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OVERLAPPED ADAPTIVE PARTITIONING FOR IMAGE CODING BASED ON THE THEORY OF ITERATED FUNCTIONS SYSTEMS

Emmanuel Reusens

Signal Processing Laboratory
Swiss Federal Institute of Technology
CH-1015 Lausanne, Switzerland

ABSTRACT

Memoryless blockwise partitioning induces blockiness artifacts highly disturbing to the human visual system. This paper presents a new partitioning with overlapped blocks in the context of image coding based on iterated transformations systems. Each block of the partition is extended by n pixels. As usual each cell is expressed as the contractive transformation of another part of the image. During the decoding, values of pixels corresponding to overlapped regions are computed as the weighted sum of the different contributions leading to that pixel. This overlapped partitioning is embedded in a quadtree segmentation of the image support. In order to avoid blurring effects in small blocks while maintaining efficiency in bigger ones, the overlapping width n is function of the block size. Simulations show a very significant improvement of the visual quality of decoded images with no increase of the bitrate request.

1. INTRODUCTION

The concept of *fractal* has been introduced by Mandelbrot [1] in the 1960's as an alternative to the traditional Euclidean geometry mainly for dealing with shapes generated by Nature. Since then, this theory attracts the interest of the scientific community in many fields of applications. Iterated contractive functions systems have shown their capability to generate fractal structures. The efficiency of such systems for representing fractals is explained by the self-similar appearance of these objects. Fractals, which can display very intricate structures, are characterized by statistical or deterministic similarities across scales.

From an image processing point of view, it turns out that some form of self-similarity does exist in many natural images. Such an observation induces naturally to use the theory of fractal for image coding purposes. First methods based on this idea have been presented by Barnsley [2] and Jacquin[3]. Since then, several schemes for still image compression have been developed [4] [5]. All the proposed techniques rest on a memoryless blockwise partitioning of the image support introducing highly annoying blocking artifacts. This paper proposes to overcome these visual distortions by using an *overlapped* block partitioning of the image support.

2. THEORETICAL FOUNDATIONS

The principle of the technique described in this paper relies on the mathematical theory of Iterated Contractive Transformations Systems. A transformation τ on a metric space $(\mathcal{M},d)$ is called contractive if there is a constant $0 \leq s < 1$ such that

$$d(\tau(\mu), \tau(\nu)) \leq s d(\mu, \nu) \quad \forall \mu, \nu \in \mathcal{M}. \tag{1}$$

The factor s is called the *contractivity factor* of the transformation τ. A fundamental property of contractive transformations in a complete metric space is that iterative application to *any* initial subset of $\mathcal{M}$ assures convergence to a unique attractor.

In the context of image coding, we are facing the *inverse* problem i.e. finding the contractive transformation whose fixed point is close to a given set. A clue to solving this problem is provided by the *Collage* theorem [2]:

Let $(\mathcal{M},d)$ denotes a complete metric space where d represents a given distance measure and let μ_{orig} be the set to approximate. If the transformation τ from $\mathcal{M}$ to $\mathcal{M}$ satisfies the following requirements:

- $\exists s < 1 : \forall \mu, \nu \in \mathcal{M}, d(\tau(\mu), \tau(\nu)) \leq s d(\mu, \nu)$
- $\exists \varepsilon : d(\mu_{orig}, \tau(\mu_{orig})) \leq \varepsilon$

then for any set $\mu_0 \in \mathcal{M}$ and for any positive integer n :

$$d(\mu_{orig}, \tau^n(\mu_0)) \leq \frac{\varepsilon}{1-s} + s^n d(\mu_{orig}, \mu_0). \tag{2}$$

This theorem tells us that we need to find a contractive transformation under which the given set is an approximate fixed point. Then, according to inequality (2), it can be seen that applying the transformation τ iteratively to *any* initial set μ_0 causes convergence to an attractor clustered around the given set μ_{orig}.

In the context of image coding, $(\mathcal{M},d)$ denotes the complete metric space of digital images where d represents the traditional mean-square-error. The self-similarity encountered in natural images differs from the one present in fractal objects. Instead of having an image formed by copies of its whole self, one has an image formed by copies of properly transformed *parts* of itself. Therefore the class of contractive transformations considered for exploiting this *piecewise* self-similarity is defined blockwise. If $\{R_i, 0 \leq i < N\}$ denotes a nonoverlapping partition of the image support into N *range cells*, the transformation τ is defined by:

$$\begin{aligned} \forall \mu \in \mathcal{M}, \tau(\mu) &= \sum_{i=0}^{N-1} \tau(\mu)_{|R_i} \\ &= \sum_{i=0}^{N-1} \tau_i(\mu_{|D_i}) \textit{ with } \tau_i : D_i \rightarrow R_i \end{aligned}$$

where $\mu_{|R_i}$ denotes the restriction of the image μ to the cell R_i. D_i is called a *domain cell* and denotes a block belonging to the image but not necessarily to the partition.

