Fractal approach to low rate video coding

Bernd Hürtgen and Peter Büttgen

Institute for Communication Engineering
Aachen University of Technology
Melatener Str. 23, 52056 Aachen, Germany
Tel.:+49–241–80 7677, Fax:+49–241–8888 196
Email: huertgen@ient.rwth-aachen.de

ABSTRACT

This paper presents a method for fast encoding and decoding of image sequences based on fractal coding theory and the hybrid coding concept. The DPCM-loop accounts for statistical dependencies of natural image sequences in temporal direction. Those regions of the original image where the prediction, i.e. motion estimation and compensatio, fails are encoded using an advanced fractal coding scheme which originally was developed for encoding of still images. Similar to conditional replenishment coders, not regions of the residual image itself but of the original image are encoded.

The introduction of a fractal coding scheme instead of the commonly used DCT turns out advantageous especially at very low bit rates (8–64 kbit/s). In order to increase reconstruction quality, encoding speed and compression ratio, some additional features as *hierarchical codebook search* and *multi level block segmentation* are proposed.

1. INTRODUCTION

With the spread of digital networks during the last years, also the interest in efficient encoding of video data has increased. An H.261–like coding algorithm is expected to be the "near-term" solution (within the next two years) especially for videophone applications on LANs, WANs and mobile networks. However these standards do not meet the requirements for future applications particularly when very low bit rates are the issue. Besides waveform- and model based schemes the 'MPEG Ad-Hoc Group on Very-Low Bitrate Audio-Visual Coding' therefore considers fractal coding techniques as one possible "far-term" solution [1].

In the case of low rate channels a hybrid coding concept which takes the temporal and spatial dependencies into account was found to perform best and therefore has been adopted in most coding proposals. Similar standards e.g. [2, 3] use motion compensated prediction in order to remove temporal redundancy and a discrete cosine transform (DCT) of the prediction error image, though it has been shown that the DCT is not optimal for this purpose. In this paper we emerge from a hybrid concept but replace the commonly used DCT by a fractal based scheme which adopts to the special characteristics of image data. Similar to conditional replenishment coders we apply the fractal transform only to those regions of the original image where prediction failed.

The fractal coding concept originates from Barnsley's idea to exploit self similar structures in real world images for compression purposes [4]. A first practical implementation also capable of encoding grey scale images has been proposed by Jacquin [5, 6]. Several improvements and modifications e.g. [7, 8, 9, 10, 11, 12] have been reported since then, but the basic concept of blockwise approximation of the entire image by parts of itself remained the same.

For a given image the encoding process consists of finding among a class of *a priori* defined contractive transformations one leaving the image nearby invariant. According to *Banach's fixed point theorem* the sequence of reconstructed images converges for any arbitrary initial image to the fixed point of the transformation which is the original image. Compression is achieved if the transformation parameters can be described more compactly than the original image.

Our paper is organized as follows: After presenting the theory for fractal encoding of grey scale images we briefly review the commonly used hybrid coding concept for image sequences. We then evaluate fractal techniques with respect to video coding and point out some possible improvements. Recent simulation results demonstrate the efficiency of the proposed algorithm. We finish with some concluding remarks and a prospect on future investigations.

2. BACKGROUND

2.1. Mathematical foundations

Let $\mathbf{x} = (x_1, x_2, \cdots, x_N)^T = (x_i)^T;\ 1 \leq i \leq N$ be an image of size $N = N_x N_y$ pixels which we consider as point in the N-dimensional euclidean vector space $\mathbb{R}^N$. The components $x_i \in \mathbb{R}$ represent the grey levels of the image. We can measure the "length" of the vector $\mathbf{x}$ by defining the *Euclidean norm*

$$\|\mathbf{x}\| := \sqrt{\sum_{i=1}^{N} |x_i|^2}\,. \tag{1}$$

The space $\mathbb{R}^N$ becomes a *normed metric space* denoted $\left(\mathbb{R}^N, \varrho\right)$ by inducing a *metric*

$$\varrho(\mathbf{x}, \mathbf{y}) := \|\mathbf{x} - \mathbf{y}\|,\ \forall\, \mathbf{x}, \mathbf{y} \in \mathbb{R}^N \tag{2}$$

based on the vector norm. The metric $\varrho(\mathbf{x}, \mathbf{y})$ can be considered as distance measure between the two vectors or images $\mathbf{x}$ and $\mathbf{y}$.

