Are outcomes driving expectations or the other way around? An I(1) CVAR analysis of interest rate expectations in the dollar/pound market

Katarina Juselius and Josh Stillwagon

University of Copenhagen, Trinity College, Hartford

Abstract

This paper uses consensus forecasts to address puzzles in international macro. The data analyzed consists of 3 months libor rates, their forecasts 3 months ahead, prices and exchange rates for the US and UK. All variables were found to exhibit pronounced persistent movements away from long-run benchmark values consistent with imperfect knowledge expectations. Results from an I(2) CVAR showed that over the medium run the nominal exchange rate has been pushing the foreign currency market away from steady state while interest rates have followed suit. Interestingly, over the long run, the nominal exchange rate has been equilibrium error correcting, whereas interest rate forecasts have primarily pushed the system away from steady state. Such evidence of self-reinforcing feed-back mechanisms among the variables in the system signals the importance of speculative bubbles.

1 Introduction

The use of expectations in economic theory is far from new. Many earlier economists, including A. C. Pigou, John Maynard Keynes, and John R. Hicks, assigned a central role to people's expectations for the determination of the business cycle. When financial actors rush to desert a currency that they expect to lose value, they contribute to its loss in value. Also, the price of a stock or bond depends partly on what prospective buyers and sellers believe it will be in the future. Keynes referred to this as "waves of optimism and pessimism" that helped determine the level of economic activity. John F. Muth was the first to formulate the theory of rational expectations as a way to model expectations in economic theory.

Rational expectations (RE) are based on the standard economic assumption that people behave in ways that maximize their utility or profits and assert that outcomes should not differ systematically from what people expected them to be. RE based theory accepts that people often make forecasting errors, but assumes that errors will not persistently occur on one side or the other. Model consistent rational expectations, often used as a way to obtain internal consistency in a theory model, is based on the assumption that the economists' model is a "true" description of the economy, that the model holds over infinite time and, hence, can be used to calculate "rational" forecasts.

Recently, as a result of the failure of RE-based models to foresee the financial and economic crisis, several alternatives have been suggested. Common to these models is that today's asset price depends on forecasted prices which, in varying degree, are derived under imperfect knowledge. For example, Hommes (2005) and Hommes et al. (2005a, 2005b) developed models for a financial market populated by fundamentalists and chartists where fundamentalists use expectations based on economic fundamentals and chartists are trend-followers using naive expectations. Positive feedback prevails when the latter dominate the market. Adam and Marcet (2011) proposed a separation of standard RE rationality into an internal and an external component. They showed that positive feedback can arise in a model where internal rationality is maintained but external rationality is relaxed due to imperfect market knowledge. Heemeijer et al. (2009), based on experiments, found that prices converge to their fundamental level under negative feedback but fail to do so under positive feedback.

Frydman and Goldberg (2007, 2011) developed a theoretical framework where agents' expectations are formed in the context of imperfect knowledge about the underlying causal mechanisms, which are assumed to be subject to structural change. They argue that expectations based on imperfect knowledge and fundamental uncertainty about future outcomes, including unpredictable structural changes, have strong consequences for individuals' optimal decision making. For example, when individuals are faced with imperfect knowledge about economic mechanisms it is optimal, and therefore rational, to use many different models to predict future outcomes. When actual outcomes tend to deviate from expectations it is also optimal to revise one's forecasting model. Also, it can be beneficial, and therefore optimal, to include psychological factors, such as the market's tendency for herd behavior, in one's expectations. Thus, under imperfect knowledge, expectations are likely to play an autonomous role in the model rather than endogenously adjust to the optimal path given by the theoretical model. In this sense, the long swings we see in asset price data may very well be due to expectations that have driven prices persistently away from (and towards) historical benchmark levels.

Therefore, the purpose of the present paper is to address the role of expectations in the long swings we see in the data by using survey expectations. More specifically, we study the role of expectations of 3 months libor rates in the dollar/pound currency market using time series on professional forecasters' consensus forecasts as a measure of the market's expectations.[1] This means we can focus on how the actual forecasts influence price setting in the foreign currency market without having to make (untestable) assumptions about how agents form expectations.

To allow the data to speak as freely as possible about the empirical relevance of competing hypotheses we base our empirical analyses on the "general-to-specific" Cointegrated VAR (CVAR) approach, rather than the "specific-to-general" approach.[2] We use this methodology to address the following questions:

1. Are expectations primarily adjusting to the interest rates and the exchange rate or is it the other way around?

2. Are the persistency properties of the data more consistent with rational expectations or imperfect knowledge expectations?

3. Are there evidence of self-reinforcing feed-back mechanisms in the system consistent with speculative bubbles?

[1] Other studies using interest rate forecast data in the FX literature include: Gourinchas and Tornell (2004) who focus on underreaction to interest rate shocks in the forward discount anomaly, and Dick, MacDonald, and Menkhoff (2015) who examine the correlation between interest rate forecasts and exchange rate forecasts.

[2] In the latter case, many untested theoretical restrictions are imposed on data from the outset making it difficult to know which findings represent true empirical facts and which reflect these a priori assumptions. In the former case no (or as few as possible) a priori restrictions are imposed without first having been tested and not rejected.

4. Are interest rate expectations causing the long persistent swings characterizing foreign currency markets?

5. How are interest rates, prices and the exchange rate affected by the consensus forecasts in the short run?

6. Have the professional forecasters been able to foresee major changes in the market?

2 Basic regularities in the foreign currency data

As discussed in Juselius (2015) and Juselius and Assenmacher (2015), rational expectations based models would in general be consistent with the real exchange rate

$$q_t = s_{12,t} - p_{1,t} + p_{2,t} \tag{1}$$

holding as a stationary or near $I(1)$ process and with the uncovered interest rate parity corrected for a stationary or near $I(1)$ risk premium holding as a market clearing mechanism

$$i_{1,t} - i_{2,t} = \Delta s^e_{12,t} + rp_t \tag{2}$$

where rp_t is a stationary or a near $I(1)$ process measuring a risk premium. The latter is typically associated with exchange rate volatility.

In contrast, (Frydman and Goldberg, 2007, 2011) introduce an uncertainty premium of order $I(1)$ or near $I(2)$ as an explanation for why unregulated financial markets have shown a tendency to drive nominal exchange rates away from long-run purchasing power parity values. This implies that the real exchange rate is no longer stationary or near $I(1)$ but, instead, a random walk with a time-varying drift term (see Frydman and Goldberg, 2007, Juselius, 2012)):

$$\Delta q_t = \omega_t + \varepsilon^q_t \tag{3}$$

where ε^q_t is stationary and the drift term, ω_t, is assumed to follow an autoregressive process:

$$\omega_t = \rho_t \omega_{t-1} + \varepsilon^\omega_t.$$

and where ε_t^{ω} is a white noise process. The parameter ρ_t may vary over different periods but its average value $\bar{\rho}$ is generally close to 1.0 provided the sample period is sufficiently long. Thus, while the differenced real exchange rate behaves like white noise in the REH-based model, it behaves like a very persistent near $I(1)$ process in the IKE-based model. In the latter case the real exchange rate will exhibit long persistent swings typical of a near $I(2)$ process. The length of the swings are not predictable and the real exchange rate will entail swings of shorter and longer duration[3].

