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Robust Computation of Minimum-Time Pacing Strategies on Realistic Road Cycling Tracks

1. Introduction

In a previous publication (Dahmen, 2012) we compared minimum-time pacing strategies for road cycling time trials for two endurance models: the 3-parameter critical power model (Morton, 1996) and an exertion model by Gordon (2005). For the former model, the problem is singular and only an approximate regularized solution for a synthetic continuously varying slope profile was computed. In this contribution, we improve the numerical methods to compute strategies on realistic tracks with complex height profiles.

2. Methods

Minimum-time pacing strategies for road cycling may be computed as solutions of optimal control problems involving a demand model for the bicycle mechanics and a bioenergetic supply model for the physical capability of an athlete.

The dynamical system for road cycling power demand (Martin et al. 1998) arises from the equilibrium of the propulsive pedaling force and resistance forces accounting for gravity, inertia, rolling friction, frictional losses in wheel bearings, and aerodynamic drag. In contrast to common practice in literature, we parametrize this system of two differential equation using the kinetic energy $e(t)$ instead of the speed $v(t)$:

$\dot{x}=q\sqrt{e}$ (1a)

$\dot{e}+q\sqrt{e}(mg\mu+b_0+mgh'(x)+b_1 q\sqrt{e}+1/2\, c_d r A q^2 e)=zeta\, P$.(1b)

subject to the boundary conditions

$x(0)=0$ and $e(0)=0$ (2)

with $q=\sqrt{2/m+I_w/r_w^2}$. Besides the kinetic energy, the pedaling power $P(t)$ as well as the distance $x(t)$ represent the continuous variables. The invariant parameters are the gravity constant g , air density r , mass of the cyclist and his bicycle m , inertia of the wheels I_w , radii of the wheels r_w , rolling friction coefficient μ , bearing friction coefficients b_0 and b_1 , shape coefficient c_d , cross-sectional area of the cyclist and his bicycle A , and chain efficiency factor zeta. For those parameters that depend on an individual athlete or a specific bicycle and cannot be measured directly, we adopted the values from Dahmen & Saupe (2011), which we estimated based on a fit of the dynamic model to real power and speed data for a rider-bicycle combination on an asphalt road.

We used a differential gps device (Leica GPX900) to record accurate height profiles of real cycling tracks. The accuracy of conventional GPS is by far to low in order to compute the slope $h'(x)$ since the differentiation of the height data introduces high frequency noise. On sections with obstacles like forest or rows of houses and particularly in gorges, however, the satellites are often occluded and cannot provide the necessary information for the differential correction. On those sections we acquired power and speed data and plugged it into the right-hand-side term that arises when (1b) is resolved for $h'(x)$. Fusing both estimations provides accurate slope profiles for realistic road cycling tracks that completes our mechanical bicycling model (Dahmen et al., 2011).

A prominent physiological supply model is the 3-parameter critical power model (Morton, 1996). It assumes that the anaerobic energy resources e_a of an athlete are filled with a fixed anaerobic capacity E_a at the beginning of a time trial when the athlete is rested

$e_a(t=0)=E_a$ (2).

The rate $\dot{e}_a(t)$ at which the resources decrease is equal to the difference between the individual critical power of the athlete P_c and his instantaneous pedaling power

$\dot{e}_a=P_c-P(t)$ (3).

Thus, the athlete may recover when the pedaling power falls below the critical power. The state variable e_a must be positive and is capped by the anaerobic capacity

$0\leqslant e_a\leqslant E_a$ (4).

Clearly, the pedaling power should be non-negative and limited. The maximum instantaneous power P_m is chosen to decrease from the absolute maximum power at rest P_{max} to critical power when the cyclist is exhausted, i.e., $e_a=0$:

$P_{min}\leqslant P\leqslant P_m$ (5)

with $P_{min}=0$ and

$P_m=P_c+(P_{max}-P_c)e_a$ (6).

For this contribution, we adopt the values of a model fit to a self-paced mimimum-time effort on a simulated course on our ergometer (Wolf & Dahmen, 2010)

As the control variable P enters only linearly into the equations, the optimal control problem is singular and a regularization is to be considered to improve the robustness of numerical algorithms. For this reason we modify the performance criterion, which is the time to complete the course, t_f , and add a small term that penalized high variations in power

$$\tilde{t}_f = t_f + epsi \int_0^{t_f} \dot{P}^2 \, dt \quad .(7)$$

We use the GPOPS-II package for Matlab (Patterson & Rao, 2014) to solve the problem numerically. It discretizes optimal control problems using spectral collocation methods and hands the discrete problem over to an NLP solver, for which we chose IPOPT. Recently, automatic differentiation, provided by the ADiGator package has vastly improved the computation efficiency. In order to use this feature, we approximate our slope profile by a cosine series.

3. Results

We computed optimal pacing strategies for the road cycling courses Ottenberg, Wachtelbühler Höhe, Pfyn, and Ermatingen in Switzerland. Figure 1 depicts the numerical result for the Pfyn course, which is the longest. We used $N=400$ coefficients to approximate the slope profile by a cosine series. The regularization parameter was $epsi=10^{-9}$, the mesh tolerance was 10^{-5} and the scaling method was set to “automatic bounds”. Other settings in GPOPS-II, AdiGator, and IPOPT were left at their default.