A linear transformation within this space is described by a linear operator $\mathbf{A} : \mathbb{R}^N \rightarrow \mathbb{R}^N$ mapping the space $\mathbb{R}^N$ onto itself. Also for every operator a norm can be defined. The smallest operator norm consistent with the Euclidean vector norm in the sense that $\|\mathbf{A}\mathbf{x}\| \leq \|\mathbf{A}\|\,\|\mathbf{x}\|$ holds, is the *spectral- or Hilbert-norm* defined by

$$\|\mathbf{A}\|_{sp} := \sup_{\lambda \in \sigma(\mathbf{A}^T\mathbf{A})} \sqrt{|\lambda|}\,. \tag{3}$$

$\sigma(\mathbf{A}^T\mathbf{A})$ is called the spectrum of the matrix $\mathbf{A}^T\mathbf{A}$ which is the set of all eigenvalues λ of $\mathbf{A}^T\mathbf{A}$.

Most implementations emerge from a blockwise defined non-linear affine transformation in $\mathbf{x}$

$$W:\ \mathbb{R}^N \rightarrow \mathbb{R}^N \quad \Rightarrow \quad \mathbf{x} \rightarrow \mathbf{A}\mathbf{x}+\mathbf{b} \tag{4}$$

of the entire image $\mathbf{x}$ consisting of a linear part $\mathbf{A}\mathbf{x}$ and an additive part $\mathbf{b}$. The transformation W is called *contractive,* if there exists a constant $s < 1$, so that

$$\varrho\,(W(\mathbf{x}), W(\mathbf{y})) \leq s\ \varrho(\mathbf{x},\mathbf{y})\ \forall\, \mathbf{x},\mathbf{y} \in \mathbb{R}^N \tag{5}$$

holds. From (4), (5) and the definition of the metric (2), we obtain the contractivity condition

$$\|\mathbf{A}\| \leq s < 1, \tag{6}$$

for the affine transformation W which constrains the determination of the matrix $\mathbf{A}$ during the encoding process.

The encoding process of the given image $\mathbf{x}$ now consists in finding a matrix $\mathbf{A}$ and a vector $\mathbf{b}$ such that the approximation error

$$\varrho(W(\mathbf{x}),\mathbf{x}) = \varrho(\mathbf{A}\mathbf{x}+\mathbf{b},\mathbf{x}) \tag{7}$$

becomes as small as possible and the contractivity condition (6) holds.

Decoding is mainly based upon *Banach's fixed point theorem:*.

Let $\mathbb{R}^N$ *be a metric space with metric* ϱ *and* $W:\mathbb{R}^N \rightarrow \mathbb{R}^N$ *a contractive transformation. Then the sequence of images* $\{\mathbf{x}_k\}$ *constructed by* $\mathbf{x}_{k+1} = W(\mathbf{x}_k)$ *converges for any arbitrary initial image* $\mathbf{x}_0 \in \mathbb{R}^N$ *to the unique fixed point*

$$\mathbf{x}_f = W(\mathbf{x}_f) = \mathbf{A}\mathbf{x}_f + \mathbf{b};\ \mathbf{x}_f \in \mathbb{R}^N \tag{8}$$

of the transformation W.

The decoder generates the sequence $(\mathbf{x}_k)$ with

$$\mathbf{x}_k = W^{\circ k}(\mathbf{x}_0) = \mathbf{A}^k\mathbf{x}_0 + \left(\sum_{i=0}^{k}\mathbf{A}^{\mathbf{i}}\right)\mathbf{b} \tag{9}$$

by iteratively applying the transformation W to some arbitrary initial image $\mathbf{x}_0$. Due to *Banach's fixed point theorem*, the sequence $(\mathbf{x}_k)$ converges to the fixed point

$$\lim_{k\rightarrow\infty}\mathbf{x}_k = (\mathbf{I}-\mathbf{A})^{-1}\mathbf{b} = \mathbf{x}_f \tag{10}$$

if the transformation satisfies the contractivity condition (6). An upper bound for the reconstruction error can be formulated with the definition of the fixed point (8) and the encoding instruction (4) by

$$\varrho(\mathbf{x}_f, \mathbf{x}) \leq \frac{1}{1 - \|\mathbf{A}\|} \varrho(W(\mathbf{x}), \mathbf{x}) \ . \tag{11}$$