The uncovered interest rate parity, corrected additionally for an uncertainty premium, up_t, is then assumed to describe a market clearing mechanism:

$$i_{1,t} - i_{2,t} = \Delta s^e_{12,t} + rp_t + up_t$$

where the up_t is assumed to be proportional to the gap between the nominal exchange rate and its long-run fundamental value. In the foreign currency market the gap effect is likely to be primarily related to the deviation from the long-run PPP value, but as shown in Juselius and Assenmacher (2016a), the gap effect can be measured by several gaps. The uncertainty adjusted UIP condition is specified as

$$i_{1,t} - i_{2,t} = \Delta s^e_{12,t} + rp_t + \gamma(s_{12,t} - p_{1,t} + p_{2,t}). \tag{4}$$

Previous studies of similar hypotheses based on other countries data have found that it is the twice cumulated interest rate shocks that are the exogenous drivers (Juselius, 2006, Johansen et al., 2011, Juselius, 2015, Juselius and Assenmacher, 2015b). It is, therefore, of particular interest to study survey expectations of interest rates to find out whether expectations adjust to actual interest rates or the other way around.

3 A graphical analysis

The upper panel of Figure 1 shows that US prices have increased relatively more than the UK ones up to the collapse of the Lehman Brothers in 2008:9, after which a reverse development took place. The real exchange rate in

[3] This means that there can be sample periods when the evidence of I(2) is quite weak. See Juselius (2012, 2014) for an illustration of the difference between the RE- and IKE-based assumptions.

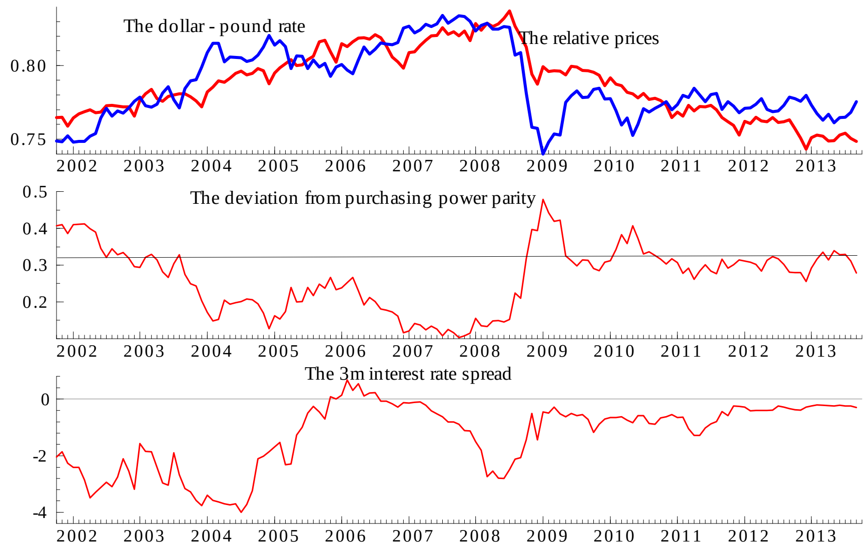

Figure 1: The graphs of the relative US-UK price together with the nominal exchange rate (upper panel), the deviation from the long-run purchasing power parity level (middel panel) and the US-UK interest rate differential (lower panel).

the middle panel shows how the dollar/pound rate has fluctuated around its fundamental value, the US-UK relative price, in long persistent swings, except for a period after the Lehman Brothers collapse. As can be seen from the upper panel, the nominal exchange rate fell rapidly and substantially more than the US-UK relative price when the crisis peaked. The lower panel shows that the interest rate differential has exhibited similar long and persistent swings as the real exchange rate. The question we address in this paper is whether interest rate expectations may have played an important role in these persistent swings.

Figure 2 show the forecast errors, $i_{t+3} - i^e_{t|t+3}$, where $i^e_{t|t+3}$ is the consensus forecast (red line) compared to a simple random walk for which $i^e_{t|t+3} = i_t$ (blue line). In the case of US (upper panel) the consensus forecast seems generally to coincide with the random walk forecast except for in the period from mid 2004 to 2007 when they were more precise. Nonetheless, many consensus forecasts seem . In the case of UK (lower panel) the consensus forecasts seem to have done worse than the random walk. But, except for

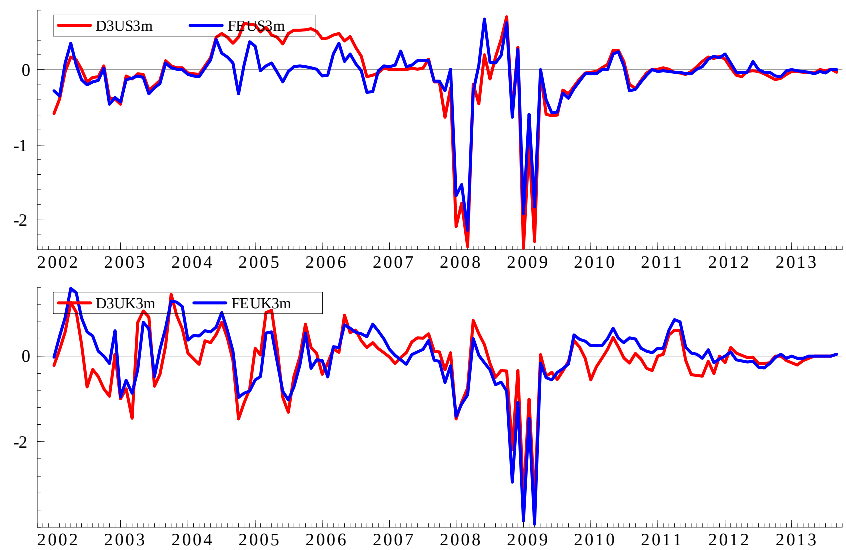

Figure 2: The graphs of the forecast errors for the US/UK 3 months treasury bill rates using consensus forecasts and using a random walk.

large forecast failures of the US rate at the beginning of 2008 and several failures in connection with the Lehman Brothers bankruptcy in both countries, the forecast errors are quite moderate. The mean squared prediction error relative to the random walk variance was 0.63 for US and 1.22 for UK supporting the visual image.

According to standard theory, model-based rational expectations can deviate from outcomes but, since agents know the correct model, expectations will adjust back towards the "true" model values. Thus, a positive forecast error should be followed by a negative one. In such a world, expectations are purely adjusting and expectational errors would have no permanent impact on the system. Imperfect knowledge based models, on the other hand, do not make such strong assumptions about how individuals adjust back to the "true" model. This is partly because in an imperfect knowledge world views on what a good model is differ and may change over time so that model parameters cannot be assumed constant. One consequence of this is that expectations can play an autonomous role in the model allowing expectational shocks to have a permanent effect on the system, and even in some cases to be exogenous to the system.

