical theories and methods, the practical implementation and application of these methods, and the ability to communicate this knowledge. In addition to imparting special mathematical knowledge, specific ways of thinking and working as well as rigour, creativity and persistence are acquired. Since these skills are in demand in wide areas of industry and commerce as well as at schools and universities and since they are of high social relevance, they represent and important goal that is automatically imparted through the study of mathematics. The intensive active engagement with mathematical content enables students to experience an openness of thought, coupled with stringency and self-criticism, which can also be extended to other areas of professional and public life. Through the active acquisition of sound mathematical knowledge, students acquire the ability to recognise analogies and basic patterns as well as the ability to recognise, formulate and solve complex problems. They practise conceptual, analytical and logical thinking and develop strategies for lifelong learning. The consecute Master's programme in Mathematics aims to expand basic mathematical knowledge as well as a specialisation that leads to contact with current research in one of the focal areas available at Konstanz (see below). Graduates of the Master's programme are able to apply mathematical methods and models and to develop them independently. The preparation of the Master's thesis strengthens students' skills to work independently and scientifically, to analyse and solve problems and to organise work. The successfully completed consecutive Bachelor's - Master's programme should, among other things, enable students

- to pursue an independent mathematical work in industry and economy,
- to lead projects that involve analysing, modelling and solving complex scientific, economic or technical problems,
- to perform planning, development and research tasks in scientific and public institutions,
- to work as a research assistant or as a research associate at a university and
- to pursue a doctoral programme.

2 Analysis and Numerics

Main module: Theory of partial differential equations II

ECTS	Duration	SWS (Weekly teaching hours)	Placement
9	1 semester	6	Main module in the study field “Analysis and Numerics” or elective module

Module grade: Written or oral exam

Learning objectives:
- Deepening of the previously acquired basic knowledge in partial differential equations
- Basis for specialization in the field of partial differential equations
- Prerequisites: Basic knowledge in partial differential equations and functional analysis, for example, obtained from the theoretical part of the course "Theory and numerics of partial differential equationsänd the course "Functional analysisïn the Bachelor’s degree program.

Learning outcomes: The students
- know and understand selected advanced topics including concepts, statements, and methods in the field of partial differential equations,
- understand the specific characteristics of individual types of partial differential equations and can apply methods of functional analysis to specific types,
- are able to independently analyse various types of partial differential equations mathematically.

Partial differential equations II

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	– Lecture 4 SWS – Tutorial 2 SWS	First semester Master’s degree	Winter semester (annually)	German or English (if desired)

Prerequisites: Basic knowledge about theoretical methods for partial differential equations, to the extent treated in the bachelor course “Theory and numerics of partial differential equations” offered in Konstanz.

Teaching contents: Building upon the foundations of partial differential equations as covered in the course "Theory and numerics of partial differential equations", selected topics in the field of partial differential equations are addressed, such as: wave equations, elliptic operators, variational evolution equations, conservation equations, coupled systems, or semigroups. Additionally, elliptic/parabolic regularity theories, eigenfunctions, maximum principles, and their applications are explored.

Form of examination: Written or oral exam

Work load: 270 h

- On-site studies in lectures and exercises
- Preparation and follow-up of the lectures
- Exercise tasks
- Exam preparation

Main module: Numerical methods for partial differential equations II

ECTS	Duration	SWS (Weekly teaching hours)	Placement
9	1 semester	6	Main module in the study field "Analysis and Numerics" or elective module

Module grade: Written or oral exam

Learning objectives: The lecture provides a comprehensive overview of numerical approximation methods for partial differential equations, with a focus on the finite element method. Additionally, modern methods from the field of numerical linear algebra are covered. In the programming aspect, the focus is on the structured implementation of extensive projects.

Learning outcomes: The students

- understand the basic idea of the finite element method (FEM), can derive weak formulations and provide the finite-dimensional representations. They have knowledge of error estimations for common finite elements,
- are able to explain the construction of finite-dimensional trial spaces and understand the derivation of a priori and a posteriori error estimations,
- can independently develop FEM algorithms on simple domains, starting from grid generation, matrix assembly, solving the equation systems, and visual representation of the solutions,
- can further analyse self-written algorithms using special cases with known solutions, identify and correct possible programming errors,
- are able to discretize and solve classical partial differential equation problems using FEM software packages,
- can assess the quality of FEM calculations and estimate the impact of different triangulations on the final result.

Numerical methods for partial differential equations II

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	— Lecture 4 SWS — Tutorial 2 SWS	Second semester Master's degree	Summer semester (annually)	German or English (if desired)

Prerequisites: Basic knowledge about theoretical and numerical methods for partial differential equations and functional analysis, e.g. to the extent of the bachelor courses "Theory and numerics of partial differential equations" and "Functional analysis" offered in Konstanz.

Teaching contents:

- Finite element method (FEM) for elliptic boundary value problems
- FEM for parabolic problems

- Finite Volume and DG methods for hyperbolic conservation equations
- Error estimations, adaptivity
- Krylov space methods for solving linear equation systems
- Preconditioning and multigrid methods

Form of examination: Written or oral exam

Work load: 270 h

- On-site studies in lectures and exercises
- Preparation and follow-up of the lectures
- Exercise tasks
- Exam preparatio

Main module: Optimization II

ECTS	Duration	SWS (Weekly teaching hours)	Placement
9	1 Semester	6	Main module in the study field “Analysis and Numerics”

Module grade: Written or oral exam

Learning objectives: The lecture deals with mathematical methods for the solution of constrained nonlinear optimization problems.

Learning outcomes: Students are familiar with mathematical methods of constrained nonlinear optimization, can apply them to corresponding optimization problems and program suitable algorithms for the numerical implementation of the learned methods.

Optimization II

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	- Lecture 4 SWS - Tutorial 2 SWS	First semester of Master’s degree	Winter semester (annualy)	German or English (if desired)

Prerequisites: Contents of the bachelor courses Numerical mathematics, Optimization I and basic programming skills in Python (or similar).

Teaching contents:
- Optimality conditions for constrained optimization
- Linear programming and interior-points method
- Quadratic programming
- Penalty and augmented Lagrangian methods
- Newton-type methods
- SQP method

Form of examination: Written or oral exam

Work load: 270 h
- On-site studies in lectures and exercises
- Preparation and follow-up of the lectures
- Exercise tasks
- Exam preparation

Specialization module: Asymptotics of nonlinear waves

ECTS	Duration	SWS (Weekly teaching hours)	Placement
3	1 semester	2	Specialization Module in the area Analysis and Numerics

Module grade: Written or oral exam

Learning objectives:
- Deepening of the previously acquired knowledge in partial differential equations in a specific direction
- Foundation for an examination as a specialization in the field of partial differential equations

Learning outcomes: The students
- possess advanced knowledge of the intricacies of the long-term behaviour of nonlinear waves in parabolic and hyperbolic-parabolic systems,
- are able to apply methods from the theory of evolutionary partial differential equations,
- recognize how the combination of different representations and techniques enables precise statements about long-term behaviour.

Asymptotics of nonlinear waves

ECTS	Teaching methods	Recommended semester	Frequency	Language
3	Lecture 2 SWS	From the second semester of the Master's program onwards	Depending on the overall offerings in Analysis	German or English (if desired)

Prerequisites: Theory of partial differential equations II

Teaching contents:
- Nonlinear wave equations and Schrödinger equations and the temporal asymptotics of their solutions are investigated in various geometries.

