Optimal control for cycling time trials: The Maronski effect

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Many researchers considered for cycling time-trials the speed minimizing time for fixed energy expenditure. The purpose of this contribution is to recall and extend a fundamental result overlooked in the literature. In 1994, Maronski proved that for moderate slope profiles, the minimum-time pacing strategy for cycling has three phases: (1) a short initial maximum-power phase, (2) a long constant-speed central phase, (3) and a short final maximal-power phase (Maronski, 1994). We call the constant-speed phase the *Maronski effect*.

It seems counter-intuitive that the optimal speed is constant during climbing and descent. The Maronski effect can be explained by the dual problem to minimize energy expenditure for fixed minimum time. Then any variation of the constant speed in the central phase (2) causes an increase in energy due to the quadratic dependence of aerial resistance on the speed.

Here we point out that the Maronski effect is not limited to the simple model used. For example, consider replacing the fixed-work constraint by the 3-parameter critical power model (Morton, 1996), introducing critical power, anaerobic capacity, and maximum power as a function of remaining anaerobic resources as parameters. We found that in this case the optimal solution, although quantitatively different, is qualitatively equivalent to that of Maronski.

Concluding, we have confirmed the Maronski effect for an extended optimal cycling control problem that includes a physiological component. It remains open to further generalize the optimization problem to more realistic physiological models for which we expect the Maronski effect to be overcome. For a first step in that direction, see (Dahmen, 2012).

Finally, we point out that the Maronski effect implies that the second phase of the solution is a singular arc. Such arcs bear principal difficulties in numerical computation because precisely on those arcs higher-order optimality conditions are required to uniquely define the solution. For computing solutions with several singular arcs we report progress following Riedinger (2013) by incorporating higher-order optimality conditions using complementarity constraints and GPOPS (Rao, 2010).

Literature

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