In the subsequent sections we shall address these ideas using formal econometric analysis.

4 Empirical model specification

As discussed above there are basically two competing hypotheses for how to explain the pronounced persistence in exchange rate data, first the RE theory claiming that the real exchange rate is a stationary process or at most a near $I(1)$ process, second the imperfect knowledge theory claiming that the *change* of the real exchange rate is a highly persistent near $I(1)$ process, i.e. the *level* of the real exchange is a near $I(2)$ process. These hypotheses will be formulated and tested in the Cointegrated VAR model.

Without loss of generality, the empirical VAR is formulated in acceleration rates, changes and levels (see Juselius 2006) as follows:

$$\Delta^2 x_t = \Gamma_1 \Delta^2 x_{t-1} + \Gamma \Delta x_{t-1} + \Pi x_{t-1} + \Phi_1 Ds_{08.09_t} + \Phi_2 t + \Phi_3 D_t + \Phi_4 S_t + \varepsilon_t. \quad (5)$$

where $x_t = [i^e_{1,t}, i^e_{2,t}, i_{1,t}, i_{2,t}, s_{12,t}, p_{1,t}, p_{2,t}]$ and i, i^e_t stands for a three months treasury bill rate with a superscript e denoting an expectation, $s_{12,t}$ stands for the log of the nominal US dollar - UK pound rate, p_t for the log of the CPI price and a subscript 1 for the US and a subscript 2 for the UK. A vector of dummy variables, D_t is included to control for extraordinary large shocks which cannot be explained by the chosen information set. It contains one transitory dummy defined as $Dt_{08.09_t}$ which is 1 in 2008.09 and -1 in 2008.10, eight impulse dummies $D_{XX.yy_t}$ which are 1 in 20XX.yy, 0 otherwise. The step dummy $Ds_{08.09_t}$ which is one for 2008:09-2013:7, 0 otherwise controls for the great recession after the Lehman Brothers collapse. The impulse dummies are effective in 2002.11, 2003.01, 20003.07, 20004.07, 2005.09, 2008.01, 2008.04, 20008.11, 2008.12 and 2009.1. The model also includes 11 seasonal dummies, S_t. Finally, ε_t is assumed $N_{iid}(0, \Omega)$ and the sample covers the period 2001:3 to 2013:7.

The hypothesis that x_t is $I(1)$ is formulated as a reduced rank hypothesis on Π:

$$\Pi = \alpha\beta' \quad (6)$$

where α is a $p \times r$ matrix of adjustment coefficients, β is a $(p+2) \times r$ matrix describing long-run relationships among the variables. The hypothesis that

x_t is $I(2)$ is formulated as an additional reduced rank hypothesis

$$\alpha'_{\perp}\Gamma\beta_{\perp} = \xi\eta', \tag{7}$$

where ξ, η are $(p-r) \times d_1$ and $\alpha_{\perp}, \beta_{\perp}$ are the orthogonal complements of α, β. Under reduced rank of Π, $\Phi_1 = \alpha\gamma_1$ and $\Phi_2 = \alpha\gamma_2$ implying that the trend, t, and the step dummy, $Ds_{08.09_t}$, are restricted to the cointegration relations.

Because the second rank condition is formulated as a reduced rank restriction on the transformed Γ matrix, its coefficients in (5) are no longer unrestricted as in the $I(1)$ model. This is why Johansen (1997) suggested a different parameterization more suitable for maximum likelihood estimation and testing of structural hypotheses:

$$\Delta^2 x_t = \alpha(\beta'\tilde{x}_{t-1} + d'\Delta\tilde{x}_{t-1}) + \zeta\tau'\Delta\tilde{x}_{t-1} + \Phi_3 D_t + \Phi_4 S_t + \varepsilon_t, \tag{8}$$

where $\tilde{x}'_t = [x_t, t, Ds_{08.09_t}]$ and $d' = -((\alpha'\Omega^{-1}\alpha)^{-1}\alpha'\Omega^{-1}\Gamma)\tau_{\perp}(\tau'_{\perp}\tau_{\perp})^{-1}\tau'_{\perp}$ is a $(p+2) \times r$ matrix of coefficients determined so that $(\beta'\tilde{x}_{t-1} + d'\Delta\tilde{x}_{t-1}) \sim I(0)$, $\Gamma = -[\alpha d' + \zeta\tau']$, $\tau = [\beta, \beta_{\perp 1}]$ is a $(p+2) \times (r + c_1)$ matrix describing stationary relationships among the differenced variables with $\beta_{\perp 1}$ being the orthogonal complement of $[\beta, \beta_{\perp 2}]$, $\tau_{\perp}$ is the orthogonal complement of τ, ζ is a $p \times (p - s_2)$ matrix of medium run adjustment coefficients, p is the dimension of the data vector, r is the number of multicointegration relations, s_1 is the number of cointegration relations that can only become stationary by differencing, s_2 is the number of $I(2)$ trends, and $p = r + s_1 + s_2$.

The VAR model is based on the assumption that ε_t is $Niid(0, \Omega)$. This works reasonably well for the two prices and the nominal exchange rate but less well for the actual and forecasted interest rates. This is of course because nominal interest rates (in particular of short maturities) seldom are well described by a Gaussian process. Because of the inclusion of the dummy variables, the estimated interest rates residuals are reasonably symmetrical but nonetheless fat-tailed and normality was rejected mostly due to excess kurtosis rather than skewness. Even though the former is less problematic for the estimation (see Gonzalo, 1994), the VAR model can only be considered a rough approximation of the true data generating process.

5 Rank determination

The correct determination of the number of stationary cointegration relations, r, the number of $I(1)$ trends, s_1, and $I(2)$ trends, s_2, is crucial for

Table 1: Determination of the rank indices

Trace test statistics for the I(2) model							
$p-r$	r		$s_2=4$	$s_2=3$	$s_2=2$	$s_2=1$	$s_2=0$
4	3		205.5 [0.00]	155.7 [0.00]	97.5 [0.03]	79.1 [0.03]	70.1 [0.08]
3	4			111.4 [0.00]	**58.2** [0.31]	38.5 [0.57]	35.0 [0.55]
2	5				28.2 [0.87]	15.6 [0.94]	12.8 [0.92]
1	6					11.5 [0.51]	1.8 [0.99]
The five largest roots of the characteristic polynomial							
Unrestricted VAR			0.99	0.99	0.96	0.92	0.92
$r=3$	$s_1=4$	$s_2=0$	1.00	1.00	1.00	1.00	0.89
$r=4$	$s_1=3$	$s_2=0$	1.00	1.00	1.00	0.93	0.85
$r=4$	$s_1=1$	$s_2=2$	1.00	1.00	1.00	1.00	1.00

all subsequent results. Therefore, we use both the trace tests by Rahbek and Nielsen (2007) and the roots of the characteristic polynomial reported in Table 1 as a basis for the final choice. The characteristic roots reported at the lower part of the table give a rough indication of the number of (near) unit roots in the VAR model and, hence, of the nonstationary directions that need to be controlled for by the choice of s_1 and s_2. The unrestricted VAR suggests two roots almost on the unit circle and three large near unit roots. As all roots are inside the unit circle, the model is stable.

