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Partitioning Complexity Issue for Iterated Functions Systems based Image Coding

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Abstract. This paper copes with the issue of adaptive partitioning in the context of Iterated Functions Systems (IFS) image coding. The main concern is to provide some insights on the optimum degree of partition adaptivity. The key point is to point out the proper balance of information between the partition representation and the transformation parameters of the system. Several systems involving different degrees of adaptivity will be compared. To that end a polygonal-based segmentation will be presented. This will allow to compare performances of partitioning using respectively square, rectangular and polygonal segments.

1. Introduction

The concept of *fractal* has been introduced by Mandelbrot in the 1960's as an alternative to the classical Euclidean geometry mainly for describing shapes generated by Nature. Since then, this theory attracts the interest of many researchers in fields ranging from biology to physics through computer imaging and image processing. Iterated contractive functions systems (IFS) have shown their capability to generate fractal structures. The efficiency of such systems for representing fractals is explained by their self-similar appearance. Fractals can display very intricate structures but they are characterized by statistical or deterministic similarities across scales.

Application of the IFS theory to image compression purposes has been first suggested by Barsnley [1] for representing fractal-like images. However, natural images generally do not display self-similarity but rather *piecewise* self-similarity. Jacquin [2] put forward that observation and proposed an automated scheme based on partitioned iterated functions system. Since then, numerous techniques presenting various improvements and complexity reduction have been proposed [3] [4] [5]. Some applications to image sequence compression purposes have also been presented [6] [7].

In the context of image coding based on the IFS theory, partitioning is an important issue [4] [5]. This paper mainly focuses on the relevance of high degree of adaptivity for partitioning the image support. To that end, an empirical approach is considered in which performances of schemes involving different degree of segmentation complexity are compared.

The paper is organized as follows. Section 2 briefly reviews the theoretical foundations of IFS applied to picture compression. Section 3 states the problem and describes different approaches to adaptive partitioning. Section 4 compares performances of the different schemes by means of simulations results while section 5 draws conclusions.

2. Theoretical Foundations

The principle of the method described in this paper relies on the mathematical theory of iterated contractive transformations systems. A transformation $\tau : \mathcal{M} \rightarrow \mathcal{M}$ on a metric space $(\mathcal{M}, d)$ is said to be contractive if there exists a constant $0 \leq s < 1$ such that:

$$d(\tau(\mu), \tau(\nu)) \leq s d(\mu, \nu) \quad \forall \mu, \nu \in \mathcal{M}. \qquad (1)$$

The factor s is called the *contractivity factor* of the transformation τ. Operating a contractive transformation on the whole space $\mathcal{M}$ results in a shrunk version (with respect to d) of $\mathcal{M}$.

A fundamental property of contractive transformations in a *complete* metric space is provided by the *Banach's fixed point theorem*. This theorem states that iterative application of a contractive transformation τ to *any* initial set assures convergence to the unique fixed point of τ.

In the context of signal coding, we are facing the *inverse* problem i.e. finding the contractive transformation whose attractor is close to a given set. A clue to solving this problem is given by a corollary to the Banach's theorem known as the *Collage theorem* [2]:

Let $(\mathcal{M},\ d)$ denotes a complete metric space where d represents a given distance and let μ_{orig} be the set to approximate. If the transformation τ from $\mathcal{M}$ to $\mathcal{M}$ satisfies the following requirements:

- $\exists s < 1 : \forall \mu, \nu \in \mathcal{M}, d(\tau(\mu), \tau(\nu)) \leq s d(\mu, \nu)$
- $\exists \varepsilon : d(\mu_{orig}, \tau(\mu_{orig})) \leq \varepsilon$

then for any set $\mu_0 \in \mathcal{M}$ and for any positive integer n:

$$d(\mu_{orig}, \tau^n(\mu_0)) \leq \frac{\varepsilon}{1-s} + s^n d(\mu_{orig}, \mu_0) \quad (2)$$

$\tau(\mu_{orig})$ is termed the *collage* of μ_{orig} with respect to τ. The Collage theorem tells us that we need to find a contractive transformation under which the given set is an approximate fixed point. Then, according to inequality (2), it can be seen that applying the transformation iteratively to any initial set μ_0 causes convergence to an attractor close to the target set. Therefore the transformation τ fully describes the attractor approximating the original set.

In the context of image coding, $(\mathcal{M}, d)$ denotes the complete metric space of digital images where d represents the traditional mean-square-error. The self-similarity encountered in natural images differs from the one present in fractal objects. Instead of having an image formed by copies of its whole self, one has an image formed by copies of properly transformed *parts* of itself. This form of redundancy is termed *piecewise* self-similarity. Therefore the class of contractive transformations considered for picture coding applications is defined blockwise. If $\{R_i, 0 \leq i < N\}$ denotes a non overlapping partition of the image support into N *range cells*, the transformation τ is defined by:

$$\begin{aligned} \forall \mu \in \mathcal{M}, \tau(\mu) &= \sum_{i=0}^{N-1} \tau(\mu)_{|R_i} \\ &= \sum_{i=0}^{N-1} \tau_i(\mu_{|D_i}) \; with \; \tau_i : D_i \rightarrow R_i \end{aligned}$$

where $\mu_{|R_i}$ denotes the restriction of the image μ to the cell R_i. D_i is called a *domain cell* and denotes a block belonging to the image but not necessarily to the partition.