As has been shown above, the approximation of the original image $\mathbf{x}$ is completely determined by the affine transformation W. So instead of storing or transmitting the original image, the parameters $\mathbf{A}$ and $\mathbf{b}$ of the transformation W are transmitted. Data compression can be achieved, if $\mathbf{A}$ and $\mathbf{b}$ can be stored more compactly (requiring less amount of data) than the image $\mathbf{x}$ itself.

Apart from very simple binary images, e.g. Barnsley's fern, no practical methods for construction of mappings of the entire image onto itself are known. Therefore we follow Jacquin's proposal and partition the image into non-overlapping blocks. For practical reasons only some simple block transformations are allowed in order to find correspondences within the image. As Jacquin pointed out, fractal coding schemes share many properties with vector quantization using image dependent codebooks. Due to the special features of the fractal transformations, the codebook itself needs not to be transmitted, but can be reconstructed iteratively during the decoding process. For further details concerning the fractal coding scheme the reader is referred to Jacquin's detailed original description [6, 13].

2.2. Image sequence coding

Most approaches for encoding of video sequences and almost all standardization activities are based upon the hybrid coding concept since it was found to be most efficient for those applications. This scheme incorporates a temporal DPCM-loop in order to account for the redundancy between successive frames of the sequence (interframe coding). Additionally some sort of transform coding or vector quantization is applied to the residual image in order to remove the spatial redundancy (intraframe coding). The combination of inter- and intraframe coding offers high compression ratio with rather low implementation effort. A further advantage of this scheme is that significant improvements can easily be achieved by incorporating a *motion compensated prediction*. Hereby the movements of single objects are measured in terms of a motion vector. Together with the encoded residual image the motion vectors for all objects are transmitted over the channel. The receiver generates a prediction image by compensating the previous image using the known motion of the objects. The reconstructed image is then obtained by adding the decoded residual image to the prediction image. A block diagram of a typical standard hybrid coding scheme is depicted in figure 1.

Though the DCT is known to be suboptimal for encoding of prediction error images, all standardization proposals are based upon the DCT, mainly because of its simplicity and its available dedicated hardware. However "far-term" solutions for video coding demand for new ideas in order to bridge the gap between available channel capacity and desired picture quality. Therefore we investigated substitution of the DCT by an advanced fractal coding scheme which already has performed well for still image coding applications.

