Square Isometries as Integral Part of Fractal Transformation - An Analysis

Isometrien als Integraler Bestandteil der Fraktalen Transformation

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Abstract

In block based methods of fractal image compression the domain-pool is frequently enlarged by using square isometries of image blocks. This does not result in higher performance of fractal codecs, as described in various works. The present paper analyses the position of isometries in fractal transformation. The influence of rotation angle of blocks to the approximation of image ranges is examined. The conclusion shows under which conditions the use of square isometries results in a better image coding performance.

Übersicht

Bei den blockbasierten Verfahren der fraktalen Transformation wird der Domain-Pool häufig durch die 8 Isometrien der quadratischen Bildblöcke erweitert. In der Literatur finden sich vielfache Hinweise darauf, daß dadurch keine Leistungssteigerung der fraktalen Coder erreichbar ist. In der vorliegenden Arbeit wird der Anteil der Isometrien an der fraktalen Transformation analysiert. Der Einfluß des Rotationswinkels auf die Approximation von Bildblöcken wird untersucht. Die abschließende Analyse zeigt, unter welchen Bedingungen die Nutzung von Isometrien zu einer Leistungssteigerung führen kann.

For documentation:

Fractal Transformation / Square Isometries / Image Compression

1. Introduction

Most of the developed methods of fractal transformation of digital images are based on the research of Jacquin in 1989 [3]. An image is divided into square parts, the so-called range blocks. Range blocks cover the whole image without overlapping each other. Further square blocks of the image, the domain blocks, will be summarized in the domain pool. Domain blocks have to be greater than range blocks, in order to satisfy the condition of contractivity. Domains can overlap each other and they had not to cover the whole image.

One range block can be approximated by scaling a domain block followed by a luminance transformation. The quality of the approximation will be determined with the help of a measure of distance, e.g. the MSE[1]. The fractal transformation defines the domain of every range block of the image, which produces the best approximation for this range block relating to the choosen measure. In practice the domain pool will be expanded through 8 square isometries of every domain block in order to improve the reachable quality of approximation.

Saupe [4] represents selected studies about the influence of isometries on the quality of fractal codecs. Afterwards an empirically comparision of the efficiency of domain pools with same size with or without using isometric blocks follows. The presented results show that the using of isometries doesn't increase the efficiency of codecs. This is valid for range blocks of constant size as well as for using quadtree-based defined range blocks. A more substantical analysis isn't made in [4].

This paper tries to give reasons for the described effect. The part of isometries in fractal transformation will be described in section 2. Afterwards the approximation of image blocks will be investigated in dependence on the angles of rotation. The interpretation of results follows in section 4.

2. Part of isometries in fractal transformation

Fractal transformation is based on self similarity of image elements. An image is described by a set of N functions w_i:

$$W = \bigcup_{i=1}^{N} w_i \qquad (1)$$

Each function w_i which contains an affine geometric and a luminance transformation, converts one part of the image into a similar part of this image at another position. The geometrical part contains scaling, trans-

[1] mean square error

lation and rotation.

The theory allows no conclusion about the assignment of image parts. For that reason a search for similar parts, which are convertable with high quality by a tranformation described above, is necessary in fractal transformation. In practice, this search in an theoretical infinite set of functions needs restrictions. This means for the geometric transformation in the method of Jacquin:

1. only to investigate square blocks
2. fixed sizes of range and domain blocks results in constant scaling
3. only rotations with multiples of 90 degrees

The rotations with multiples of 90 degrees together with negative scaling (corresponding to reflections) represent the 8 isometries of a square. It's to investigate, whether the restriction of the rotation allowes a block approximation with the best quality or not.

3. Influence of rotation to the approximation of image blocks

Two square blocks of same size are assumed and arranged one above the other and rotated against their common center point. The CCF[2] and AMDF[3] along the largest common circumference and MSE over this circle will be recorded. In order to get whole center points, blocks with odd size are used only.

The first study compares rotation between two identical blocks (ideal case) and between two different blocks. All calculations are executed with and without luminance transformation. Results for images with size of 255 x 255 pixels are presented in Figure 1 .

All characteristic curves show the same qualitative dependence on rotation angle. The similarity for identical blocks is detectable for rotation angles within a sector of only ±15 degrees around the optimal angle. Outside of this bounds all curves show about the same characterization. For that reason detection of identity or similarity are impossible.

The dynamic range of the gray-values inside the blocks are modified by using luminance transformation. Modifications of internal dependencies do not proceed. Consistent a variance reduction of the curves is the result of luminance transformation (Figure 1). The qualitative behaviour does not change.

Using MSE for following studies is sufficient, because of the same statement of all characteristic curves. All studies are executed with luminance transformation additionaly.