Form of examination: Written or oral exam

Work load: 90 h
- In-person attendance in lectures
- Preparation and follow-up of the lectures
- Exam preparation

Specialization module: Dynamical systems I

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 Semester	3	Specialization module in the study field “Analysis and Numerics”, or elective module

Module grade: Written or oral exam

Learning objectives: Consolidation and deepening of previously obtained knowledge in the study field Analysis, and basis for an examination in Analysis as area of specialization.

Learning outcomes: Students understand and internalize basic concepts, statements and methods of the theory of dynamical systems, can apply the obtained knowledge to important nonlinear systems, and recognize the importance of the topic for non-mathematical questions.

Dynamical systems I

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- Lecture 2 SWS - Tutorial 1 SWS	1st semester of Master’s degree or later	Depending on the overall course offer in Analysis	German or English (if desired)

Prerequisites: Background knowledge in the scope of the bachelor courses Analysis I-III und Funktionalanalysis offered in Konstanz.

Teaching contents:

- Invariant manifolds
- Stability of equilibrium points and periodic orbits

Form of examination: Written or oral exam

Work load: 135 h

- Attendance in lectures and tutorials
- Preparation and follow-up of the lectures
- Solving of exercises
- Exam preparation

Specialization module: Dynamical systems II

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 Semester	3	Specialization module in the study field "Analysis and Numerics", or elective module

Module grade: Written or oral exam

Learning objectives: Consolidation and deepening of previously obtained knowledge in the study field Analysis, and basis for an examination in Analysis as area of specialization.

Learning outcomes: Students understand and internalize basic concepts, statements and methods of the theory of dynamical systems, can apply the obtained knowledge to important nonlinear systems, and recognize the importance of the topic for non-mathematical questions.

Dynamical systems II

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- Lecture 2 SWS - Tutorial 1 SWS	2nd semester of Master's degree or later	Depending on the overall course offer in Analysis	German or English (if desired)

Prerequisites: Dynamical systems I

Teaching contents: Topics may vary, but could for instance be one of the following:
- Spectral stability of nonlinear waves
- Geometriec singular perturbation theory

Form of examination: Written or oral exam

Work load: 135 h
- Attendance in lectures and tutorials
- Preparation and follow-up of the lectures
- Solving of exercises
- Exam preparation

Specialization module: Model reduction with proper orthogonal decomposition

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	3	Specialization module in the field of Analysis and Numerics

Module grade: Written or oral exam

Learning objectives:

- The main objective of the lecture is to introduce the field of model reduction using proper orthogonal decomposition (POD) as an example. In addition to the theoretical foundations, the numerical implementation of the methods with the help of the computer is also of great importance.

Learning outcomes: The students

- can name exemplary applications of model reduction,
- are able to clearly present the POD method and to choose an appropriate topology for computing the POD basis,
- can assess whether the POD model reduction is suitable for a given initial value problem or not,
- are able to determine a POD basis numerically efficiently
- can transform a given initial value problem into a reduced problem and solve the reduced model with the help of the computer,
- can assess various variants of the POD method (such as the choice of snapshots, of topology) and select the appropriate method for a given problem.

Model reduction with proper orthogonal decomposition

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	– Lecture 2 SWS – Tutorial 1 SWS	From the first semester of the Master's degree program onwards	Every 2-3 years	German or English (if desired)

Prerequisites: Main module Optimization II

Teaching contents:

- Introduction to Proper Orthogonal Decomposition (POD)
- Application as model reduction
- Analysis of reduced models

Form of examination: Written or oral exam

Work load: 135 h

- Attendance in the lectures and tutorials
- Preparation and follow-up of the lectures
- Exercise tasks
- Programming exercises
- Exam preparation

Specialization module: Nonlinear Cauchy problems

ECTS	Duration	SWS (Weekly teaching hours)	Placement
3	1 semester	2	Specialization module within the field of Analysis and Numerics

Module grade: Written or oral exam

Learning objectives:
- Deepening of the previously acquired knowledge in partial differential equations in a specific direction
- Basis for an examination as a specialized area in the field of partial differential equations

Learning outcomes: The students
- are familiar with and understand methods for obtaining global solutions in nonlinear initial value problems of partial differential equations,
- understand the peculiarities that arise in various systems of mathematical physics,
- apply results from functional analysis and the theory of partial differential equations to concrete Cauchy problems.

Nonlinear Cauchy problems

ECTS	Teaching methods	Recommended semester	Frequency	Language
3	Lecture 2 SWS	From the second semester of the Master's degree onwards	Depending on the overall course offering in the field of analysis	German or English (if desired)

Prerequisites: Main module Partial differential equations II

Teaching contents:
- A method for treating general nonlinear Cauchy problems is presented using nonlinear wave equations as examples. This method is applied to various models in mathematical physics.

Form of examination: Written or oral exam

Work load: 90 h
- Attendance in the lectures
- Preparation and follow-up of the lectures
- Exam preparation

Specialization module: Numerical methods for stochastic differential equation

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 Semester	3	Specialization module in the study fields “Analysis and Numerics” or “Stochastics”, or elective module

Module grade: Written or oral exam

Learning objectives:

- Basic point measure approximations of common probability measures in high dimensions.
- Algorithms for the approximate solution of stochastic differential differential equations and graphical representation of solutions in a meaningful way.

Learning outcomes: Students

- can name basic ideas in the discretization of stochastic differential equations and have knowledge of the convergence properties of the treated methods,
- are able to translate abstract probabilistic models into concrete models for algorithmic purposes and to implement them in the form of programs,
- can explain advantages and disadvantages of different dimensional approximations,
- are able to trace the development of different approximation algorithms and to give mathematical reasons for the justifications for properties of the methods,
- can analyze self-written algorithms using special cases with known solutions and recognize and eliminate programming errors.
- can assess the quality of calculations with pseudorandom number-based algorithms and assess the effect of different dimensional approximations on the final result.

Numerical methods for stochastic differential equation

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- Lecture 2 SWS - Tutorial 1 SWS	1st semester of Master’s degree or later	Winter semester (annualy)	German or English (if desired)

Prerequisites: Background knowledge in the scope of the bachelor courses Analysis I-III, Linear Algebra I and Numerical Mathematics I offered in Konstanz

Teaching contents:

- Theory and numerics of stochastic differential equations
- Difference methods for Black-Scholes and heat equation
- Numerical realization of the methods on a computer

Form of examination: Written or oral exam

Work load: 120 h

- Preparation and follow-up of the lectures
- Exercise tasks
- Programming tasks
- Exam preparations

Specialization module: Numerical methods for constrained optimization

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	3	Specialization module in the field Analysis and Numerics

Module grade: Written or oral exam

Learning objectives:
- The main objective of the lecture is to study algorithms for constrained optimization methods. In addition to the theoretical foundations, the numerical implementation of these methods with the help of a computer is also of great importance.

Learning outcomes: The students
- are able to represent optimality conditions for optimization problems with constraints and distinguish between different numerical methods for constrained optimization based on optimality conditions,
- are able to implement specifically SQP and interior point methods on a computer,
- are able to select appropriate globalization strategies (such as trust-region or line search methods),
- are able to verify theoretical convergence properties using numerical examples,
- are able to present numerical results and interpret them in the context of the given optimization problem.