4. Discussion

Using kinetic energy instead of speed vastly improves the efficiency and robustness of the computations. In fact, we could not compute any pacing strategy for a realistic track when the problem was parametrized with speed. We suppose that this is due to the scaling of the optimization problem. Although automatic scaling methods are available and implemented in GPOPS-II, they rely on heuristics and may fail for a particular problem.

It is remarkable that when using kinetic energy, the regularization (7) is not essential. Even without, the algorithm converges, although in particular at discontinuities in the pedaling power overshoots caused by Gibb's phenomenon are visible.

Previously, we parametrized the problem with speed only and used saturation functions to approximate the singular problem by a regular problem (Dahmen, 2012). However, this approach only allowed to compute strategies for synthetic slope profiles that were much simpler than the realistic ones considered here. Together with the improvements recently introduced in the GPOPS-II software package, in particular automatic differentiation, we have achieved a substantial improvement in the computation of optimal pacing strategies on realistic road cycling courses.

Finally, we discuss the example given in Figure 1 in detail: The necessary condition for a local extremum in optimal control states that at any point in time either one of the inequality constraints (4) or (5) must be active or the optimality criterion (Euler-Lagrange Equation) that any small variation of the strategy leads to an increase of the objective functional (7), must hold. Earlier, we pointed out that for the problem without regularization, this criterion states that on unconstrained arcs the kinetic energy, and thus the speed, is constant (Saupe & Dahmen, 2013). Due to the power constraint (5), e_a can only approach zero asymptotically but never vanish completely. Thus

the optimal solution consists of a sequence of intervals, on which one of the following condition holds: $e_a = E_a$, $P = P_{min}$, $P = P_m$, or $\dot{v} = 0$. Table 1 summarizes the switching structure that may be read off the numerical solution in Figure 1.

The minimum-time pacing strategy for the Pfyn course has a complex switching structure. Between the initial $P = P_m$ arc, that accelerates the cyclist to 7.3 m/s , and 1141 m, three singular arcs, two $e_a = E_a$ arcs and one $P = P_{min}$ at a descent from 502 m to 802 m alternate.

From 1141 m to 4133 m two singular arcs with significantly lower constant speed between only 4.4 m/s and 4.7 m/s dominate. They are interrupted by a descent where $P = P_{min}$ at 2422 m – 2572 m and a minor E_a arc at 2989 m – 3032 m . There is a very steep section from 3800 m to 4330 m . Here, the suggested strategy is to save energy on the preceding low-speed singular arcs so that the cyclist is almost rested before the maximum slope is encountered. Now, the anaerobic resources are consumed rapidly and thus the maximum instantaneous pedaling power declines. At 4133 m it has fallen to 315 W and the cyclist can go on only on a $P = Pm$ arc until x=4390 m . It may be assumed that the strategy on this section is particularly important since a pronounced descent follows where the athlete recovers up to 80% of his initial anaerobic resources.

On the following ascent at 4817 m he changes almost instantaneously from $P = P_{min}$ to the final $P = P_m$ spurt that is hold till the end although there is a short downhill section just 200 m before the target line.

5. Conclusion

We believe that this approach has the potential to improve intuitive pacing strategies in particular on courses with a complex slope profile. The mechanical model that is incorporated into the optimization is certainly more accurate regarding the estimation of power demand than an estimation of a human based on his experience and visual impression of the track can be. Clearly the accuracy of the physiological model and its parameter calibration remains the bottleneck and requires further efforts and validation until athletes may benefit from our approach in practice.