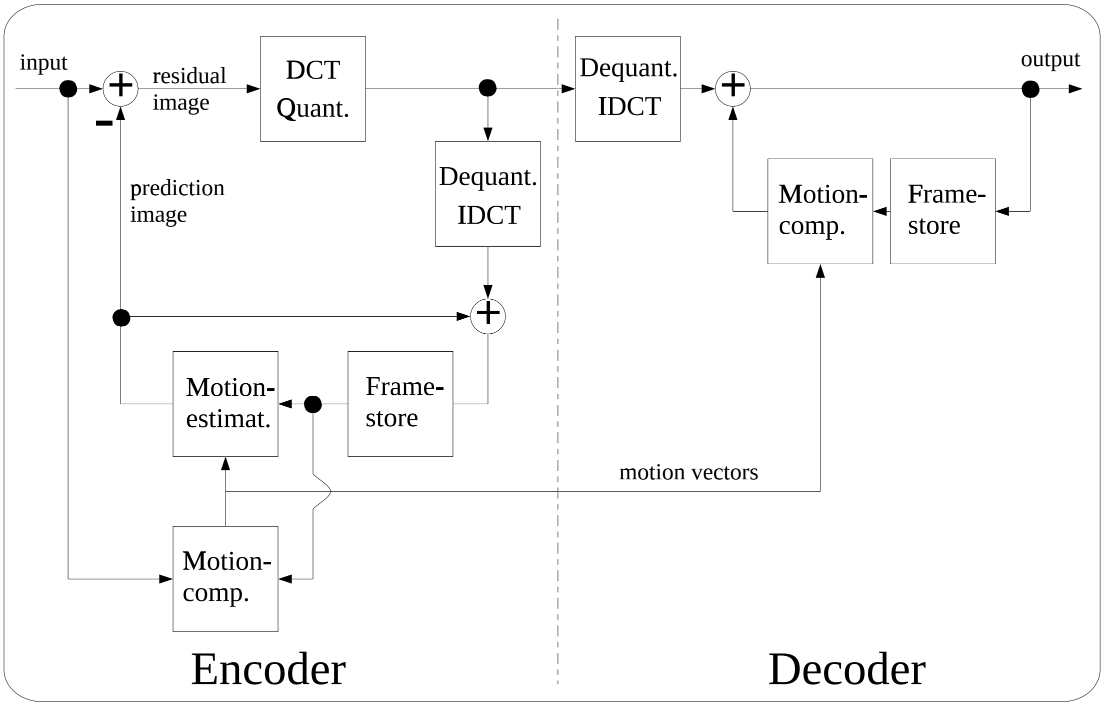

Figure 1: Standard hybrid coding scheme. Left: Encoder with motion estimation and compensation, DCT-coding and quantization of the residual image. Right: Decoder with motion compensation, dequantization and inverse DCT-coding of the residual image.

3. FRACTAL VIDEO CODING

Former investigations performed by others and ourselves indicate that fractal coding schemes are suited for applications with a demand for extremely high compression ratios. Due to the increasing encoding complexity for visual loss-free coding at low compression ratios, e.g. digital HDTV, fractal coding schemes cannot play of its advantages compared with advanced subband or transform coders. For this reason our investigations are directed to low rate video coding with applications in mobile video telephony, teleconferencing and narrow band ISDN distributed audio-visual services which rise from 4.8 to 64 kbit/s. Typical for these applications are head-and-shoulder sequences consisting of a slowly moving foreground and an approximately static background.

Beaumont [14] showed that straight forward extension of the 2–D fractal encoding scheme to three dimensions in order to encode image sequences only results in poor reconstruction quality. We therefore adopt the hybrid coding concept with temporal DPCM-loop as described above to our fractal scheme.

We found out that direct encoding of the prediction error image by a standard fractal scheme does not work satisfactorily due to the special statistical properties of prediction error images. Instead of direct encoding of those regions in the residual image where the prediction failed the corresponding regions of the original image are encoded. For this we employ an advanced fractal coding scheme suited to still images. A block diagram of the modified encoder is depicted in figure 2.