Numerical methods for constrained optimization

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	– Lecture 2 SWS – Tutorial 1 SWS	From the first semester of the Master's degree onwards	every 2-3 years	German or English (if desired)

Prerequisites: Main module Optimization II

Teaching contents:
- Globally convergent descent methods
- Netwon-like methods
- SQP method

Form of examination: Written or oral exam

Work load: 135 h
- Attendance in the lectures and tutorials

- Preparation and follow-up of the lectures
- Exercise tasks
- Programming tasks
- Exam preparations

Specialisation module: Optimization III

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 Semester	4	Specialisation module in the study field Änalysis and Numerics"

Module grade: Oral exam

Learning objectives: Building on basic knowledge of mathematical optimization, the course introduces various other areas of optimization, which can be deepened in subsequent special courses.

Learning outcomes: The students have theoretical knowledge of advanced methods of mathematical methods, can apply them to corresponding problems and program algorithms for their practical implementation.

Optimization III

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- Lecture 2 SWS - Tutorial 1 SWS	1st to 3rd semester of Master's degree	About every second year	German or English (if desired)

Prerequisites: Contents of the bachelor courses on Numerical analysis and Optimization I, ideally also of the master course Optimization II, and programming skills in Python (or similar)

Teaching contents: The lecture treats the following topics:
- Finite dimensional optimal control
- Model reduction by means of Proper Orthogonal Decomposition
- Multi-objective optimization
- Linear-quadratic optimal control
- Stochastic gradient method

Form of examination: Oral exam

Work load: 135 h
- Attendance in lectures and tutorials
- Self-study
- Preparation and follow-up of the lecture
- Exercises
- Exam preparation

Specialization module: Optimal control of elliptic PDE

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	3	Specialization module in the field of Analysis and Numerics

Module grade: Written or oral exam

Learning objectives:

- The main objective of the lecture is to introduce the field of optimal control of partial differential equations, using elliptic equations as an example. Topics covered include the existence of optimal controls and the derivation of optimality conditions. The overall goal is to provide a methodology that can be applied to much more general optimal control problems.

Learning outcomes: The students

- can provide examples of application as well as state existence and uniqueness results for (non-)linear elliptic differential equations,
- are able to analyse optimal control problems for (non-)linear elliptic differential equations using the theory of infinite-dimensional optimization,
- can derive optimality conditions,
- are able to provide example problems and explain the theoretical results obtained based on those examples,
- can specify numerical algorithms for solving optimal control problems and implement them on a computer.

Optimal Control of Elliptic PDEs

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	— Lecture 2 SWS — Tutorial 1 SWS	From the first semester of the Master's degree onwards	every 2-3 years	German or English (if desired)

Prerequisites: Main modules Optimiziation II, Partial differential equations II

Teaching contents:

- Key concepts in the finite-dimensional case
- Linear-quadratic elliptic problems
- Nonlinear elliptic problems
- Optimality conditions

Form of examination: Written or oral exam

Work load: 135 h

- Attendance in the lectures and tutorials
- Preparation and follow-up of the lectures
- Exercise tasks
- Programming tasks
- Exam preparation

Specialization module: Optimal control of parabolic PDE

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	3	Specialization module in the field of Analysis and Numerics

Module grade: Written or oral exam

Learning objectives:

- The main objective of the lecture is to address optimal control problems for parabolic differential equations. The topics covered include the existence of optimal controls and the derivation of optimality conditions. A meta-objective is to provide a methodology that allows for the treatment of much more general optimal control problems.

Learning outcomes: The students

- can provide motivating application examples as well as state existence and uniqueness results for (non)linear parabolic differential equations,
- are able to investigate optimal control problems for (non)linear parabolic differential equations using the theory of infinite-dimensional optimization,
- are able to derive optimality conditions, particularly the adjoint equation,
- are able to provide example problems and explain the theoretical results found using those examples,
- are able to specify numerical algorithms for solving optimal control problems and implement them on a computer.

Optimal control of parabolic PDE

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	– Lecture 2 SWS – Tutorial 1 SWS	from the second Master's semester onwards	every 2-3 years	German or English (if desired)

Prerequisites: Main modules Optimization II, Partial Differential Equations II and specialization module Optimal control of elliptic PDE

Teaching contents:

- linear-quadratic parabolic problems
- nonlinear parabolic problems
- Optimality conditions of first and second order

Form of examination: Written or oral exam

Work load: 135 h

- Attendance in the lectures and the tutorials
- Preparation and follow-up of the lectures
- Exercise tasks
- Programming tasks
- Exam preparation

Specialization module: Optimization methods in Banach spaces

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	3	Specialization module in the field of Analysis and Numerics

Module grade: Written or oral exam

Learning objectives:

- The main objective of the lecture is to examine algorithms for infinite-dimensional optimization problems and their application to optimal control problems for partial differential equations. In addition to the theoretical foundations, the numerical implementation of these methods on a computer plays an important role.

Learning outcomes: The students

- are able to formulate and analyze infinite-dimensional optimization problems,
- are able to demonstrate the existence of optimal solutions,
- are able to derive necessary and sufficient optimality conditions,
- are able to apply the theoretical results to optimal control problems for partial differential equations,
- are able to identify optimization methods for infinite-dimensional optimization problems and implement optimization algorithms on a computer,
- are able to verify theoretical convergence results through numerical examples and provide meaningful explanations of numerical results in relation to the given optimization problem.

Optimization methods in Banach spaces

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	– Lecture 2 SWS – Tutorial 1 SWS	from the second Master's semester onwards	Winter semester (every 2-3 years)	German or English (if desired)

Prerequisites: Main module Optimization II

Teaching contents:

- globally convergent descent methods
- Netwon-like methods
- SQP methods

Form of examination: Written or oral exam

Work load: 135 h

- Attendance in the lectures
- Preparation and follow-up of the lectures
- Exercise tasks
- Programming tasks
- Exam preparation

Specialization module: Parabolic boundary value problems

ECTS	Duration	SWS (Weekly teaching hours)	Placement
3	1 semester	2	Specialization module within the field of Analysis and Numerics

Module grade: Written or oral exam

Learning objectives:
- Deepening of previously acquired knowledge in partial differential equations in a specific direction
- Basis for an examination as a specialization in the field of partial differential equations

Learning outcomes: The students
- are able to identify parabolic and parameter-elliptic boundary value problems and can describe central solution approaches such as Fourier and Laplace transformations,
- can develop a solution approach based on these methods and apply it to specific equations in mathematical physics,
- are able to derive a priori estimates using the previously developed solution approach, utilizing the Mikhlin's theorem,
- can transfer the developed elliptic concepts to parabolic boundary value problems and compare the obtained regularity results with other approaches, such as the weak solution concept.

Parabolic boundary value problems

ECTS	Teaching methods	Recommended semester	Frequency	Language
3	Lecture 2 SWS	from the second Master's semester onwards	Depending on the overall course offerings in Analysis	German or English (if desired)

Prerequisites: Main module Partial differential equations II

Teaching contents:
- The module discusses parabolic and parameter-elliptic boundary value problems and their associated solution methods. Based on Fourier and Laplace transformations, explicit solution formulas are developed. Additional topics include the Shapiro-Lopatinskii condition, the Mikhlin's theorem, a priori estimates, and maximal regularity.

Form of examination: Written or oral exam

Work load: 90 h

- Attendance in the lectures
- Preparation and follow-up of the lectures
- Exam preparation

Specialization module: Pseudodifferential operators

ECTS	Duration	SWS (Weekly teaching hours)	Placement
3	1 semester	2	Specialization module in the field of Analysis and Numerics

Module grade: Written or oral exam

Learning objectives:
- Deepening of previously acquired knowledge in partial differential equations in a specific direction
- Basis for an examination as a specialization in the field of partial differential equations

Learning outcomes: The students
- are familiar with the concept and applications of pseudodifferential operators,
- recognize connections to differential operators and the Fourier transform,
- are able to apply and further develop the calculus of pseudodifferential operators, and analyze the solvability of partial differential equations using them,
- are able to construct solution approaches for partial differential equations using the parametrix method,
- are able to evaluate the concept of pseudodifferential operators in comparison with other methods of partial differential equations.