[2]Cross Correlation Function

[3]Average Magnitude Difference Function

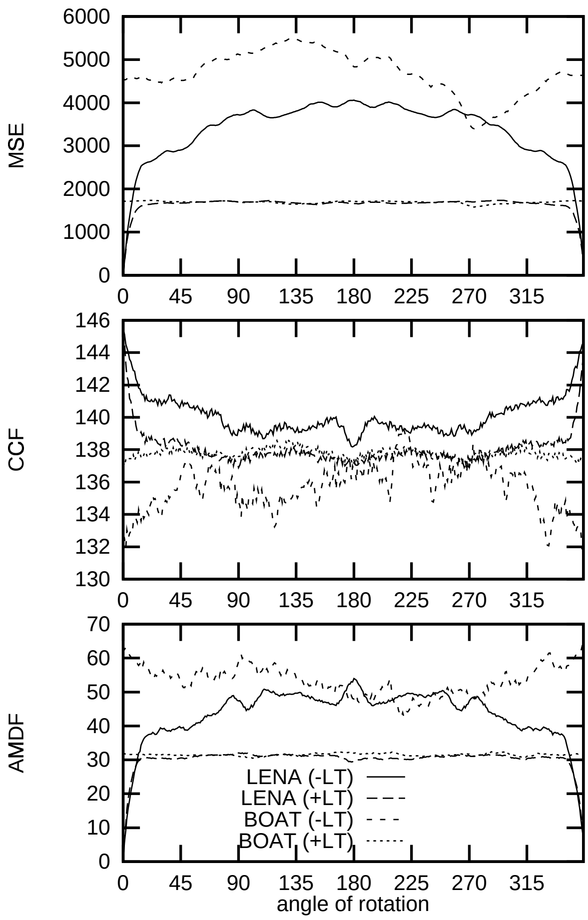

Figure 1: MSE, CCF and AMDF between original image *LENA* and a rotated image above (*LENA* und *BOAT*) with (+LT) and without (-LT) luminance transformation

In the next study different modifications of the first block are used as the second one. Following operations are used to create the second block from the first one:

1. Minimum-Operator (size 3x3)
2. Median-Operator (size 3x3)
3. Maximum-Operator (size 3x3)
4. Gaussian-Lowpass (size 7x7, $\sigma = 3.0$)

Figure 2 presents results for angles between 0 and 60 degrees.

Morphological operators modify the internal structure of images. The optimal angle is still recognicable, but MSE increases and approximation quality decreases. The use of lowpass filter results in small increase of MSE within the sector around the optimal angle only.

The third study uses an assigned pair of blocks (range and domain) from the classical fractal transformation of image *Lena* (Figure 3). Block size is set to 5 x 5 pixels.

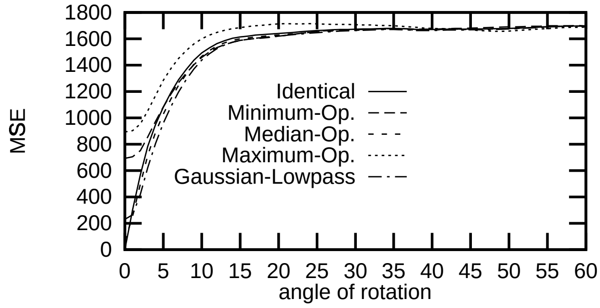

Figure 2: Dependence of MSE and rotation angle for different versions of the second block (image *LENA*)

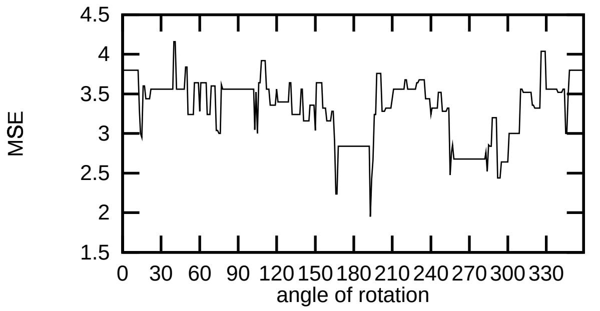

Figure 3: Dependence of MSE and rotation angle for an classical assigned pair of range and domain (image *LENA*)

Figure 3 presents the curve of MSE. Because of small block size and approximation to whole coordinates in the rotation process, many angles result in same values. By using isometries, instead of all angles, values of 0, 90, 180 and 270 degrees are available only. It's recognizable, that the minimal MSE can't be decided in this case.

4. Interpretation

An internal similarity between range and domain under a defined angle is an important condition for an approximation with best quality. Using square isometric blocks leads to a detection of optimal angles in only a small and random number of cases. A use of the rotation part at the search for similar blocks is sensible for steps of angles smaller than ±15 degrees, in principle. Such a search needs higher costs (computational performance, additional coding of angles). Whether the results of such expensive search justify this costs or not, must be examined in a further study.

5. Conclusion

This investigation shows, why the performance of usual fractal codecs does not increase by additional use of square isometric blocks in domain-pool.

A sufficient detection of the optimal angle between blocks isn't reachable with isometric blocks. An originally larger domain pool instead of enlarging it with square isometric blocks is recommended.

6. References

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