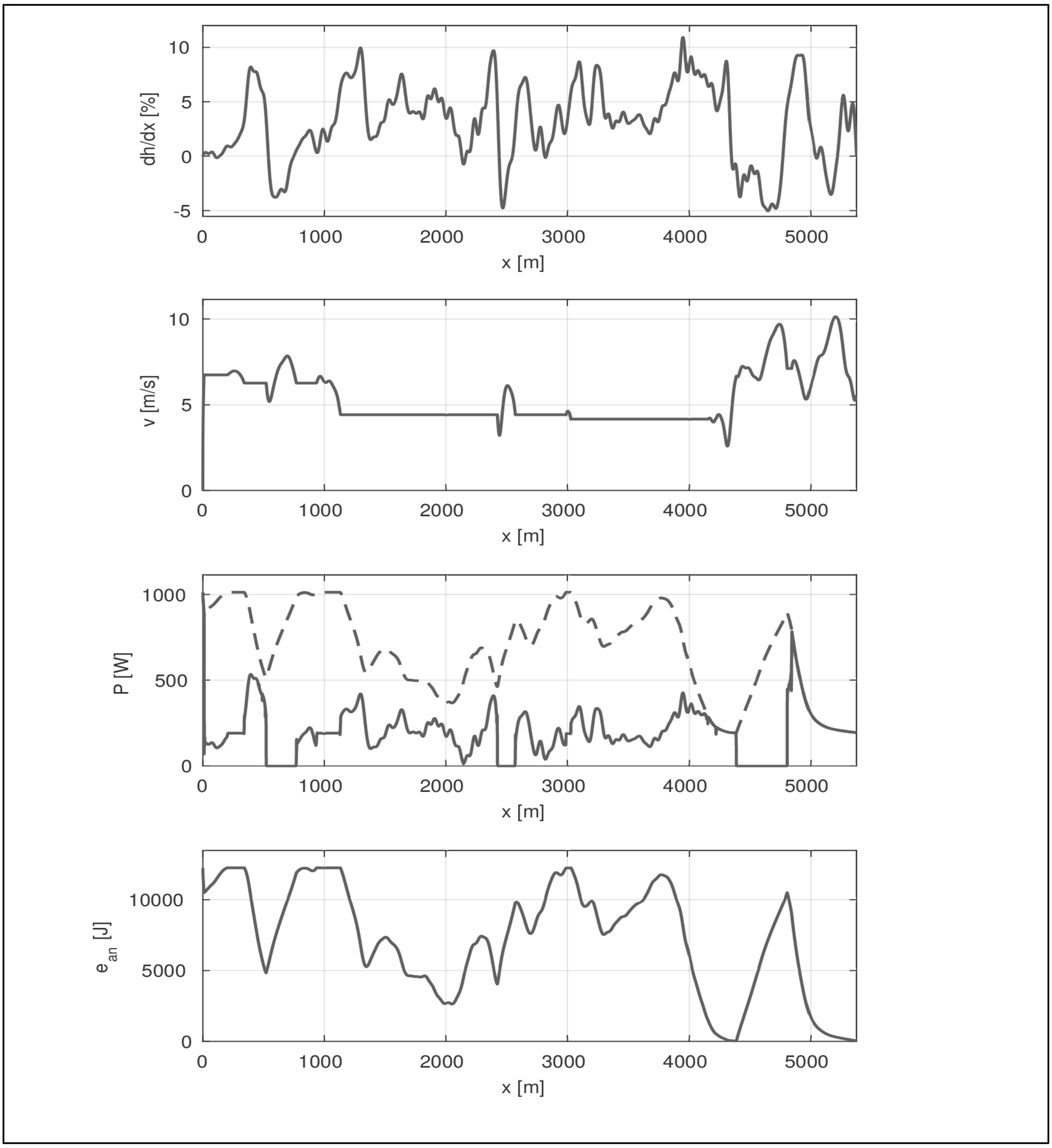

Fig. 1: Minimum-time pacing strategy for the real slope profile of a 5369 m long cycling course near Pfyn in Switzerland. The dashed line shows the instantaneous maximum power P_m .

Table 1: Switching Structure of the solution. The singular condition is denoted by v_{sing} .

Distance [m]	0	22	192	358	502	802	1141	2422
Solution candidate	P_m	v_{sing}	E_a	v_{sing}	P_{min}	E_a	v_{sing}	P_{min}
Distance [m]	2572	2989	3032	4133	4390	4817	5369	
Solution candidate	v_{sing}	E_a	v_{sing}	P_m	P_{min}	P_m		

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Literature

Dahmen, T. (2012). Optimization of pacing strategies for cycling time trials using a smooth 6-parameter endurance model. In Pre-Olympic Congress on Sports Science and Computer Science in Sport 2012. International Association of Computer Science in Sport (IACSS).

Dahmen, T., Byshko R., Saupe, D., Röder, M., and Mantler, S. (2011). Validation of a model and a simulator for road cycling on real tracks. Sports Engineering, 14(2-4):95-110.

Dahmen, T. & Saupe, D. (2011). Calibration of a power-speed-model for road cycling using real power and height data. International Journal of Computer Science and Sport, 10(2):18-36.

Gordon, S. (2005). Optimising distribution of power during a cycling time trial. Sports Engineering, 8(2), 81-90.

Martin, J. C., Milliken D. L., Cobb J. E., McFadden K. L., and Coggan A. R. (1998). Validation of a mathematical model for road cycling power. Journal of Applied Biomechanics, 14:276-291.

Morton, R. H. (1996). A 3-parameter critical power model. Ergonomics, 39(4), 611-619.

Saupe, D. & Dahmen, T. (1994). Optimal Control for cycling time trials: The Maronski effect. 21. Sportwissenschaftlicher Hochschultag der Deutschen Vereinigung für Sportwissenschaft. KOPS Institutional Repository University of Konstanz, http://nbn-resolving.de/urn:nbn:de:bsz:352-246429.

Patterson, M. A., & Rao, A. V. (2014). GPOPS-II: A MATLAB software for solving multiple-phase optimal control problems using hp-adaptive Gaussian quadrature collocation methods and sparse nonlinear programming. ACM Transactions on Mathematical Software (TOMS), 41(1), 1-37.

Wolf, S. & Dahmen, T. (2010). Optmierung der Geschwindigkeitssteuerung bei Zeitfahrten im Radsport. In D. Link and J. Wiemeyer, editors, Sportinformatik trifft Sporttechnologie, 8. Symposium der dvs-Sektion Sportinformatik, pages 235-239. Feldhaus Edition Czwalina.