Pseudodifferential operators

ECTS	Teaching methods	Recommended semester	Frequency	Language
3	Lecture 2 SWS	from the second Master's semester onwards	Depending on the overall course offerings in analysis	German or English (if desired)

Prerequisites: Main module Partial differential equations II

Teaching contents:
- Pseudodifferential operators and their application to partial differential equations are discussed. Pseudodifferential operators are generalizations of differential operators that are defined using the Fourer transofmr. Topics covered include composition, algebraic properties, symbol classes, parametrices, and norm estimates.

Form of examination: Written or oral exam

Work load: 90 h
- Attendance in the lectures
- Preparation and follow-up of the lectures

- Exam preparation

Specialization module: Stability and spectrum

ECTS	Duration	SWS (Weekly teaching hours)	Placement
3	1 semester	2	Specialization module in the area of Analysis und Numerik

Module grade: Written or oral exam

Learning objectives:
- Deepening of previously acquired knowledge in analysis
- Basis for an examination as a specialization in analysis

Learning outcomes: The students
- understand concepts and statements of spectral theory and stability theory,
- can apply methods of spectral theory to stability questions,
- recognize the connection between the theory and its application.

Stability and spectrum

ECTS	Teaching methods	Recommended semester	Frequency	Language
3	Lecture 2 SWS	from the second Master's semester onwards	Depending on the general course offerings in Analysis	German or English (if desired)

Prerequisites: Knowledge in the scope of the bachelor courses Analysis I-III and Functional analysis

Teaching contents:
- Spectrum and stability of nonlinear waves in evolutionary systems.

Form of examination: Written or oral exam

Work load: 90 h
- Attendance in the lectures
- Preparation and follow-up of the lectures
- Exam preparation

Specialization module: Thermoelastic systems

ECTS	Duration	SWS (Weekly teaching hours)	Placement
3	1 semester	2	Specialization module in the field Analysis and Numerics

Module grade: Written or oral exam

Learning objectives:

- Deepening of previously acquired knowledge in partial differential equations in a specific direction
- Foundation for an examination as a specialization in the field of partial differential equations

Learning outcomes: The students

- know and understand coupled systems of partial differential equations of the thermoelastic type,
- understand the peculiarities that arise from the coupling of different types of equations,
- apply theorems from functional analysis and the theory of partial differential equations to concrete coupled systems.

Thermoelastic systems

ECTS	Teaching methods	Recommended semester	Frequency	Language
3	Lecture 2 SWS	from the second Master's semester onwards	As needed in Analysis	German or English (if desired)

Prerequisites: Main module Partial differential equations II

Teaching contents:

- Various models of partial differential equations for systems that couple elasticity equations with parabolic or hyperbolic heat conduction equations are analyzed, particularly in terms of temporal dynamics

Form of examination: Written or oral exam

Work load: 90 h

- Attendance in the lectures
- Preparation and follow-up of the lectures
- Exam preparation

3 Real Geometry and Algebra

Main module: Real algebraic geometry I

ECTS	Duration	SWS (Weekly teaching hours)	Placement
9	1 semester	6	Main module in the specialization "Real Geometry and Algebraör elective module

Module grade: Written or oral exam

Learning objectives:

- The students will be introduced to central areas of real algebraic geometry. The methods of this field have gained significant importance for both theory and practical applications in recent decades.

Learning outcomes: The students

- understand the basic properties of ordered fields,
- understand the conceptual and algorithmic aspects of the Tarski-Seidenberg theorem and its implications, and can apply this central theorem to the study of the properties of semialgebraic sets and varieties,
- are able to examine the field-theoretical background of the Artin-Schreier theory and analyze the role of this theory for real closed fields,
- can derive the geometric properties of semialgebraic sets using their algebraic properties,
- are able to mathematically differentiate between archimedean and non-archimedean real closed fields.

Real algebraic geometry I

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	– Lecture 4 SWS – Tutorial 2 SWS	First semester Master's degree	Winter semester (annually)	German or English (if desired)

Prerequisites: Advanced knowledge in Algebra, e.g. to the extent of the bachelor courses “Algebra I” and “Algorithmic algebraic geometry” offered in Konstanz

Teaching contents: Ordered fields and real closure, Tarski-Seidenberg elimination, Artin-Lang theorem, real spectrum, connection with semialgebraic sets, semialgebraic geometry

Form of examination: Written or oral exam

Work load: 270 h

- On-site studies in lectures and exercises
- Preparation and follow-up of the lectures
- Exercise tasks

- Exam preparation

Main module: Real algebraic geometry II

ECTS	Duration	SWS (Weekly teaching hours)	Placement
9	1 semester	6	Main module in the specialization "Real Geometry and Algebraör elective module

Module grade: Written or oral exam

Learning objectives:
- The knowledge in real algebraic geometry will be further deepened, building upon the first part, and the students will be introduced to selected results from current research. The module aims to prepare the students to work independently on current issues in real algebraic geometry.

Learning outcomes: The students
- understand the basic properties of positive polynomials and sums of squares,
- understand the conceptual properties of Hilbert's and Artin-Schreier's theorem and their implications, and can apply these central theorems to the study of the properties of semialgebraic sets and varieties,
- are able to analyze the functional analytic background,
- can derive the topological properties of quadratic preorders using the algebraic properties of semialgebraic sets,
- are able to mathematically differentiate between finitely generated and non-finitely generated preorders.

Real algebraic geometry II

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	– Lecture 4 SWS – Tutorial 2 SWS	Second semester Master's degree	Summer semester (annually)	German or English (if desired)

Prerequisites: Main module Real algebraic geometry I

Teaching contents: Positive polynomials and sums of squares, archimedean property, representation theorem, discussion of selected applications

Form of examination: Written or oral exam

Work load: 270 h
- On-site studies in lectures and tutorials
- Preparation and follow-up of the lectures
- Exercise tasks

- Exam preparation

Main module: Model theory

ECTS	Duration	SWS (Weekly teaching hours)	Placement
9	1 semester	6	Main module in the specialization "Real Geometry and Algebraör elective module

Module grade: Written or oral exam

Learning objectives:

- The students are introduced to central areas of model theory and familiarized with selected results from current research. The methods of this field have gained significant importance and applications in algebra, analysis, and geometry in recent decades. The module aims to prepare the listeners to work independently on current questions in model theory of algebraic structures.

Learning outcomes: The students

- understand basic abstract structures and models,
- understand theories that allow quantifier elimination,
- apply abstract theorems and methods of model theory to concrete mathematical problems,
- are able to analyze algebraic situations using abstract model-theoretic methods,
- can independently prove the main results of model theory and are capable of justifying the correctness of a statement through a proof or refuting it with counterexamples.

