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Image compression using fractals and discrete cosine transform

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A new image data compression method using fractals and the discrete cosine transforms(DCT) is presented. The original image is first encoded by fractals in the DCT domain, then the error image is encoded using the DCT. Experiments show that the method can achieve high fidelity at a high compression ratio.

Introduction: Recently a new compression method using fractal theory has been proposed by Jacquin[1] and widely investigated[2-6]. Experiments have shown that, although the method can achieve high compression, the quality of the decompressed image is not very good. Some measures (such as two-level image partition) have been taken to improve the quality[1]. In this Letter, we propose a method, combining fractals and DCT to improve the quality. It can offer high quality at a high compression ratio.

Coding scheme:

(i) *Compression procedure*: The basic theory of the fractal-based coding method can be found in [1]; here, we just introduce our method.

The original image is first partitioned into two kinds of blocks whose sizes are 8×8 and 16×16. They are then transformed by the DCT. The smaller are called range blocks and the larger are called domain blocks. They are denoted as $F_R(u,v)$ and $F_D(u,v)$, respectively.

We then classify $F_R(u,v)$ into two kinds of range blocks according to its AC coefficients:

$$\text{If } |F_R(0,1)| + |F_R(1,0)| + |F_R(1,1)| = \begin{cases} < T & F_R(u,v) \text{ is a simple range block} \\ \geq T & F_R(u,v) \text{ is a complicated range block} \end{cases}$$

where T is a threshold.

For a simple range block, we just approximate it by storing its DC coefficient $F_R(0,0)$. 10 bits are needed to store the coefficient.

For a complicated range block, we approximate it by

$$F_R(u,v) = \tau \circ \varphi(F_D(u,v))$$

where φ is a contractivity operator which maps a 16×16 domain block $F_D(u,v)$ onto a 8×8 range block $F_R(u,v)$. It takes the low frequency part of $F_D(u,v)$ shown as in Fig. 1.

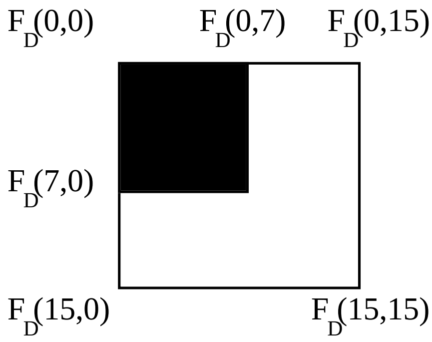

Fig. 1. Contractivity operator and its result. The black part is its mapping result.

τ is the compound transformation composed of an isometry, a scaling and a luminance shift of the form

$$F_R(u,v) = \tau \circ \varphi(F_D(u,v)) = \begin{cases} \Delta g = F_D(0,0) - F_R(0,0) & u = v = 0 \\ \iota_n(\alpha(\varphi(F_D(u,v)))) & \text{otherwise} \end{cases}$$

where α is a scaling factor which takes values in the set {0.2, 0.3, …, 0.9}, Δg is the luminance shift and ι_n is one of the eight isometries which include reflection rotation[1]. In the DCT domain, the transformation changes(see Table 1).

The changes are shown in table 1.

Table 1. Changes of the isometries from spatial domain to DCT domain.

	Spatial Domain	DCT Domain
1	identity	$\iota_0(F(u,v)) = F(u.v)$
2	reflection about mid-vertical axis	$\iota_1(F(u,v)) = (-1)^V F(u,v)$
3	reflection about mid-horizontal axis	$\iota_2(F(u,v)) = (-1)^u F(u,v)$
4	reflection about first diagonal	$\iota_3(F(u,v)) = F(v,u)$
5	reflection about second diagonal	$\iota_4(F(u,v)) = (-1)^{u+v} F(v,u)$
6	rotation through +90°	$\iota_5(F(u,v)) = (-1)^u F(v,u)$
7	rotation through +180°	$\iota_6(F(u,v)) = (-1)^{u+v} F(u,v)$
8	rotation through -90°	$\iota_7(F(u,v)) = (-1)^v F(v,u)$

The derivation process of the isometries' changes from spatial domain to DCT domain is similar to that presented by Bracewell et al.[7], Here the detailed derivation is omitted.

The coding procedure of the complicated range block is to search for the best matching domain block among all the domain blocks to minimise the distortion. The distortion used here is

$$d = \sum_{\substack{u,v=0 \\ u,v\neq 0 \text{ at the same time}}}^{7} \left[F_R(u,v) - \tau \circ \varphi(F_D(u,v))\right]^2$$

There are four parameters needed to be stored; the bit allocation is as follows:

1. The co-ordinates of the best matching domain block 5+5=10
2. α 3
3. ι_n 3
4. Δg 10

Next, we calculate the error image between the rage block and its fractal approximation:

$$E(u,v) = F_R(u,v) - \tau \circ \varphi(F_D(u,v))$$

For the error image $E(u,v)$, we quantise it and encode it using the Huffman code, then store it. When every range blocks is encoded, the compression procedure is finished.

(ii) *Decompression Procedure*:The decompression procedure is relatively simple. First we decode the fractal parameters and then transform an arbitory image iteratively by fractals. It needs about eight iterations to converge. This procedure is similar to that proposed by Jacquin[1]. In contrast to the Jacquin method, our method is carried out in DCT domain. This procedure produces the fractal approximation of the original image; On the other hand, we decode the Huffman code, dequantise it and transform it by 2-D inverse DCT. Thus we obtain the error image. Finally, the addition of the the fractal approximation and the error image is the decompressed image.

Experimaental Results and conclusion: We have compressed the standard test image ‘Lena’ using the method. The results is satisfactory. The compression ratio is 12.4. The quality of the decompressed image is very good, with SNR=32.3dB. The original image and the decompressed image are shown in Figs 2 and 3.

Fig 2. Original image ‘Lena’

Fig3 Decompressed imade

The method presented in this Letter has the following features:

(i)The fractal coding procedure is all carried out in the DCT domain. It can provide high compression.

(ii)Although fractals can achieve high compression, the detailed the original image have been lost. In this Letter, we use the DCT to encode the image details, so the method can provide high quality at high compression ratio.

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