A Practical Investigation of Optimal Strategies on the Flüela Pass

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Introduction

During recent years, the optimization of pacing strategies in the field of endurance sports based on mathematical models has become more and more popular. Strategies have been developed for running [1] as well as for cycling [2],[3]. But most of these studies are of theoretical nature and lack of practical relevance.

In this work we want to present an experiment where the optimal strategy has been applied on a simulated real world track by providing feedback to the athlete on-the-fly. We will present the underlying mathematical models, the parameter estimation, the experiment setting and first results.

Methods

All tests are performed on a bike simulator based on a Cyclus2 brake and our own software [4]. In cases where a strategy is simulated, the riders get a constant visual feedback of the difference in the travelled distance and are advised to keep that gap as small as possible.

The experiment consists of five tests: At first, the subject performs an incremental step test to obtain an estimate of his anaerobic lactate threshold (AT). The other tests are rides on a real track, namely the eastern climb of the Flüela Pass in Switzerland. The first ride (I) is for familiarization purposes. In order to obtain a decent benchmark, subjects were advised to ride as close to their AT as possible but were free in the selection of their power output. In the second ride (II) subjects rode with their own predefined pacing strategy. In ride I and II subjects were instructed to perform with maximal effort.

The third ride (III) is performed with an optimal strategy feedback and the fourth ride (IV) with a validation strategy feedback.

The optimal strategy is calculated based on a physical model describing the equilibrium of the rider's pedal force and the forces induced by aerodynamic drag, friction, gravitation and inertia (see [2]). The physiological capabilities of the athlete are modelled by a dynamic version of Morton's critical power model [5]. The resulting optimal control problem is solved by the state-of-the-art solver GPOPS II [6].

The validation strategy is determined by adding a constant power offset to the rider's own strategy of ride II in order to achieve the same time than with the optimal strategy. This trial was performed to give information on whether only the optimal strategy or if also the modified own strategy with supportive visual feedback can lead to an improved performance. Obviously the energy demand of this strategy is higher than in ride II.

The parameters of the physiological model are determined with the step test and rides I and II by assuming that the athlete is fully exhausted at the end of each test. Therefore parameters are chosen in a way that the anaerobic work capacity level is zero at the end of the race by minimizing its squared error.

Results

All subjects were able to perform the optimal strategy ride as proposed and the total race time improved by around 1.9% compared to the self-paced ride II (Table 1). Three out of six subjects were able to follow the validation strategy IV. One reason for this could be that they were not fully exhausted in ride II and therefore could maintain the more energy consuming validation task until the end. Also day to day variations in performance could lead to such a result. Nevertheless, the three subjects that could not follow the validation strategy show that the optimal strategy offers a benefit over the self-paced strategy.

Table 2 shows the root mean square error between proposed power output and power output during the experiment. The error is significantly higher in the validation ride as in the ride with optimal strategy feedback.

rider	II (hh:mm:ss)	III (hh:mm:ss)	III (%)	IV (hh:mm:ss)	IV (%)
1	00:43:03	00:42:43	-0.77	00:42:43	-0.77
2	01:00:12	00:59:26	-1.27	00:59:26	-1.27
3	00:44:28	00:43:55	-1.24	00:44:44	+0.59
4	01:19:01	01:16:51	-2.74	01:18:42	-0.40
5	00:58:17	00:57:00	-2.20	00:57:49	-0.80
6	00:53:55	00:52:10	-3.24	00:52:10	-3.24

Table 1: Total race-times for rides II, III and IV. Additionally for rides III and IV the improvement compared to test II is provided in relative values.

Conclusions

Our experiment showed that the calculated strategy is feasible, which means that all athletes were able to follow it until the end. Furthermore, it provides an advantage over the strategy the athletes chose on their own. However, it should be noted that three riders could improve their performance with a different pacing strategy. This leaves the question whether the strategy or the external feedback given to the subjects allows for an improvement in performance.

rider	III		IV	
	RMS	Mean P	RMS	Mean P
1	13 W	391 W	18 W	391 W
2	13 W	266 W	16 W	266 W
3	22 W	337 W	25 W	331 W
4	4 W	175 W	12 W	170 W
5	9 W	264 W	19 W	260 W
6	16 W	293 W	20 W	294 W

Table 2: Root mean square error in power output between proposed strategy and experiment for rides III and IV and mean power output during the ride in Watt.

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