Model theory

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	– Lecture 4 SWS – Tutorial 2 SWS	First semester Master's degree	Every 1-2 years	German or English (if desired)

Prerequisites: Advanced course in Algebra, course in set theory

Teaching contents:

- Fundamental concepts of propositional logic, languages, structures, models, and theories,
- Completeness and compactness theorems, Loewenheim-Skolem theorems,
- Model completeness and quantifier elimination, elementary extensions,
- Ultrafilters and ultraproducts, homomorphisms and isomorphisms, categoricity,
- Applications of some mathematical theories (order theory, group theory, field theory, number theory, set theory)

Form of examination: Written or oral exam

Work load: 270 h

- On-site studies in lectures and exercises
- Preparation and follow-up of lectures
- Exercise tasks
- Exam preparation

Main module: Valuation theory

ECTS	Duration	SWS (Weekly teaching hours)	Placement
9	1 semester	6	Main module in specialisation "Real Geometry and Algebraör elective module

Module grade: written or oral exam

Learning objectives:

- The students will be introduced to central areas of valuation theory and to selected results of current research. Methods in this field have grown in importance and applications in algebra, analysis and geometry. The module aims to prepare the listeners to work independently on current questions in valuation theory and model-theoretic algebra.

Learning outcomes: The students

- know Hahn groups as well as Hahn bodies and understand the theories of Hensel bodies,
- apply abstract theorems and methods of valuatio theory to concrete mathematical problems,
- are able to analyse geometric facts with abstract evaluation methods of evaluation theory,
- are able to independently prove the main statements of valuation theory and are able to justify the correctness of a statement with evidence or refute it with counterexamples.

Valuation theory

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	– Lecture 4 SWS – Tutorial 2 SWS	second semester Master's programme	Irregularly	German or English (if desired)

Prerequisites: Advanced module in algebra. Set theory and model theory lectures are recommended.

Teaching contents: Fundamentals of commutative algebra, places and valuation rings, ordered abelian groups, valuations, rank of a valuation, extensions, the fundamental inequality, Hensel's valuations, definable valuations, model theory of valued solids.

Form of examination: Written or oral exam

Work load: 270 h

- Attendance study in lecture and tutorial
- Preparation and follow-up of the lectures
- Exercise sheets

- Exam preparation

Specialization module: Positive polynomials and optimization

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	3	Specialization module in the field "Real Geometry and Algebraör elective module

Module grade: Written or oral exam

Learning objectives:

- The students should understand how to naturally relax polynomial optimization problems into a primal-dual pair of semidefinite optimization problems.
- The students are familiar with results on the convergence and accuracy of relaxations, as well as their connection to results on square sum representations of positive polynomials.
- The students should test the effectiveness of the relaxation method on specific examples.

Learning outcomes:

- The students should be informed about the relatively new connections between polynomial optimization problems and real algebraic geometry.
- They should learn how to translate nonlinear optimization problems exactly or approximately into semidefinite optimization problems using real algebra.
- The acquired knowledge can be applied in a thesis or in future professional endeavours.

Positive polynomials and optimization

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	– Lecture 2 SWS – Tutorial 1 SWS	from the fifth Bachelor's semester or the first Master's semester onwards	In general, the course is offered during the winter semester. Please note that the scheduling may vary slightly from year to year, but there is typically a similar offering annually	German or English (if desired)

Prerequisites: Main module Real algebraic geometry I/II

Teaching contents: Semidefinite optimization (SDP), duality in SDP, Gram matrix method, polynomial optimization problems, Lasserre relaxations, square sum representations (with degree bounds), (truncated) moment problem, spectrahedra, and semidefinite representations

Form of examination: Written or oral exam

Work load: 135 h

- Attendance in the lectures
- Attendance in the tutorials
- Exercise tasks
- Exam preparation

Specialization module: Quadratic forms

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	3	Specialization module in the field "Real Geometry and Algebraör elective module

Module grade: Written or oral exam

Learning objectives:
- The participants will be introduced to the fundamentals of the theory of quadratic forms. It is a research-intensive central area of algebra with significant connections to real algebraic geometry.
- Therefore, it is an important complement to the main module "Real Algebraic Geometry"but can also be beneficially taken by students from other specialization areas.

Learning outcomes:
- The students have a fundamental knowledge in the field of quadratic forms theory and are able to apply it, for example, to problems related to various field invariants, particularly in connection with quadratic sums.

Quadratic forms

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	– Lecture 2 SWS – Tutorial 1 SWS	Master's degree, any semester	every 2-3 years	German or English (if desired)

Prerequisites: Algebra I (bachelor studies)

Teaching contents: Basics, Witt ring, invariants, signatures, Pfister's local-global principle, field extensions, Pfister forms, level, Pythagoras number

Form of examination: Written or oral exam

Work load: 135 h
- Attendance in the lectures
- Attendance in the tutorials
- Exercise tasks
- Exam preparation

Specialization module: Toric varieties

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	3	Specialization module in the field "Real Geometry and Algebraör elective module

Module grade: Written or oral exam

Learning objectives: Introduction to the fundamentals of toric varieties. This field is a particularly explicit specialization of algebraic geometry and has close connections to discrete polyhedral geometry, which plays an important role in many applications of real algebraic geometry. Thus, this module serves as a valuable complement to the main module "Real Algebraic Geometry", where students are introduced to research-oriented techniques.

Learning outcomes: The students

- understand the theoretical foundations of toric varieties and recognize connections to the field of discrete and combinatorial geometry,
- are able to apply the developed concepts to various compactifications of Tori.

Toric varieties

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	— Lecture 2 SWS — Tutorials 1 SWS	3rd or 4th semester Master's degree	every 3-4 years	German or English (if desired)

Prerequisites: Advanced knowledge in Algebra in the scope of the bachelor courses “Algebra I-II” and “Algorithmic algebraic geometry” in Konstanz

Teaching contents: Affine toric varieties, projective and general toric varieties, fans, orbit-cone correspondence, toric morphisms. Additional topics such as Weil and Cartier divisors on toric varieties, quotient constructions, toric singularities and their resolution. Depending on the participants' prior knowledge, supplementary topics may include algebraic geometry, fundamentals of algebraic tori, basics of convex cones or polytopes and their facets.

Form of examination: Written or oral exam

Work load: 135 h

- Attendance in the lectures
- Attendance in the tutorials
- Exercise tasks
- Exam preparation

Specialization module: Representation theory and invariant theory of finite group

ECTS	Duration	SWS (Weekly teaching hours)	Placement
9	2 semesters	6	Specialization module in the specialization track "Real Geometry and Algebraör elective module

Module units:

- Representation theory of finite groups
- Invariant theory of finite groups

Module grade: Written or oral exam

Learning objectives:

- The students learn the basics of ordinary representation theory and invariant theory of finite groups, as well as some of their most important applications. They should acquire the ability to recognize symmetries in mathematical problems and learn the necessary theoretical tools to exploit these symmetries. The lecture is independent of the module "Real Algebraic Geometry", but provides knowledge that can potentially be adapted specifically to problems with a lot of symmetry.

Learning outcomes: The students

- are familiar with abstract algebraic structures such as groups, which can be used to describe symmetries,
- understand how a group can act linearly on a vector space and are able to articulate what these actions reveal about the group,
- can apply theoretical insights about symmetries to describe symmetric objects,
- analyze processes that transform an object in itself,
- are able to categorize objects based on their invariants,
- are able to mathematically assess the extent to which symmetries can be helpful in a given problem.