The maximum likelihood procedure in Nielsen and Rahbek (2007) is a standard procedure for determining the number of r, s_1, and s_2. The test statistics in the upper part of the table corresponds to the joint tests of the $I(1)$ and the $I(2)$ rank conditions given by (6) and (7). Because the tests of $r = 0, 1, 2$ were all strongly rejected, we have omitted the first three rows from the table. The standard test procedure starts with the most restricted model ($r = 3, s_1 = 0, s_2 = 4$) in the upper left hand corner, continues to the end of the row ($r = 3, s_1 = 4, s_2 = 0$), and proceeds similarly row-wise from left to right until the first acceptance. The first acceptable case ($r = 3, s_1 = 4, s_2 = 0$) has a fairly low p-value (0.08) and would leave a near unit root (0.89) in the model. The next case ($r = 4, s_1 = 1, s_2 = 2$) is our preferred choice. It has a p-value of 0.31 and accounts for all large roots

Table 2: Tests of no levels feedback and unit vector in alpha

r	DGF	$5\%C.V.$	$i^e_{1,t}$	$i^e_{2,t}$	$i_{1,t}$	$i_{2,t}$	$p_{1,t}$	$p_{2,t}$	$s_{12,t}$
Test of a zero row in α									
3	3	7.81	**1.13** [0.77]	17.98 [0.00]	44.86 [0.00]	32.96 [0.00]	**6.93** [0.07]	**6.26** [0.10]	**0.99** [0.80]
$\Longrightarrow$ **4**	**4**	9.49	**2.79** [0.59]	30.14 [0.00]	44.91 [0.00]	45.93 [0.00]	**9.03** [0.06]	**7.46** [0.11]	10.51 [0.03]
5	5	11.07	**4.54** [0.47]	35.74 [0.00]	45.76 [0.00]	56.75 [0.00]	**9.59** [0.09]	15.63 [0.01]	21.03 [0.00]
Test of unit vector in α									
3	4	9.49	27.85 [0.00]	**8.39** [0.08]	**5.83** [0.21]	**7.04** [0.13]	31.46 [0.00]	32.72 [0.00]	19.34 [0.00]
$\Longrightarrow$ **4**	**3**	7.81	16.44 [0.00]	8.36 [0.04]	**5.46** [0.14]	**6.66** [0.08]	19.52 [0.00]	16.25 [0.00]	**3.10** [0.38]
5	2	5.99	12.22 [0.00]	7.32 [0.03]	**3.67** [0.16]	6.64 [0.04]	8.10 [0.02]	**4.54** [0.10]	**0.15** [0.93]

in the model[4]. This choice corresponds to four polynomially cointegrating relations combining $\beta'\tilde{x}_t$ with a linear combination of the differences, $d'\Delta\tilde{x}_t$ and one relation, $\beta'_{\perp 1}\Delta\tilde{x}_t$, which can only become stationary by differencing.

6 Are expectations pulling or pushing?

A crucial question much debated among economists is whether outcomes drive expectations or the other way around, one of the most difficult questions to test based on the observed data. As a starting point we shall use simple test procedures to get a first picture of which variables, if any, have primarily pushed the system away from long-run equilibrium states and which variables, if any, have been pulling the system back towards its long-run equilibrium. In the former case we test the null of a zero row in α, which if not rejected implies that the variable in question is not reacting to a disequilibrium in the multicointegration relation $\beta'\tilde{x}_t + d'\Delta\tilde{x}_t$. However, Rahbek and Paruolo (1999) show that a test for weak exogeneity involves an additional test of a corresponding zero row in ζ in the medium-run relations, $\zeta\tau'\Delta\tilde{x}_t$. Table 2 reports the first set of tests. For the preferred choice of $r = 4$, the US interest rate forecast is a good candidate for no levels feed-back. While the US and UK prices might also have been considered such candidates, the joint

[4]The largest unrestricted root is 0.73.

tests of any combination of the three variables were rejected. As a sensitivity check, the results for $r = 3$ and 5 are also reported. The conclusion is robust to either choice, except for nominal exchange rate that would have exhibited no levels feed-back for $r = 3$.

The test of a unit vector in α implies, if not rejected, that the variable in question has been primarily adjusting to the multi-cointegrating relations. Hence, they can be given a connotation of an endogenous variable. For the preferred choice of $r = 4$ the two interest rates and the nominal exchange rate cannot be rejected as purely adjusting. This result is robust to either the choice of $r = 5$ or $r = 3$. However, when testing them jointly all possible combinations were rejected.

Based on this, the US expectational shocks seems primarily to be pushing over the long run, whereas the actual interest rates and the nominal exchange rate seems to be primarily adjusting. As the subsequent results will show, both actual and expected interest rates exhibit in addition strong feed-back effects from the medium-run relations, $\zeta\tau'\Delta\tilde{x}_t$.

7 Integration properties of the data

To derive the time-series properties of nominal interest rates assuming that the market is demanding an uncertainty premium for holding an asset in any of the two currencies, Juselius (2016) (building on Frydman and Goldberg (2007)) suggested the following data generating process:

$$\Delta i_{j,t} = \omega_{j,t} + \varepsilon_{j,t}, \text{and } \varepsilon_{j,t} \sim Niid(0, \sigma^2_{\varepsilon,j}) \qquad j = 1, 2 \tag{9}$$

where $\varepsilon_{j,t}$ stands for unanticipated interest rate shocks and $\omega_{j,t}$ measures a drift term that can be interpreted as a change in the domestic uncertainty premium. As in (3) the latter is assumed to follow an AR(1) process:

$$\omega_{j,t} = \rho_{t,j}\omega_{j,t-1} + \varepsilon^{\omega}_{j,t}, \text{ and } \varepsilon^{\omega}_{j,t} \sim (0, \sigma^2_{\varepsilon^{\omega},j}) \qquad j = 1, 2 \tag{10}$$

where $\rho_{t,j} \approx 1.0$ in periods when the PPP gap is moderately sized (i.e when the proportion of chartists is high) and $\rho_{t,j} \ll 1.0$ when the gap is large (i.e. when the proportion of fundamentalists is high). Since the periods when $\rho_{t,j} \ll 1.0$ are likely to be short compared to the ones when $\rho_{t,j} \approx 1.0$, the average $\bar{\rho}_j$ is assumed to be close to 1.0 so that $\omega_{j,t}$ can be considered a near $I(1)$ process.

Integrating (9) over t gives:

$$i_{j,t} = i_{j,0} + \sum_{s=1}^{t} \varepsilon_{j,s} + \sum_{s=1}^{t} \omega_{j,s}, \qquad j = 1, 2 \tag{11}$$

The drift component, $\omega_{j,t}$, is supposed to capture smooth momentum trading along the trend. Under the near $I(1)$ assumption, $\sum_{s=1}^{t} \omega_{j,s}$ is a near $I(2)$ process implying that nominal interest rates are near $I(2)$. Such a process would describe persistent swings of shorter and longer durations such as those pictured in Figures **??**.