In that context, the *Collage* theorem states that the function τ defined by the set of contractive transformations $\{\tau_i, 0 \leq i < N\}$ under which the original image is an approximate fixed point possesses an attractor close to this image. Therefore the encoding process implies to find for each *range cell* R_i of the N blocks partition the contractive transformation τ_i leading to the best matching in a mean-square-error sense[1].

[1] It must be noted that the specification of the position of the domain cell D_i is included in the description of τ_i.

Based on the work of Ramamurthi and Gersho [8], range and domain blocks are classified into three categories of cells named *shade*, *midrange* and *edge*. The motivation of such a classification is to reduce the exhaustive search among domain blocks D_i of the same class as R_i and to vary the complexity of the transformations τ_i according to the block statistics. Shade range blocks are approximated by absorption to their average value. Midrange blocks are coded through a contractive transformation involving a contrast scaling α and a brightness shift Δg. Edge blocks are represented trough a transformation involving a contrast scaling α, a brightness shift Δg and 8 possible isometries. For further details, the reader is referred to [2].

3. Partition adaptivity issue

Since natural images display non-stationarity, adaptivity of the partitioning is required in order to reach good performances. It yields an image dependent partition which allows to vary the size and/or the shape of the blocks according to the local statistics of the signal.

The question which remains unsolved is to point out the proper degree of partition adaptivity. In other terms, is it worth building partitions displaying higher and higher degree of adaptivity ? Since the encoding data consists in the partition representation and a transformation for each block of the partition, the key point is to determine the optimum balance of information between these two sets of data. Consider the case of a fixed partition, no segmentation information is necessary while numerous transformations parameters need to be transmitted. On the other hand, in the case of a segmentation with cells of arbitrary shape, a large amount of bit is necessary to specify of the partition while very few transformations will be required. None of those solutions is optimal such that we perceive that a trade-off must exist.

To cope with that issue, only a empirical approach is possible. We compare performances of systems involving different degrees of adaptivity, namely segmentation with square, rectangular and polygonal cells. In the following, we briefly describe the three approaches. The partitioning with rectangular cells will be considered as a special case of polygonal segmentation.

3.1 Quadtree partitioning

In the field of fractal image coding, a widely used technique for image support partitioning is the quadtree segmentation [4] [5]. Starting from a coarse grid, cells are recursively split into four equally-sized subblocks until a minimal predefined image quality is reached. Quadtree partitioning is attractive for its simplicity and for the low cost of its representation. It requires 1 bit per node for specifying whether the node is terminal or not. However it suffers from the inflexibility of regular decomposition since it allows only adaptation in size to the local statistics.

3.2 Polygonal Segmentation

In order to remove the constraint of fixed shape cell imposed by the quadtree partition, we introduce a polygonal segmentation. This partitioning enables one to vary the size of the blocks as well as their shape (within a certain range). The segmentation technique relies on the work of Wu and Yao [9] in the context of piecewise constant approximation of images.

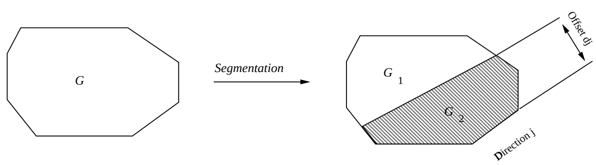

Figure 1. Optimum partitioning

The partitioning is produced recursively from top to bottom starting from a coarse grid and follows a binary tree structure. The root of the tree represents the entire image whereas the leaves symbolize the partition blocks. The decision to split a block is based on a maximum mean-square-error allowance of its approximation by $\tau_i(D_i)$. The split can be performed along four different directions (0, 45, 90 and 135 degree) and and at any location within the block G. This procedure is illustrated in figure 1. Cutting direction j as well as cutting offset d_j of the partition are determined in order to maximize the uniformity within each block G_1 and G_2. Formally expressed, we minimize:

$$\sum_{i=1}^{2} \sum_{(x,y)\in G_i} (g(x,y) - \mu(G_i))^2 \qquad (3)$$

where g(x,y) represents the intensity function and $\mu(G_i)$ the average intensity of segment G_i.

The choice of such a segmentation technique rests on two major reasons. Since the split is restricted to 4-way-cuts, the cell will only have the shape of *canonical* polygons possessing interesting properties such as convexity and special edge orientation allowing a simple shape description. This point makes this segmentation technique tractable for the manipulation of segments in the context of IFS image coding. Moreover the segmentation representation cost remains weak. Indeed, the partition code has only two fields per node i.e. the cutting direction j and the cutting offset d_j.