Module unit: Representation theory of finite groups

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	– Lecture 2 SWS – Tutorial 1 SWS	Third semester Master's degree	irregularly	German or English (if desired)

Prerequisites: Advanced knowledge in Algebra, e.g. as treated in the bachelor course "Algebra I" in Konstanz

Teaching contents: Linear representations of groups, Wedderburn and Maschke theorems, invariant subspaces and complete reducibility, Schur orthogonality, decomposition of the group algebra, Fourier transformation of a finite group, decomposition of a representation, character table, central characters, computation of the character table from the structure constants

Form of examination: Written or oral exam

Work load: 150 h
- Attendance in the lectures
- Attendance in the tutorials
- Exercise tasks
- Exam preparation

Module unit: Invariant theory of finite groups

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	– Lecture 2 SWS – Tutorial 1 SWS	Fourth semester Master's degree	irregularly	German or English (if desired)

Prerequisites: Advanced knowledge in Algebra, e.g. as treated in the bachelor course "Algebra I" in Konstanz

Teaching contents: Noether's degree bound, decomposition into isotypic components and modules of covariants, modules over the invariant ring, graded algebras and graded modules, regular parameter systems, Poincaré and Molien series, reciprocity for invariants of cyclic groups, semi-invariants of finite reflection groups, Cohen-Macaulay property of the invariant ring.

Form of examination: Written or oral exam

Work load: 150 h
- Attendance in the lecture
- Attendance in the tutorial
- Exercise tasks
- Exam preparation

4 Differential Geometry and Topology

Main module: Differential geometry III

ECTS	Duration	SWS (Weekly teaching hours)	Placement
9	1 Semester	6	Main module in the study field "Differential geometry"

Module grade: Written or oral exam

Learning objectives: Description of abstract manifolds and basic structures such as tangent bundle and tensor bundles, tensor calculus, abstract general concepts of curvature, coordinate invariant definitions

Learning outcomes: Students
- are able to describe abstract manifolds and master the necessary calculus to perform analysis on manifolds,
- are proficient in definitions in coordinates, invariant definitions, index notation and abstract notation for tensors on manifolds,
- can establish connections between embedded and abstract manifolds.

Differential geometry III

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	- Lecture 4 SWS - Tutorial 2 SWS	1st semester of Master's degree	irregularly (following Diff. geometry II)	German or English (if desired)

Prerequisites: Basic knowledge about embedded manifolds and elementary differential geometry as treated e.g. in the bachelor courses Differentialgeometrie I-II in Konstanz.

Teaching contents: Geodesics on submanifolds, abstract manifolds, tensor analysis, tangent bundles, vector fields, connections, curvature, geodesics on abstract manifolds.

Form of examination:
- written or oral exam
- succesful participation in the exercises

Work load: 270 h
- Attendance in the lecture and tutorial
- Preparation and follow-up of the lectures in self-study
- Solving of exercises
- Exam preparation

Main module: Classical solutions of partial differential equations

ECTS	Duration	SWS (Weekly teaching hours)	Placement
9	1 Semester	6	Main module in the study field “Analysis and Numerics”

Module grade: Written or oral exam

Learning objectives: In-depth knowledge of classical solutions of partial differential equations.

Learning outcomes: Students

- recognize partial differential equations that can be solved smoothly are smooth,
- understand conditions on data that allow a priori estimates allow,
- can apply the regularity theory they have learned to geometrically or physically motivated problems and are able to determine the maximum possible regularity,
- can modify given equations by approximation in such a way that classical solutions exist,
- are able to deduce existence results from a priori estimates.

Classical solutions of partial differential equations

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	- Lecture 4 SWS - Tutorial 2 SWS	1st semester Master’s degree or later	irregularly	German or English (if desired)

Prerequisites: Basic knowledge about the theory of partial differential equations to the extend covered in the bachelor course “Theory and numerics of partial differential equation” in Konstanz.

Teaching contents:

- General objective: Investigation of classical solutions of elliptic and parabolic partial differential equations
- Exemplary: Schauder theory, Krylov-Safonov estimates, Monge-Ampère equations, De Giorgi-Nash-Moser theorem

Form of examination: Written or oral exam

Work load: 270 h

- Attendance in the lecture and tutorial
- Preparation and follow-up of the lectures in self-study
- Solving of exercises
- Exam preparation

Specialization module: Geometric analysis

ECTS	Duration	SWS (Weekly teaching hours)	Placement
9	1 Semester	6	Specialization module in the study fields “Analysis and Numerics” or “Differential geometry”

Module grade: Written or oral exam

Learning objectives: Primary objective of the course is the investigation of curvature flows and manifolds of prescribed curvature using partial differential equations

Learning outcomes: Students
- can define geometric flow equations as well as compare intrinsic and extrinsic flows and quantities,
- are able to apply the formalism for the calculation of evolution equations of given quantities,
- can build on this to analyze whether properties such as convexity or positive sectional curvature are preserved under a flow,
- are able to explain how reaction and diffusion components of of an equation influence the geometric behavior,
- can argue which flow equations are suitable to show geometric statements.

Geometric analysis

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	- Lecture 4 SWS - Tutorial 2 SWS	2nd semester of Master’s degree or later	irregularly	German or English (if desired)

Prerequisites: Background knowledge in differential geometry and partial differential equations

Teaching contents:
- General objective: mathematical analytical description of manifolds
- Specific objectives: Investigation of partial differential equations which describe manifolds whose curvature is prescribed or whose evoluation in time is curvature-dependent
- Exemplary: Evolution equations such as (graphical) mean curvature flow, Gaussian curvature flow, Ricci flow, minimal surfancces, hypersurfaces of prescribed mean curvature, Monge-Ampère equations

Form of examination:
- typically oral examination

- succesful participation in the exercises classes

Work load: 270 h

- Attendance in the lecture and tutorial
- Preparation and follow-up of the lectures
- Exercise tasks
- Exam preparation

5 Stochastics

Main module: Mathematical statistics II

ECTS	Duration	SWS (Weekly teaching hours)	Placement
9	1 semester	6	Main or specialisation module in the field of "Stochastics"

Module grade: Written or oral exam

Learning objectives: Students are able to analyze the asymptotic behavior of statistically relevant stochastic processes. Students are able to assess the quality of statistical methods and independently analyze questions with regard to the development of optimal procedures and answer them using mathematical statistics.

Learning outcomes: The students

- can establish an appropriate framework to investigate the convergence of distributions of stochastic processes,
- can apply suitable criteria to demonstrate the convergence of distributions of stochastic processes,
- are familiar with the asymptotic behaviour of some important empirical processes,
- are able to compare different criterial for evaluating a decision rule,
- can categorize, evaluate, and contrast the quality of statistical procedures,
- are capable of independently investigating questions related to the development of optimal procedures and answering them using the methods of mathematical statistics.

Hauptmodul: Mathematical Statistics II

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	- Lecture 4 SWS - Tutorial 2 SWS	1st or 3rd semester of the Master's degree	Winter semester (annually)	German or English (if desired)

Prerequisites: Mathematical statistics I

Teaching contents: Convergence of probability measures, transformations, weak convergence in C, the space D, metrization of D, weak convergence in D, criteria for tightness, empirical processes, Donsker's theorem, Brownian bridge, decision problems, consistency, unbiasedness, minimax and Bayes principle, admissibility, sufficiency, minimal sufficiency, completeness, factorization theorem, exponential families, MVUE estimator, Rao-Blackwell theorem, Fisher information, Cramer-Rao bound, efficiency, asymptotic efficiency, super-efficiency, Stein estimator, Jackknife, MLE, MDE and M-estimators and their asymptotic distribution, robustness distribution, robustness

Form of examination:

- Written or oral exam

- successful completion of the exercise tasks

Work load: 270 h

- Attendance in the lectures and the tutorials
- Self-study
- Preparation and follow-up of the lectures
- Exercises
- Exam preparation

Main module: Stochastic analysis

ECTS	Duration	SWS (Weekly teaching hours)	Placement
9	1 Semester	6	Main module in the study field "Stochastics"

Module units:

- Stochastic analysis
- Selected topics of stochastic analysis

The two module units are taught in the first and second half of the lecture period and build on each other.