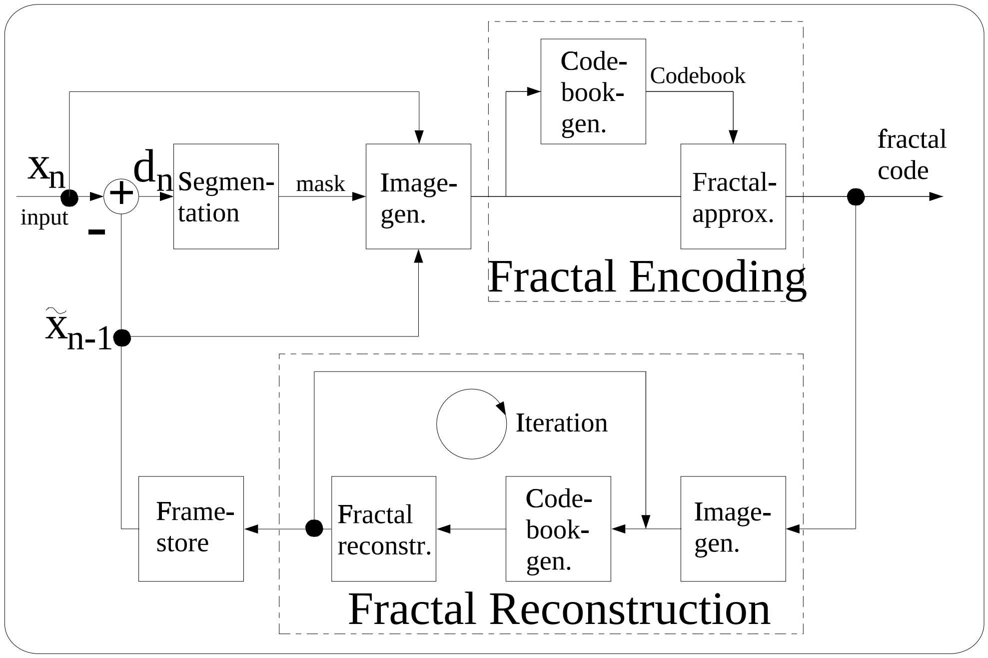

Figure 2: Block diagram of a hybrid coding scheme with fractal encoding of those parts of the original image where prediction failed.

The encoding procedure works as follows: The old reconstructed frame $\tilde{\mathbf{x}}_{n-1}$ is subtracted from the new input frame denoted by $\mathbf{x}_n$. The resulting residual image $\mathbf{d}_n = \mathbf{x}_n - \tilde{\mathbf{x}}_{n-1}$ is used to create a mask determining those areas of prediction failure. By use of this mask the original input image is segmented into two parts referred to as foreground and background. The error power in the residual image corresponding to the background is only determined by camera noise and contains all those areas of the image for where prediction worked satisfactorily. The rest of the image is unpredictable foreground. Since the background is approximately static, it is already known to the receiver and therefore needs not to be transmitted. So only the foreground region of the image has to be encoded and transmitted. In order to exclude the influence of the various motion estimation schemes for prediction, we employ a simple frame store as predictor though we believe that proper motion estimation can improve the encoding results significantly.

In contrast to [15] where only the foreground regions contribute to the fractal codebook we use the entire image (consisting of foreground and background). The authors of [15] argue that when additionally considering the background for codebook construction the scheme will fall out of the domain of fractal coding. We do not agree on this, because in our scheme there are still many codebook entries taken solely from the foreground image so that the substantial characteristics of fractal coding are preserved. Besides this we explored that the partial construction of the codebook from the background image increases the flexibility of the encoding scheme and even found it necessary when the foreground is small (this is if prediction worked almost satisfactorily).

Multi level block segmentation

The main intention of the proposed segmentation algorithm is to divide the prediction error image in one area of high error power (foreground) and one of small error power mainly caused by camera noise (background). We perform the segmentation before and independently from the encoding of the image so that segmentation and encoding are decoupled. By this way the processing speed can be increased significantly, because only those blocks have to be regarded which are chosen by the segmentation.

Due to the fact that channel capacity and the overhead information for the segmentation is nearly constant, the remaining data rate for encoding of the image regions is also constant. As has been mentioned above, the employed fractal coding scheme is block oriented. This means for each block a transformation independent from the other blocks is determined. If fixed word length coding is applied to the transform parameters of the image blocks, each block can be represented by the same amount of data. So the number of blocks which can be encoded of each frame is constant. The input parameters of the segmentation algorithm are therefore the number of blocks to be encoded and the prediction error image. In order to keep the segmentation overhead feasible we employ a quadtree structure. The blocks have variable size 4x4, 8x8, and 16x16 and are alined in a predefined block raster. An example of a typical quadtree segmentation is shown in figure 3.