In foreign exchange markets, the short term noise component, ε_t, is likely to be large relative to the drift component, ω_t, implying a small signal-to-noise ratio, i.e. $\sigma^2_{\varepsilon^\omega} \ll \sigma^2_\varepsilon$. When this ratio is small, econometric testing can have difficulties to detect the second large root associated with the drift term (10). For example, Juselius (2014) shows by simulations that the univariate Dickey-Fuller tests essentially never finds the second large root when $\bar{\rho} = 0.9$ and $\sigma_\omega/\sigma_\varepsilon = 0.15$ (a typical value for many foreign exchange markets) whereas the multivariate trace tests finds it in the majority of all cases.

Starting from (9), Juselius (2016) derives the time-series properties of the remaining variables and shows that the deviations from basic parities such as the PPP, the Fisher parities, and the terms spreads are all likely to be near $I(2)$. Thus, the parities are assumed to be one degree more persistent under imperfect knowledge than under REH, where they would generally be stationary, or at most near $I(1)$.

In the $I(1)$ model, the test of a unit vector in β corresponds to testing whether a variable is stationary, whereas in the $I(2)$ model whether the variable is $I(1)$. See Johansen et al. (2010). In the latter model, the test is formulated as a known vector k_1 in τ, i.e. $\tau = (k_1, k_{1\perp}\varphi)$ where $k_{1\perp}\varphi$ defines the remaining unrestricted vectors to lie in the orthogonal space of k_1. For example $k_1 = [0, 0, 0, 0, 0, 1, -1, 0, 0]$ is a test whether the relative price is a unit vector in τ. If not rejected, it implies that $p_1 - p_2$ can be considered $I(1)$.

The results are reported in Table 3 for all the variables in the vector process as well as relevant transformations of them. The hypotheses that the relative price, the nominal and the real exchange rate, expected and actual interest rates, expected and actual interest rate differentials can be considered $I(1)$ are all rejected, whereas the hypotheses that the differential between the US interest rate and its 3 months ahead expected value is $I(1)$

Table 3: Testing hypotheses of $I(1)$ versus $I(2)$

	i_1^e	i_2^e	i_1	i_2	s_{12}	p_1	p_2	$D_{s,08}$	t	$\chi^2(v)$	$p-val$
$\mathcal{H}_1$:	0	0	0	0	0	1	−1	0	0	19.4(4)	0.00
$\mathcal{H}_2$:	0	0	0	0	1.0	0	0	0	0	21.0(4)	0.00
$\mathcal{H}_3$:	0	0	0	0	−1	1	−1	0	0	11.3(4)	0.02
$\mathcal{H}_4$:	1	0	0	0	0	0	0	0	0	33.4(4)	0.00
$\mathcal{H}_5$:	0	1	0	0	0	0	0	0	0	35.1(4)	0.00
$\mathcal{H}_6$:	0	0	1	0	0	0	0	0	0	28.9(4)	0.00
$\mathcal{H}_7$:	0	0	0	1	0	0	0	0	0	16.4(4)	0.00
$\mathcal{H}_8$:	1	0	−1	0	0	0	0	0	0	3.6(4)	0.46
$\mathcal{H}_9$:	0	1	0	−1	0	0	0	0	0	19.3(4)	0.00
$\mathcal{H}_{10}$:	1	−1	0	0	0	0	0	0	0	16.1(4)	0.00
$\mathcal{H}_{11}$:	0	0	1	−1	0	0	0	0	0	15.6(4)	0.00

cannot be rejected with a p-value of 0.46. Altogether, the results suggest a considerable degree of persistence in the nominal exchange rate, prices and interest rates and in co-movements between them. This is broadly consistent with imperfect knowledge economics and lends support to the conclusion in Johansen et al. (2010) that the *change* of the variables is a highly persistent near $I(1)$ process. This persistence, however, is likely to be associated with the shocks to the drift term, $\omega_{j,t}$, in (9). When theses are tiny compared to the shocks to the process itself, as is usually the case with the nominal exchange rate and the interest rates, the drift term might be hard to catch sight of as it is hidden in large short-run volatility.

A few more results are worth mentioning. First, the test statistics for the two interest rate forecasts are much higher than the ones for the actual interest rates, suggesting more persistence in the consensus forecasts than in the actual interest rates. Second, the hypothesis that the differential between the UK interest rate and its expected 3 months ahead value is $I(1)$ was rejected contrary to what was found for the US rate. Based on the magnitude of the test statistics, the UK interest rate was closer to a non-rejection of I(1) than the US interest rate, whereas their forecasts were both strongly rejected as $I(1)$.

8 The pulling force

In the maximum likelihood $I(2)$ model (8) polynomial cointegration implies that $\beta'\tilde{x}_{t-1} \sim I(1)$ and $d'\Delta\tilde{x}_{t-1} \sim I(1)$ cointegrate to $I(0)$. It is notable that the basic relation in international macro given imperfect knowledge economics has exactly this feature of combining levels and differences of the variables into a dynamic long-run relationship. In this case, one can interpret the coefficients α and d as two levels of equilibrium correction: The α adjustment describes how the acceleration rates, $\Delta^2 x_t$, adjust to the dynamic equilibrium relations, $\beta' x_t + d'\Delta x_t$ and the d adjustment describes how the growth rates, Δx_t, adjust to the long-run equilibrium errors, $\beta' x_t$. Note, however, that the interpretation of d as a medium-run adjustment is conditional on $\alpha \neq 0$. The signs of β, d, and α determine whether the variable $x_{i,t}$ is error increasing or error correcting in the medium and/or the long run. If $\alpha_{ij}\beta_{ij} < 0$ or/and $\alpha_{ij}d_{ij} < 0$, then the acceleration rate, $\Delta^2 x_{i,t}$, is equilibrium correcting to $(\beta_j' x_t + d_j'\Delta x_t)$; if $d_{ij}\beta_{ij} > 0$ (given $\alpha_{ij} \neq 0$), then $\Delta x_{i,t}$, is equilibrium error correcting to $\beta_j' x_t$; if $\zeta_{ij}\beta_{ij} < 0$ then $\Delta^2 x_{i,t}$ is equilibrium correcting to $\beta_j'\Delta x_{t-1}$. In all other cases the system is equilibrium error increasing. See Juselius and Assenmacher (2016) for further details. Note, however, that the error-correcting behavior has to counteract the error-increasing behavior for the system to be stable. For this to be the case all characteristic (eigenvalue) roots have to be inside the unit circle.