Module grade: Typically a written or examinations for each the two module units.

Learning objectives: The lecture gives an introduction to stochastic analysis. The students learn how new stochastic processes can be obtained by stochastic integration and stochastic differential equations. They also recognize the importance of this construction for the modeling of real phenomena, especially in financial mathematics.

Learning outcomes: The students

- can construct stochastic integrals and are able to transfer the concepts they have learned to modeling using stochastic differential equations,
- master the mathematical foundations for the analysis of of stochastic financial market models

Module unit: Stochastic analysis

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- Lecture 2 SWS - Tutorial 1 SWS (taught compactly during the first half the semester)	1st semester of the Master's degree or later	Winter semester (annualy)	German or English (if desired)

Prerequisites: Basic knowledge about stochastic processes as treated in the bachelor course Stochastic processes in Konstanz.

Teaching contents:

- Stochastic integration theory: Itô formula, quadratic variation variation
- Stochastic differential equations
- Maś alternation and Girsanov's theorem
- Martingale representation theorem

Form of examination:

- written or oral exam
- succesful participation in the exercises

Work load: 135 h
- Attendance in the lecture and tutorial
- Preparation and follow-up of the lectures
- Exercise tasks
- Exam preparation

Module unit: Selected topics of stochastic analysis

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- Lecture 2 SWS - Tutorial 1 SWS (taught compactly during the first half the semester)	1st semester of the Master's degree or later	Winter semester (annualy)	German or English (if desired)

Prerequisites: Module unit Stochastis analysis

Teaching contents: Possible teaching contents are
- Pathwise integration theory
- Fractional Brownian motion
- Solution concepts for stochastic differential equations
- Long-time behavior of stochastic differential equations
- Stochastic backward differential equations

Form of examination:
- written or oral exam
- succesful participation in the exercises

Work load: 135 h
- Attendance in the lecture and tutorial
- Preparation and follow-up of the lectures
- Exercise tasks
- Exam preparation

Specialization Module: Bayesian statistics

ECTS	Duration	SWS (Weekly teaching hours)	Placement
5	1 semester	4	Specialization module in the field of SStochastics"

Module grade: Written exam

Learning objectives: The students can describe Bayes models and identify situations that naturally lead to Bayes models. They can engage with questions regarding the choice of prior distributions and are familiar with important inference techniques. They are acquainted with robustness concepts and, within a specific model framework, the asymptotic behaviour of posterior estimates. They are also familiar with computational approaches for inference in Bayes models.

Learning outcomes: The students

- can determine Bayes risks and Bayes rules,
- can specify some important properties of exchangeable families,
- are able to provide motivations for choosing certain prior distributions,
- can specify inference techniques in Bayesian models,
- can describe local and global sensitivity measures for posterior distributions,
- are familiar with connections between frequentist and Bayesian models in terms of asymptotic behavior and can establish these connections,
- can apply computational or simulation methods in Bayesian inference.

Bayesian statistics

ECTS	Teaching methods	Recommended semester	Frequency	Language
5	– Lecture 2 SWS – Tutorial 2 SWS	2nd semester Master's degree	Irregularly	German

Prerequisites: Mathematical statistics II

Teaching contents: Bayes models, Bayes risks and rules, decision theory, exchangeable processes, de Finetti's theorem, conjugate classes, non-informative prior distributions, Bayesian point and interval estimation, Bayesian hypothesis testing, consistency, Bernstein-von-Mises theorem, Markov Chain Monte Carlo methods

Form of examination: Written exam

Work load: 150 h

- Attendance in lectures and exercises

- Self-study
- Preparation and follow-up of lectures
- Exercise tasks
- Exam preparation

Specialisation module: Financial Mathematics

ECTS	Duration	SWS (Weekly teaching hours)	Placement
9	1 semester	6	Specialisation module in the study field SStochastics"

Module grade: Written or oral exam

Learning objectives: This lecture presents the basic models of a financial market. The mathematical description of these models is based on basic methods of stochastic analysis. Classical results such as the Black-Scholes model are derived in a contemporary, mathematically exact form. New results on arbitrage theory, portfolio management, hedging in incomplete markets, interest rate models and risk measures are presented.

Learning outcomes: Students

- can apply stochastic processes for modelling financial markets and apply stochastic processes to model financial markets and understand arbitrage or incomplete markets in the context of mathematical modelling,
- are able to apply central concepts such as Numéraire or martingale measures to portfolio optimisation,
- are able to evaluate under which conditions different approaches are suitable for dealing with portfolio management or hedging problems.

Financial Mathematics

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	- Lecture 4 SWS - Tutorial 2 SWS	2nd/4th semester Master's degree	Summer semester (annual)	German or English (if desired)

Prerequisites: Main module: Stochastic analysis

Teaching contents: General market model, self-financing strategies, Numéraire, Martingale measures, arbitrage, complete markets, portfolio optimisation, interest rate models, risk measures

Form of examination:

- Written or oral exam
- successful participation in the exercises

Work load: 270 h

- Attendance in lectures and tutorials
- Preparation and follow-up of the lecture in self-study
- Solving exercises
- Exam preparation

Specialization module: Time series analysis

ECTS	Duration	SWS (Weekly teaching hours)	Placement
9	1 semester	6	Main or specialization module in the field SStochastics"

Module grade: Written or oral exam

Learning objectives: A systematic introduction to time series analysis is given, with emphasis on understanding mathematical foundations and their implications for data analysis. The spectral representation of stationary processes leads to an elegant theory in the Hilbert space of square integrable variables. Parametric and nonparametric statistical inference and forecasting are discussed in the time and frequency domain. The practical application of the methods is illustrated by data examples.

Learning outcomes: Students

- know the fundamental probabilistic principles of time series analysis,
- know stationarity concepts, approaches to seasonal and trend adjustment, and the basic theory of stochastic integration,
- are able to apply spectral theory and analyze time series with the help of spectral distribution and spectral representation to analyze the properties time series,
- are able to combine the acquired concepts when estimating in the spectral domain,
- are able to evaluate different estimators for trend terms and identify stationary time series,
- know the theory of basic linear models, including long-memory variants (ARIMA, FARIMA),
- know selected non-linear models (SV).

Time series analysis

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	– Lecture 4 SWS – Tutorial 2 SWS	2nd or 4th semester Master's degree	Summer semester (annually)	German or English (if desired)

Prerequisites: Advanced module in Stochastics

Teaching contents:

- fundamental probabilistic principles of time series analysis
- weak and strong stationarity
- ergodicity
- Birkhoff's ergodic theorem

- Wold decomposition
- autocovariance function, spectral distribution and spectral density
- spectral representation of univariate and multivariate weakly stationary processes, stochastic integration
- ARMA processes, FARIMA processes
- generalized Autoregressive Processes
- prediction
- parametric and non-parametric estimation methods, asymptotics
- model identification

Form of examination: Final exam, exercise performance may contribute to the final grade

Work load: 270 h
- On-site studies in lectures and tutorials
- Self-study
- Preparation and follow-up of lectures
- Exercise tasks
- Exam preparation

Elective module: Linear models

ECTS	Duration	SWS (Weekly teaching hours)	Placement
6	1 Semester	4	Elective module in the study field "Stochastics"

Module grade: Written exam

Learning objectives: Linear and generalized linear models are examined the respective model assumptions are interpreted. Further methods for estimating and testing model parameters are introduced and the asymptotic behavior of estimators is examined.