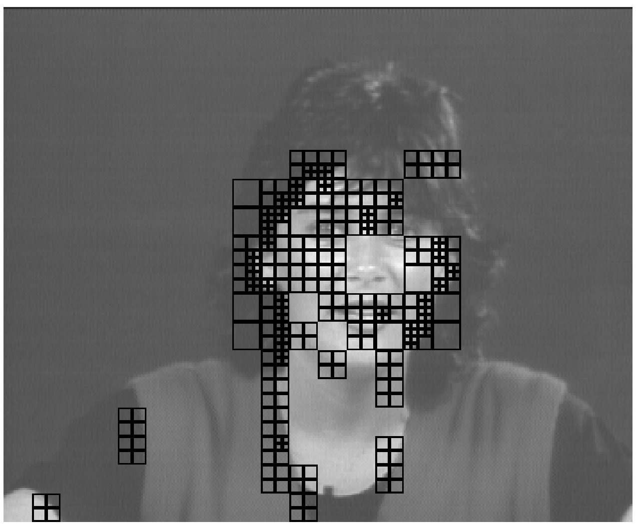

Figure 3: Quadtree segmentation of the foreground image

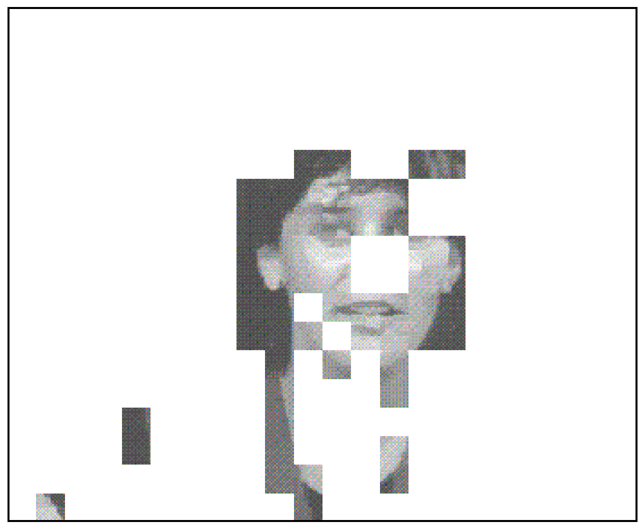

Figure 4: Fractal encoded foreground regions

Since segmentation is performed independently from the encoding process, one has to consider the special characteristics of the fractal coding scheme by incorporating a priori knowledge into the segmentation algorithm. In our case this is done by employing a gain/cost criterion which can be viewed as trade off between the costs for encoding an image block and the gain due to the reduced residual error. The aim of the segmentation is then to find the foreground mask maximizing the total gain/cost relation. Since the costs for encoding are the same for every block only the gain which is the reduction of the error power due to coding of this block has to be determined.

A priori knowledge is incorporated by estimating the reconstruction error for each block. We found out, that for fractal schemes the reconstruction error of the block is highly correlated with the block size. Therefore we estimate the reconstruction error of a block solely from the corresponding block size. Some improvements can be achieved if in addition to the block size also the grey level distribution is regarded. This can be done by calculating an activity measure for each block and determining an estimate of the reconstruction error from this activity measure.

The segmentation process can be described as follows:
Let $\mathbf{x} \in \mathbb{R}^N$; $\mathbf{x} = (x_i)^T$; $x_i \in \mathbb{R}$ be an image consisting of N pixels and $\mathbf{b}^{(j)}(\mathbf{x}) = (x_{j_k})^T$; $x_{j_k} \in \mathbb{R}$; $j_k \in \mathbb{Z}^{(j)}$ be the j-th block within the image $\mathbf{x}$. $\mathbb{Z}^{(j)}$ denotes the set of all indices of elements belonging to the j-th block. The index k runs from 1 to M with $M = M_{bx}M_{by} < N = N_x N_y$ being the number of elements within the block. Using the *Euclidean norm* as defined in eq. (1) the power of an image $P(\mathbf{x})$ or image block $P\left(\mathbf{b}^{(j)}(\mathbf{x})\right)$ can be written as:

$$\begin{aligned} P(\mathbf{x}) &= \sum_{i=1}^{N} |x_i|^2 = \|\mathbf{x}\|^2 \\ P\left(\mathbf{b}^{(j)}(\mathbf{x})\right) &= \sum_{j_k \in \mathbb{Z}^{(j)}} |x_{j_k}|^2 = \left\|\mathbf{b}^{(j)}(\mathbf{x})\right\|^2 . \end{aligned} \tag{12}$$

The segmentation emerges from the prediction error image $\mathbf{d}_n = \mathbf{x}_n - \tilde{\mathbf{x}}_{n-1}$ which is obtained within the DPCM-loop by subtracting the prediction image $\tilde{\mathbf{x}}_{n-1}$ from the original image $\mathbf{x}_n$. Additionally the number of blocks N_B which can be encoded is determined by the total data rate reduced by the segmentation overhead information.