Table 4 reports the estimates of four identified multi-cointegrating relations with six over-identifying restrictions accepted with a p-value of 0.94. [5]The d coefficients are uniquely identified by being proportional to $\tau_\perp$. Hence one cannot impose zero restrictions (say) on d without violating the proportionality condition. Therefore, insignificant coefficients have instead been replaced by a *. To distinguish between equilibrium error-increasing (positive feed-back) and error-correcting behavior (negative feed-back) significant α, d and ζ coefficients that are defining error-increasing behavior are indicated in italics.

The first β relation corresponds to the theoretically expected one. The second β relates the US interest rate and its forecast to relative US and UK prices. The third β relates the UK interest rate and its forecast to the UK dollar price. The fourth describes a homogeneous relationship between actual

[5]The standard errors of β are derived in Johansen (1997) and those of d by the delta method in Doornik (2016).

Table 4: An identified structure of polynomially cointegrating relations

	$i^e_{1,t}$	$i^e_{2,t}$	$i_{1,t}$	$i_{2,t}$	$p_{1,t}$	$p_{2,t}$	$s_{12,i}$	$Ds_{08.09}$	$t \times 10^{-3}$
Test of over-identifying restrictions $\chi^2(6) = 1.78[0.94]$									
β_1	1.00	−1.00	−1.00	1.00	−0.012 [−6.8]	0.012 [6.8]	0.006 [12.8]	−0.002 [−10.2]	0.03 [16.4]
d_1	−0.19 (−15.0)	*	−0.20 (−39.1)	−0.05 (−33.9)	−0.15 (−4.5)	0.65 (40.3)	0.05 (1.9)	0.02 (3.6)	0.004 (2.3)
α_1	−	0.38 [9.4]	*−0.27* (−4.5)	−	3.27 [2.6]	−5.58 [−6.6]	*79.8* [7.8]		
β_2	1.00	−	−0.94 [−161.7]	−	−0.008 [−5.4]	0.008 [5.4]	−	0.0001 [2.1]	−
d_2	−0.17 (−14.7)	*	−0.17 (−36.7)	−0.04 (−31.6)	−0.13 (−4.5)	−0.56 (37.0)	0.07 (3.1)	*	0.005 (5.9)
α_2	−	−1.34 [−5.8]	0.93 (7.2)	−2.31 [−7.7]	*−7.59* (−2.9)	*12.2* [7.0]	−152.9 [−7.3]		
β_3	−	1.00	−	−0.806 [−51.6]	−	−0.009 [−13.9]	−0.009 [−13.9]	0.003 [12.5]	−
d_3	0.07 (8.0)	*	0.08 (9.4)	0.02 (9.3)	0.06 (4.1)	−0.25 (−9.4)	*	−0.02 (−5.9)	0.04 (20.5)
α_3	−	−	−0.22 (−4.2)	*−0.36* [−6.3]	1.95 (1.8)	*−4.25* [−5.6]	70.6 [7.8]		
β_4	1.00	0.26 [37.6]	−0.88 [−77.1]	−0.38 [−31.8]	−0.015 [−6.8]	0.010 [4.6]	−	−	−
d_4	*−0.23* (−14.6)	*	−0.23 (−32.7)	−0.06 (−29.0)	−0.18 (−4.5)	0.75 (32.8)	0.09 (3.0)	*	0.03 (19.7)
α_4	*0.20* [7.2]	*0.80* [5.3]	*−0.21* (−2.8)	1.86 [8.5]	3.73 (2.6)	−6.49 [−6.7]	68.4 [11.9]		
$\beta_{\perp 1}$	0.93	0.29	1.00	0.35	0.35	0.70	0.08	0.01	−0.00

interest rates and their forecasts as a function of the two prices.

Regarding the α adjustment, the estimates show that the US interest rate forecast has only adjusted significantly to the fourth homogeneous β relation. The UK interest rate forecast has been significantly adjusting to all relations except the third β relation. Actual interest rates, both US and UK, have shown strong adjustment to the four β relations. The two prices have also been adjusting to all β relations, but US prices less significantly so. Nominal exchange rate has been significantly adjusting to all β relations.

All variables, except the US interest rate forecast, show evidence of both error-increasing and error-correcting behavior in α. It is notable that the US interest rate forecast is strongly equilibrium error-increasing in both α and d in the fourth, homogeneous, relationship while it shows no levels feed-back

Table 5: The estimated adjustment coefficients to tauDxt

	$\zeta_1(\beta_1'\Delta\tilde{x}_t)$	$\zeta_2(\beta_2'\Delta\tilde{x}_t)$	$\zeta_3(\beta_3'\Delta\tilde{x}_t)$	$\zeta_4(\beta_4'\Delta\tilde{x}_t)$	$\zeta_5(\beta_{\perp 1}'\Delta\tilde{x}_t)$
$\Delta^2 i^e_{1,t}$	-13.8 [-12.3]	*28.7* [11.4]	-9.9 [-13.1]	-15.6 [-5.1]	-0.23 [-10.9]
$\Delta^2 i^e_{2,t}$	*−11.2* [-8.1]	20.8 [6.6]	-9.8 [-10.8]	*−9.15* [-5.1]	-0.18 [-7.4]
$\Delta^2 i_{1,t}$	*−18.8* [-23.4]	41.6 [22.9]	-13.0 [-24.3]	*−22.3* [-21.7]	-0.37 [-25.3]
$\Delta^2 i_{2,t}$	-14.3 [-6.5]	27.9 [5.6]	*−10.8* [-7.5]	*−12.6* [-4.4]	-0.25 [-6.4]
$\Delta^2 p_{1,t}$	79.0 [7.8]	*−172.0* [-7.5]	54.8 [8.3]	92.9 [7.1]	-0.25 [-1.3]
$\Delta^2 p_{2,t}$	−	−	−	−	-0.31 [-4.9]
$\Delta^2 s_{12,t}$	−	−	−	−	−

to any of the other relations. This is strong evidence of the important role expectations to the US rate plays for the long swings we see in the data.

Table 5 reports the estimates of the adjustment coefficients to the medium-run relations, $\zeta\tau'\Delta\tilde{x}_t$. The first four columns correspond to the differenced β relations and the fifth column to the $\beta_{\perp 1}'\Delta\tilde{x}_t$ relation. These medium run relations are all describing relationships between changes in the process and are, therefore, likely to capture how interest rates, exchange rates and prices respond to short-run price movements in the market. A striking feature in $\zeta_1, ..., \zeta_5$ is that actual as well as forecasted interest rates in both US and UK move in the same direction (either positively or negatively). This, of course, implies a mixture of error-increasing and error-correcting behavior (the former indicated with italics) typical of momentum trading. Thus, trading in the foreign currency market seems to have had similar effects on all the interest rates and, therefore, to have caused the structure of interest rates to move in the long persistent swings shown in the middle and lower panel of Figure **??**.