Learning outcomes: Students

- know classical normal regression models,
- can perform MLE estimations in normal regression models and know the distribution theory of the resulting estimators,
- can formulate and test linear hypotheses in the normal regression model,
- know generalized linear models (GLMs) and an approach for MLE estimates in these,
- know the problem of overdispersion and possible remedies,
- know the asymptotic behavior of MLEs and asymptotic tests.

Linear models

ECTS	Teaching methods	Recommended semester	Frequency	Language
6	- Lecture 2 SWS - Tutorial 2 SWS	3rd semester of Master's degree	Irregularly	German or English (of desired)

Prerequisites: Main modules Stochastic analysis and Mathematical statistics II

Teaching contents: Linear normal regression models, MLEs and their distribution theory, asymptotics in the normal regression model, theorem of Gauss-Markov theorem, sum-of-squares decompositions, MLEs under linear restrictions, tests for linear hypotheses, generalized linear models (GLMs), MLE in GLMs, Fisher scoring method, asymptotic tests of linear hypotheses in GLMs, model selection, deviance, overdispersion

Form of examination: Written exam

Work load: 150 h

- Attendance in lectures and tutorials
- Preparation and follow-up of the lecture in self-study
- Solving exercises
- Exam preparation

Elective module: Multivariate statistics

ECTS	Duration	SWS (Weekly teaching hours)	Placement
6	1 Semester	4	Elective module in the study field “Stochastics”

Module grade: Written exam

Learning objectives: If more than one characteristic is observed in a random experiment observed, the experimenter is already operating in a "multivariate world". The participants learn and apply essential techniques of (normally distributed) multivariate statistics. They can derive estimators and tests for the most important variables and investigate linear relationships using regression analysis.

Learning outcomes: Students

- know the most important properties of the multivariate normal normal distribution and a multivariate form of the central limit theorem limit theorem,
- understand the approach of the maximum likelihood concept and can apply it to the derivation of estimation and selected test statistics,
- can use it to investigate basic variance analysis questions and are able to apply the developed estimation and test concepts to multivariate regression analysis,
- can transfer the techniques learned to other questions, such as the estimation of partial covariances.

Multivariate statistics

ECTS	Teaching methods	Recommended semester	Frequency	Language
6	- Lecture 2 SWS - Tutorial 2 SWS	2nd/4th semester Master’s degree	Summer semester (annualy)	German or English (if desired)

Prerequisites: Main module Stochastic analysis

Teaching contents: The course introduces the statistics of the multivariate (p-dimensional) normal distribution. After studying some essential properties of this distribution, methods for the estimation of the most important functions of the parameters of the distribution are discussed. These basic principles are then used in applications such as variance analysis, multivariate regression analysis or principal component analysis.

Form of examination: Written exam

Work load: 180 h

- Attendance in lectures and tutorials
- Preparation and follow-up of the lecture in self-study

- Solving exercises
- Exam preparation

Elective module: Acturial mathematics

ECTS	Duration	SWS (Weekly teaching hours)	Placement
6	1 Semester	4	Elective module in the study field "Stochastics"

Module grade: Written exam

Learning objectives: Actuarial mathematics touches on an elementary question in dealing with random events: Can one "exchange" random losses (insured losses) for more or less deterministic payments (premiums)? If so, how? The participants learn basic questions and techniques of life and non-life insurance mathematics. They will be able to calculate essential variables such as net premiums or calculate or estimate (approximate) ruin probabilities for given data.

Learning outcomes: Students

- know the most important rules for the valuation of deterministic cash flows and can apply these to the premium and benefit payments of typical life insurance policies,
- can use the probabilities given by mortality tables to combine the learned valuation methods for the determination of net premiums and net actuarial reserves,
- know and understand the collective model with renewal process as a fundamental model of property insurance mathematics,
- know the most important asymptotic properties of the overall loss process and simple methods for determining the overall loss distribution,
- know the most important statements of ruin theory, which revolve around grouped around the Lundberg inequality, and can apply these to the to the evaluation of various loss models with associated premium models,
- understand the key aspects of the major loss problem,
- are familiar with Bayes estimators and linear Bayes estimators for expected values and are able to assess the applicability of the linear estimator to certain heterogeneity models.

Acturial mathematics

ECTS	Teaching methods	Recommended semester	Frequency	Language
6	- Lecture 2 SWS - Tutorial 2 SWS	3rd Semester of Master's degree	Winter semester (annual)	German or English (if desired)

Prerequisites: Main module Stochastic analysis

Teaching contents: The lecture provides an introduction to the fields of life and non-life insurance mathematics. In the part on life insurance mathematics, first the basics of financial mathematics are treated, afterwards net premiums for various endowment and annuity insurance policies are discussed on the basis of annuity insurance policies and the actuarial reserve is determined. In the part on non-life insurance mathematics, models and methods for describing the overall loss distribution are introduced and some aspects of the total loss distribution are discussed. Furthermore the probability of ruin of a portfolio is examined and premium principles are discussed. In addition, the experience rating is also addressed.

Form of examination: Written exam

Work load: 150 h

- Attendance in lectures and tutorials
- Preparation and follow-up of the lecture in self-study
- Solving exercises
- Exam preparation

6 General modules

Specialized seminar

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 Semester	2	Pflicht Master Mathematik

Module grade: typically without grade, or grade given for presentation

Learning objectives: Independent acquiry of knowledge about an advanced specialized scientific topic, for example through literature research (typically in English), and presenting the topic in front of an audience.

Learning outcomes:
- mastery of basic work organization techniques
- ability to speak freely and present vividly
- ability to formulate appropriate technical questions
- confidence in dealing with technical questions
- willingness and ability to constructively criticize a presentation

Specialized seminar

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	2 SWS	2nd or 3rd semester of Master's degree	at least annualy	German or English (if desired)

Prerequisites: At least one main module and in-depth knowledge related to the respective topic.

Teaching contents:
- Students are given a challenging speciazlized topic or an advanced project assignment for independent work according to recommended literature.
- A presentation lasting 45-75 minutes is prepared for each topic and presented in plenary.
- The content of the presentation and the presentation itself will be will be discussed in plenary.
- A handout or paper may be prepared for each presentation using a scientific text typesetting system (usually LaTeX) and distributed in plenary.

Form of examination: Oral presentation, active participation, possibly written elaboration

Work load: 135 h
- Attendance in the seminar dates
- Independent work on the given topic
- Preparation of a talk on the given topic
- Possibly preperation of a handout / paper

Master thesis

ECTS	Duration	SWS (Weekly teaching hours)	Placement
30	6 Monate	individual supervision	Master thesis (obligatory)

Module grade: Reviews of the thesis by two professors or lecturers, at least one of whom must be from the department

Learning objectives: Research-oriented mathematical work

Learning outcomes: The Master's thesis should demonstrate that students are are able to work on a research-oriented mathematical topic and present the results in a comprehensible form.

Master thesis

ECTS	Teaching methods	Recommended semester	Frequency	Language
30	individual supervision	4th semester of Master's degree	annualy	German or English

Prerequisites: Main and specialization modules from within the master program in one ore more study fields.

Teaching contents: Building on knowledge from one or more modules of the Master's degree program, a research-oriented topic is agreed between the student and the supervisor. A suitable selection of the scientific methods to be used in scientific methods to be used will be agreed jointly.

Form of examination:

- Master thesis
- Presentation of the master thesis

Work load: 900 h self study