In that context, the *Collage* theorem states that the function τ defined by the set of contractive transformations $\{\tau_i, 0 \leq i < N\}$ under which the original image is an approximate fixed point possesses an attractor close to this image. Therefore the encoding process implies to find for each *range cell* R_i of the N blocks partition the contractive transformation τ_i leading to the best matching in a mean-square-error sense[1]. For further details, the reader is referred to [2] [3].

3. ADAPTIVE OVERLAPPED BLOCK PARTITIONING

The motivation for using an adaptive partitioning in the context of blockwise image coding is twofold. Such a partition is image-dependent and allows to use large blocks for nearly uniform areas and smaller ones to reproduce details in more intricate regions. Moreover, flexibility in term of bitrate is obtained. Any budget constraint or picture quality can be reached by controlling the partition accuracy.

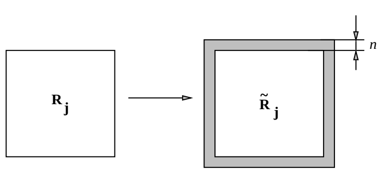

Figure 1. Block extension

The partitioning is produced recursively from top to bottom starting from a coarse grid. The root of the tree represents the entire image while the leaves symbolize the partition blocks. At each node, the decision to split a block into four equally-sized blocks is based on a maximum mean-square-error allowance. This partitioning is attractive because of its simplicity and its low cost representation.

In basic schemes of fractal image coding [3] [4] [5], the memoryless blockwise partitioning of the image support induces blockiness artifacts. Such artifacts are very annoying for the human visual system due to their highly structured appearance. These distortions are mainly located near sharp edges under the form of a staircase effect and artifacts taking the form of sharp transitions between adjacent blocks will be visible in smoothly varying areas. In order to overcome these artifacts , we propose to use an *overlapped* block partitioning. Let $\{\tilde{R}_i\}_{0<i\leq N}$ denotes the overlapped partitioning defined by:

$$\begin{aligned} \bigcup_i & \quad \tilde{R}_i = I, \\ \tilde{R}_i \bigcap & \quad \tilde{R}_j \neq \emptyset \textit{ if } \tilde{R}_i \textit{ and } \tilde{R}_j \textit{ neighbors.} \\ \tilde{R}_i \bigcap & \quad \tilde{R}_j = \emptyset \textit{ otherwise.} \end{aligned}$$

where I denotes the image support.

The encoding procedure is much similar to the one corresponding to non-overlapped partitioning. Along the quadtree partitioning, each block R_j corresponding to a node is extended by $2n$ pixels in each direction producing the block $\tilde{R}_j$ as described by figure 1. Then the search of the contractive transformation achieving the best approximation of $\tilde{R}_j$ is performed. If this approximation does not respect the maximum mean-square error allowance, the block R_j is split into four subblocks and the operation is repeated on the four subblocks.

During the decoding procedure, the transformations computed during the encoding procedure are applied iteratively on an arbitrary initial image. The values of pixels corresponding to overlapped regions are computed as the weighted sum of the outputs of the different adjacent blocks leading to these pixels. This procedure is illustrated in figure 2. In order to avoid blurring effects and loss of details in small blocks while maintaining efficiency in bigger ones, the overlapping width n is function of the block size.

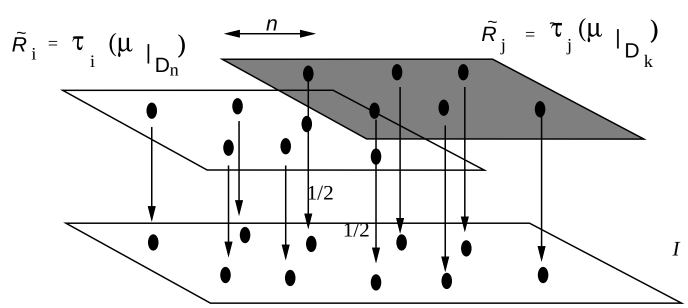

Figure 2. Combination of overlapped pixels for the decoding procedure (I=image support, n=overlapping width)

The scheme described above fulfills the requirements of the Collage theorem. The demonstration trick consist in viewing the overlapped partition as non-overlapped partition where some regions result from one contractive transformation while some others result from the contribution of 2 or 4 transformations. From this point of view we simply need to prove that the weighted sum of contractive transformations is still a contractive transformation. The demonstration is given here, without loss of generality,

[1] It must be noted that the specification of the position of the domain cell D_i is included in the description of τ_i.

for the case of the combination of two transformations and equal weighting factors:

Proposition 1 *Let $\mu, \nu \in (\mathcal{M}, d)$ and τ_1, τ_2 be contractive transformations in $(\mathcal{M}, d)$. If $\tau(\mu) = (\tau_1(\mu) + \tau_2(\mu))/2$ then τ is contractive.*

The proof is given in appendix.

Extension to the general case involving the combination of more than two transformations with any weighting factors can be simply derived by repeated application of the above proposition.