The US interest rate forecast reacts strongly and very significantly to changes in the long-run equilibrium errors, $\tau'\Delta\tilde{x}_t$, but shows little evidence of levels feed-back in α. Interestingly, the nominal exchange rate exhibits the opposite reaction pattern: while it almost exclusively α adjusts to the *levels* relations, $\beta'\tilde{x}_t + \delta'\Delta\tilde{x}_t$, it has not been significantly affected by any *changes* in the long-run equilibrium errors, $\tau'\Delta\tilde{x}_t$. Thus, over the medium run the nominal exchange rate seems to have been pushing the foreign exchange market, while interest rates have followed suit. Over the long run, the nominal exchange rate has been adjusting to the *levels* relations, $\beta'\tilde{x}_t$, whereas interest rates forecasts have been pushing.

It is also notable that prices which are not generally subject to speculative trading have reacted to changes in the estimated equilibrium errors ($\beta'\tilde{x}_t$) in a way consistent with equilibrium error correcting behavior, except for US inflation rate to $\beta_2'\Delta\tilde{x}_t$.

9 The long persistent swings and the estimated I(2) trends

The moving average representation of (5) subject to (6) and (7) expresses the variables x_t as a function of once and twice cumulated errors and stationary and deterministic components. It is given by:

$$\begin{aligned} x_t &= C_2 \sum_{j=1}^{t}\sum_{i=1}^{j}(\varepsilon_i + \Phi D_i + \mu_0) + C_1 \sum_{j=1}^{t}(\varepsilon_j + \Phi D_j + \mu_0) \\ &\quad + C^*(L)(\varepsilon_t + \Phi D_t + \mu_0) + A + Bt. \end{aligned} \tag{12}$$

The parameters are complicated functions of the parameters in (8), defined in Johansen (1992). For the purpose of this paper it suffices to focus on the matrix C_2 :

$$C_2 = \beta_{\perp 2}(\alpha_{\perp 2}'\Psi\beta_{\perp 2})^{-1}\alpha_{\perp 2}', \tag{13}$$

where $\beta_{\perp 2}, \alpha_{\perp 2}$ are $(p \times m_2)$ matrices which are orthogonal to $\beta, \beta_{\perp 1}$ and $\alpha, \alpha_{\perp 1}$, respectively, and Ψ is a function of the parameters of VAR model. It is useful to decompose C_2 as:

$$C_2 = \breve{\beta}_{\perp 2}\alpha_{\perp 2}'. \tag{14}$$

where $\breve{\beta}_{\perp 2} = \beta_{\perp 2}(\alpha_{\perp 2}'\Psi\beta_{\perp 2})^{-1}$. One can now interpret the double summation $\alpha_{\perp 2}'\sum_{j=1}^{t}\sum_{i=1}^{j}\varepsilon_i$ as an estimate of the s_2 second order stochastic trends which load into the variables x_t with the weights $\breve{\beta}_{\perp 2}$.

From (12) it follows that an unrestricted constant will cumulate twice to a quadratic trend and similarly for the dummies. Thus, the coefficients of the deterministic components need to be appropriately restricted to avoid undesirable effects in the process, such as quadratic trends by cumulating t and broken trends by cumulating $Ds_{08.09,t}$ (see Rahbek et. al, 1999). By

Table 6: The estimated common trends and their loadings

	$i^e_{1,t}$	$i^e_{2,t}$	$i_{1,t}$	$i_{2,t}$	$p_{1,t}$	$p_{2,t}$	$s_{12,t}$
The estimated loadings to the I(2) trends							
$\tilde{\beta}'_{\perp 2,1}$	0.00	0.02	0.00	0.01	0.02	-0.13	0.99
$\tilde{\beta}'_{\perp 2,2}$	-0.27	0.05	-0.27	-0.07	-0.22	-0.89	0.13
The estimated I(2) trends							
$\alpha'_{\perp 2,1}$	1.00	0.00	-0.88	0.03	$*$	$*$	$*$
$\alpha'_{\perp 2,2}$	-0.54	1.00	-0.57	-0.13	$*$	$*$	$*$

restricting the linear trend and the step dummy to lie in the polynomial cointegration relations, $\beta' \tilde{x}_t + d' \Delta \tilde{x}_t$, as specified in (8) the process is restricted not to contain any broken or quadratic trends.

The estimates reported in Table 6 are calculated for the identified β structure in Table 4. The first I(2) trend is basically describing the twice cumulated shocks to the spread between the US 3 months interest rate and its forecast with a smaller weight to the actual rate. It is primarily loading into the nominal exchange rate. The second trend is basically picking up the UK interest rate forecast relative to the US interest and its forecast and is primarily loading into the interest rates, prices and the nominal exchange rate. Thus, the US expectational shocks seem to be much more "pushing" than the UK ones, the effect of which are important only in relation to how the rest of the system behaves.

Altogether the results suggest that expectational shocks have a significant effect on the level of interest rates as well as prices in both countries. This can be interpreted as supporting the hypothesis that it is the expectations in the financial markets that drive prices away and towards long-run benchmark values in long persistent swings.

10 Expectations formation in the short-run

Expectations and outcomes are likely to be simultaneously interdependent. In forming their expectations, financial actors have strong incentives to use forecasting rules that work well. Higher yields accrue to someone who acts on the basis of better forecasts. Therefore, one would expect people to adjust their forecasting rules to eliminate avoidable errors and one would also expect

Table 7: The estimated short-run impact of expectations on the system

	$\Delta i_{1,t}$	$\Delta i_{2,t}$	$\Delta p_{1,t}$	$\Delta p_{2,t}$	$\Delta s_{12,t}$
$\Delta i^e_{1,t}$	**0.50**	−0.02	−0.52	−0.19	**−29.36**
	[13.44]	[−0.12]	[−0.73]	[−0.35]	[−3.87]
$\Delta i^e_{2,t}$	**0.12**	**1.01**	−0.29	−0.28	8.93
	[4.26]	[10.12]	[−0.54]	[−.69]	[1.83]
$\Delta i^e_{1,t-1}$	−0.01	**0.54**	1.54	0.95	−0.18
	[−0.14]	[3.86]	[2.05]	[1.67]	[−0.03]
$\Delta i^e_{2,t-1}$	**0.10**	**0.26**	0.22	0.24	−3.85
	[3.74]	[2.69]	[0.42]	[0.61]	[−0.83]

to see feedback from past outcomes to current expectations.

In section 6, table 2, the test of a unit vector in α showed that both the US and the UK interest rates and the nominal exchange rate could be considered as purely adjusting variables. The weak exogeneity results showed that the US 3 months treasury bill forecast has not been affected by levels feed-back, while this was not the case with the UK forecast. Also, one of the estimated $I(2)$ trends was essentially the twice cumulated shocks to the US interest rate forecast, and the other the twice cumulated shocks to the UK forecast relative to the actual interest rates. Taken together this suggests that actual outcomes have been strongly influenced by market expectations, thereby emphasizing the crucial role expectations have played for the long persistent swings in the foreign currency market.

It is, therefore, of some interest to study the short-run impact of expectations on the actual interest rates and to what extent the extraordinary large shocks to the interest rates can be associated with large changes in interest rate expectations. We have, therefore, estimated the CVAR model as a partial system conditional on the interest rate forecasts. It is notable that all evidence of I(2) disappeared in this model, suggesting that the two interest rate forecasts contain all information about the common I(2) trends.