The motivation for using a uniformity criterion is twofold. First the partitioning will maximize the amount of blocks classified as shade. The fact that this category of blocks requires only a few bits to be represented by their average value allows to reduce the total amount of bits. Moreover coding techniques based on memoryless blockwise partitioning induces highly annoying staircase effects along edges. The proposed partition, beeing adaptive to image semantics since it aligns range cells with image edges, yields better edge rendering.

In the context of IFS image coding, the uniformity criterion is not optimal. The goal of adaptive partitioning is to provide smaller cells around complex areas while allowing larger ones for simple regions. However, the term *complexity* must be compounded with the model used, namely the assumption of piecewise self-similarity. It is acknowledged that, within this model framework, no tractable criterion for evaluating whether a region can be properly represented by this model is available. The only alternative would be the exhaustive "try-and-decide" approach but this solution is not feasible.

4. Simulation results

Simulations have been carried out on Lena (256×256, 8 bit/pel) at different bitrates and for the three approaches. Partitioning with rectangular cells has been obtained by restricting the cutting direction of the polygonal segmentation to 0 and 90 degree. Decoding requires the knowledge of the transformation class and parameters for each partition block along with the partition representation. The partition representation counts approximatively for 7%, 20% and 25% of the total bitrate respectively for the segmentation with square, rectangular and polygonal cells. In order to take into account the non uniform probability distribution of the transform parameters after quantization, an arithmetic coder of order zero has been used.

Figure 2 shows the rate-distortion behavior for IFS-based image coders using three different levels of segmentation complexity. According to this graph, the lower the partition complexity, the better the rate-distortion behavior. However the visual quality follows the inverse trend as illustrated in figures 3 and 4.

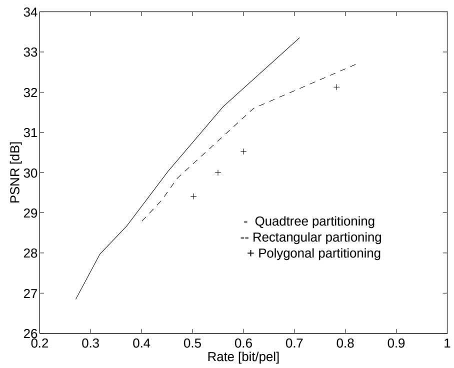

Figure 2. Rate-Distortion behavior for three levels of partitioning complexity

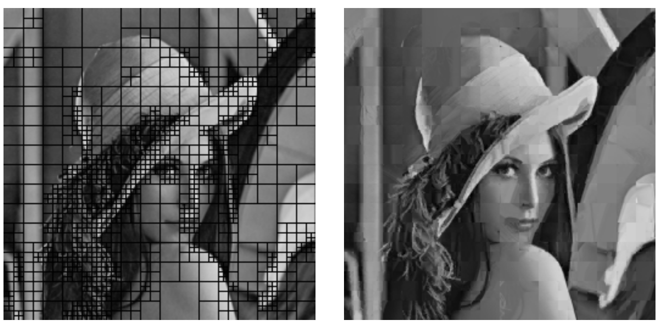

Figure 3. Lena 256×256 (a) Quadtree partition (b) Decoded image at 0.50 bit/pel (30.03 dB).

5. Conclusions

This paper reports investigations on the partitioning issue in the context of IFS applied to still image compression. It focus on the relevance of high degree of adaptivity of partitioning procedures. We compared performances of schemes involving different segmentations namely partitioning with square, rectangular and polygonal cells.

It turns out that the rate-distortion behavior decreases in performance with increasing partition complexity. In other terms, the gain in the number of transformations is lost due to the increase of the partition representation cost. The main reason of such a behavior is the absence of a tractable criterion for segmenting the image support within the model of piecewise self-similarity. However the visual quality is improved when higher adaptivity of the partitioning is considered even when using a maximal uniformity criterion.

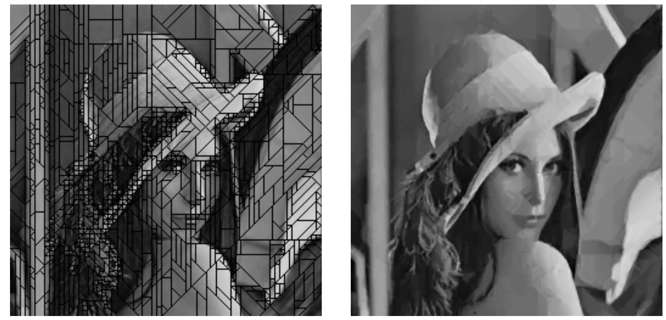

Figure 4. Lena 256×256 (a) Polygonal partition (b) Decoded image at 0.50 bit/pel (29.40 dB).

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