4. SIMULATION RESULTS

Simulations have been carried out on Lena (256x256, 8 bit/pel). The encoding data (i.e. the transform parameters) must be mapped into a finite number of codewords in order to generate the bitstream representing the compressed data. The quantization stage is performed during the partitioning scheme since the quality of each block approximation needs to be evaluated after quantization. In order to take into account the non uniform probability distribution of the transform parameters after quantization, an arithmetic coder of order zero has been used.

Figure 4 compares the schemes with overlapped block partitioning with the one using on a memoryless blockwise segmentation. Both have been coded at 0.50 bit/pel for a peak signal-to-noise ratio (PSNR) of 29.1 dB and for the same partition. The quadtree partitioning allows the block size to vary from 32 to 4. The overlapping width n has been set to 1 for blocks of size 32, 16 and 8 and to zero for blocks of size 4. As expected, the scheme with overlapped partitioning does not improve the basic scheme in terms of signal-to-noise ratio but reduces appreciably the visual distortions. Indeed noise around sharp edges completely disappears and thanks to a smoother transition between adjacent blocks, blockiness artifacts are much less disturbing.

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REFERENCES

[1] Mandelbrot B. *The Fractal Geometry of Nature*. Freeman and co, San Francisco, 1982.

[2] M.F. Barnsley. *Fractals Everywhere*. Academic Press Inc., San Diego, 1988.

[3] A.E. Jacquin. Image coding based on a fractal theory of iterated contractive image transformations. *IEEE Transactions on Image Processing*, 1:18–30, January 1992.

[4] Y. Fisher. Fractal image compression. *SIGGRAPH'92 course notes*, 1992.

[5] D. Monro and F. Dudbridge. Fractal block coding of images. *Electronics Letters*, 11:1053–1055, November 1992.

APPENDIX

Proof of proposition 1:

Let $\mu, \nu \in (\mathcal{M}, d)$ and τ_1, τ_2 be contractive transformations in $(\mathcal{M}, d)$ of respective contractivity factor s_1 and s_2. If $\tau(\mu) = (\tau_1(\mu) + \tau_2(\mu))/2$ then

$$d(\tau(\mu), \tau(\nu)) = \sum_i \sum_j \left(\frac{(\tau_1(\mu_{i,j}) + \tau_2(\mu_{i,j})}{2} - \frac{(\tau_1(\nu_{i,j}) + \tau_2(\nu_{i,j}))}{2} \right)^2$$

$$= \tfrac{1}{4}\Big[\sum_i \sum_j (\tau_1(\mu_{i,j}) - \tau_1(\nu_{i,j}))^2 + \sum_i \sum_j (\tau_2(\mu_{i,j}) - \tau_2(\nu_{i,j}))^2 + \sum_i \sum_j 2(\tau_1(\mu_{i,j}) - \tau_1(\nu_{i,j})) \times (\tau_2(\mu_{i,j}) - \tau_2(\nu_{i,j}))\Big]$$

Let define $s = max(s_1, s_2)$. Since τ_1 and τ_2 are contractive, we can write:

$$\sum_i \sum_j (\tau_1(\mu_{i,j}) - \tau_1(\nu_{i,j}))^2 \leq s_1 d(\mu, \nu) \leq s d(\mu, \nu) \quad (3)$$

$$\sum_i \sum_j (\tau_2(\mu_{i,j}) - \tau_2(\nu_{i,j}))^2 \leq s_2 d(\mu, \nu) \leq s d(\mu, \nu) \quad (4)$$

Due to the contractivity of τ_1 and τ_2 for any block size and in particular for a size of 1, we can write that:

$$(\tau_1(\mu_{i,j}) - \tau_1(\nu_{i,j}))^2 \leq s_1(\mu_{i,j} - \nu_{i,j})^2 \leq s(\mu_{i,j} - \nu_{i,j})^2 \quad (5)$$

$$(\tau_2(\mu_{i,j}) - \tau_2(\nu_{i,j}))^2 \leq s_2(\mu_{i,j} - \nu_{i,j})^2 \leq s(\mu_{i,j} - \nu_{i,j})^2 \quad (6)$$

Since we know that:

$$\left.\begin{array}{l} a^2 \leq b^2 \\ c^2 \leq b^2 \end{array}\right\} \Rightarrow ac \leq b^2$$

it implies that

$$(\tau_1(\mu_{i,j}) - \tau_1(\nu_{i,j}))(\tau_2(\mu_{i,j}) - \tau_2(\nu_{i,j})) \leq s(\mu_{i,j} - \nu_{i,j})^2 \quad (7)$$

and thus:

$$\sum_i \sum_j (\tau_1(\mu_{i,j}) - \tau_1(\nu_{i,j}))(\tau_2(\mu_{i,j}) - \tau_2(\nu_{i,j})) \leq s d(\mu, \nu) \quad (8)$$

According to equations (3), (4) and (8), one can deduce that:

$$d(\tau(\mu), \tau(\nu)) \leq s d(\mu, \nu) \quad (9)$$

and therefore τ is contractive.

Figure 3. Comparison of the two schemes (a) Lenna original picture (b) Adaptive Partitioning (c) Decoded image with non overlapping partition (0.50 bit/pel) (c) Decoded image with overlapping partition (0.50 bit/pel)