Table 7 reports the estimated effects of current and lagged changes in interest rate forecasts on the other variables in the system. The estimated effects of the lagged changes of the system variables are small and mostly insignificant and are, therefore, not reported in the table. When interpreting the results it is important to remember the time lag between US and UK which may explain why the US rate is influenced primarily by the US and UK forecast (the latter is likely to contain additional information not present in the US forecast) and why the UK rate is not influenced by the current

Table 8: Estimated t-ratios of dummy variables in the full CVAR model and in the partial model conditional on the forcasted interest rate

	$D_{02.11}$	$D_{03.01}$	$D_{03.07}$	$D_{04.07}$	$Dt_{05.03}$	$D_{05.09}$	$D_{08.01}$	$D_{08.04}$	$Dtr_{08.09}$	$D_{08.11}$	$D_{08.12}$	$D_{09.01}$
$\Delta i^e_{1,t}$	−4.31	*	2.37	5.02	*	*	−13.12	5.79	3.63	−8.46	8.49	−11.16
$\Delta i^e_{2,t}$	*	*	−4.49	2.26	3.71	*	−23.52	6.05	4.79	−8.72	14.84	−17.65
$\Delta i_{1,t}$	−6.23	*	2.57	*	*	*	−23.52	6.05	5.61	−11.13	12.40	−20.43
CM	−4.32	*	2.53	−4.31	*	−2.39	−11.32	3.29	−4.54	−3.25	3.21	−6.69
$\Delta i_{2,t}$	*	−8.02	−6.30	3.96	4.67	*	−6.91	*	2.88	−2.98	9.42	−9.29
CM	*	−9.37	−4.15	2.83	2.85	*	*	*	*	*	*	*
$\Delta p_{1,t}$	*	*	*	*	*	5.27	*	*	*	−3.08	*	*
CM	*	*	*	*	*	5.52	*	*	3.92	−3.39	*	*
$\Delta p_{2,t}$	*	*	*	*	*	*	*	*	*	*	−3.94	*
CM	*	*	*	*	2.36	*	*	*	*	*	*	*
$\Delta s_{12,t}$	*	*	−2.72	*	*	*	*	*	*	−4.53	−2.35	−5.03
CM	*	*	*	*	*	*	−3.10	*	4.13	−3.56	*	−3.41

value of the US forecast, only by its lagged value. In terms of the magnitude of the estimated coefficients both interest rates seem to react strongly to the expectations in the market, but the UK rate more strongly so. The dollar rate has appreciated as a result of an expected increase in the US rate and depreciated as a result of an expected increase in the UK rate. The inflation rates are not very significantly affected by changes in the interest rate forecasts.

11 Extraordinary shocks

The 12 dummy variables in the full CVAR model were included to control for a number of extraordinary large shocks mostly to the interest rates or their forecasts. Only one dummy in 2005:9 is exclusively needed to control for a big shock to US inflation. Except for the Lehman Brothers dummy in 2008:9, all dummies enter only at time t (i.e without lags) and hence account for large unanticipated shocks (given the chosen information set) at time t. At time $t+1$ they are no longer unanticipated and become part of the model dynamics. The estimated coefficients to the dummy variables are generally small in absolute size and, as such, of minor interest. This is why Table 8 only reports their t-ratios as an indication of how strong the shocks are. The dummy effects are estimated partly based on the full CVAR model (5), partly on a partial CVAR conditional on the two interest rate forecasts. The latter estimates are reported in the rows labelled CM. Insignificant coefficients ($t < 1.5$) are indicated with a *, very significant effects ($t > 3.0$) with bold face.

The results show that in 2002:11, 2008:01, 2008:04, 2008:09-2008:10, 2008:11, 2008:12 and 2009:1 a big shock to the US forecast made at time t coincides with a similarly big shock to the actual interest rate at time t. By conditioning on the professional forecasts, $i^e_{t|t+3}$, the strength of the big shocks declines very significantly (except for the dummy in 2002:11), suggesting that the actual values of the interest rate are affected by the forecasts. For the UK, big shocks to the professional forecasts coincide with big shocks to actual rates at time 2003:7, 2005:3, 2008:1, 2008:09-2008:10, 2008:11, 2008:12 and 2009:1. Contrary to the US case, the shocks to the actual rates are less significant than the shocks to the forecasts for the period after 2008:1. Furthermore, when conditioning on the professional forecasts, the big shocks to the actual UK rates become insignificant. For the period before 2008:1, the significance of the dummies only declines somewhat which is similar to the US case. Only in 2003:1, the UK rate is hit by a big shock without a corresponding shock to the forecast and in 2008:4 the forecast has a big shock whereas the actual UK rate has none. Thus, in the majority of cases both the forecast, $i^e_{t|t+3}$, and the actual interest rate, i_t, have experienced a big shock at the same time. Also when conditioning on the forecast, $i^e_{t|t+3}$, the significance of the shock declined and frequently became insignificant in almost all cases. This seems to suggest that professional forecasts may have affected the actual interest rates in, at least partly, an exogenous manner. This conclusion is supported by the no levels feedback in α, by the strong weight of forecast shocks in the two I(2) trends and the fact that these shocks seem to drive the variables away from long-run benchmark values for extended periods of time.

12 Conclusions

A number of questions were raised in the introduction which have been addressed within the rich structure of the $I(2)$ CVAR model. Generally the results showed that the two interest rates and the exchange rate have adjusted quite strongly to expectations, but also that fundamentals have affected expectations. In spite of the strong support for a mutual interdependence between expectations and outcomes in the foreign currency market, the former seemed, nonetheless, to have been more significant as a driving force. In particular, expectational shocks to the US 3 months interest rate seemed to have played a crucial role in the long persistent swings typical of the dollar/pound market.

We found that US interest rate forecasts showed little or no significant levels feed-back effects, whereas they showed strong and significant feed-back effects from medium run changes in the equilibrium errors. Interestingly, the nominal exchange rate showed the opposite reaction pattern. Thus, over the medium run, changes in the nominal exchange rate seem to have been pushing the foreign currency market, whereas interest rate changes have followed suit. But over the long run, the nominal exchange rate has been adjusting and interest rate expectations have been primarily pushing.

All variables (expected and actual interest rates as well as their spreads, the nominal and real exchange rate and the two prices) were found to be (near) $I(2)$. Thus, they exhibited a pronounced persistence consistent with imperfect knowledge based theories whereas not with standard rational expectations' based models. This conclusion was further confirmed by self-reinforcing feed-back mechanisms in the dynamic adjustment structure. The latter was evidenced by equilibrium error increasing behavior both in the medium run and the long run, a strong sign of persistent speculative bubbles.

Altogether the results suggest that when the market expects interests to increase, say, the actions of the market will make them increase, thus confirming the expectations. Thus financial actors will continue to believe in growth even when interest rates move away from their long-run benchmark values. No doubt, interest rate expectations play a crucial role in the foreign currency markets.

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