In a first step for each possible block $\mathbf{b}^{(j)}(\mathbf{d}_n)$ within the prediction error image, the power $P_d\left(\mathbf{b}^{(j)}(\mathbf{d}_n)\right)$ is determined according to eq. (12). Then the reconstruction error $P_c\left(\mathbf{b}^{(j)}(\mathbf{x}_n)\right)$ after encoding of the block in the original image $\mathbf{x}_n$ is estimated by

$$P_c\left(\mathbf{b}^{(j)}(\mathbf{x}_n)\right) = p_1 M^{p_2} \tag{13}$$

and only depends upon the corresponding block size M. The parameters p_1 and p_2 are determined in a way that reconstruction quality is maximized. While p_1 is just a scaling parameter, p_2 affects the distribution of the block sizes. We found that these parameters only need to be adjusted once, being well suited for a wide variety of test sequences.

For each block we determine its corresponding coding improvement

$$\Delta P^{(j)} = P_c\left(\mathbf{b}^{(j)}(\mathbf{x}_n)\right) - P_d\left(\mathbf{b}^{(j)}(\mathbf{d}_n)\right). \tag{14}$$

If $P_c\left(\mathbf{b}^{(j)}(\mathbf{x}_n)\right) \geq P_d\left(\mathbf{b}^{(j)}(\mathbf{d}_n)\right)$, i.e. the error introduced by encoding this block is larger than the prediction error, this block is not taken into consideration. Only for $P_c\left(\mathbf{b}^{(j)}(\mathbf{x}_n)\right) < P_d\left(\mathbf{b}^{(j)}(\mathbf{d}_n)\right)$ an improvement can be obtained by encoding this block. We therefore sort all blocks in descendent order of improvement $\Delta P^{(j)}$ and mark those N_B blocks which are encoded afterwards. This process is performed iteratively regarding the fact that the choice of blocks is additionally constrained by the quadtree structure of the block partitioning. The result of this process is depicted in figure 3 showing the quadtree segmentation and figure 4 showing the corresponding fractal encoded foreground regions.

4. SIMULATION RESULTS

Simulations have been carried out with several standard CIF-sequences. A coarse block diagram of the employed coding scheme is depicted in figure 2. In order to exclude the influence of various motion estimation schemes we skipped this part and instead we performed a simple prediction by providing a frame store. Encoding of the foreground image is performed in accordance with Jacquin's original proposal [5] for still image coding, except for two major modifications: Firstly a fast hierarchical codebook search which strongly increases encoding speed and nearly yields full search performance in terms of reconstruction quality is employed. A detailed description can be found in [12]. Secondly, instead of Jacquin's original unconstrained two level partitioning we apply a quadtree based multi level partitioning as described above. By this way reconstruction quality can be increased significantly.

Additional compression is achieved by reducing temporal resolution. In our scheme only every third frame is taken into consideration at the encoding side. The skipped frames can be interpolated at the receiver. The block sizes rise from 4x4 to 16x16 pels. Since high compression is desired, the application of smaller blocks is not feasible. On the other side larger block sizes than 16x16 are visually annoying. Table 1 briefly summarizes the encoding specifications.

sequence definition:	CIF-test sequences
sequence dimension:	352 horiz. 288 vert.
frame-rate	25 Hz
temporal subsampling	1:3
block sizes	4x4, 8x8, 16x16
target data rate	32, 64, 128 kbit/s

Table 1: Encoding specifications of the proposed image sequence coding scheme

If we consider a standard 64 kbit/s ISDN B-channel, a frame rate of 25 Hz and only encode every third frame, the resulting available amount of data for each frame is 7680 bit. From our investigations resulted that approximately 1000 bits have to be reserved for the segmentation overhead if a quadtree structure is applied. So there are about 6700 bits which can be used to encode the foreground image. This results for the test sequence "Miss America" in an average reconstruction quality of approximately 34–35 dB. The reconstruction quality we obtained for three different target data rates are depicted in figure (5). The corresponding grey level images are shown in figure 6.

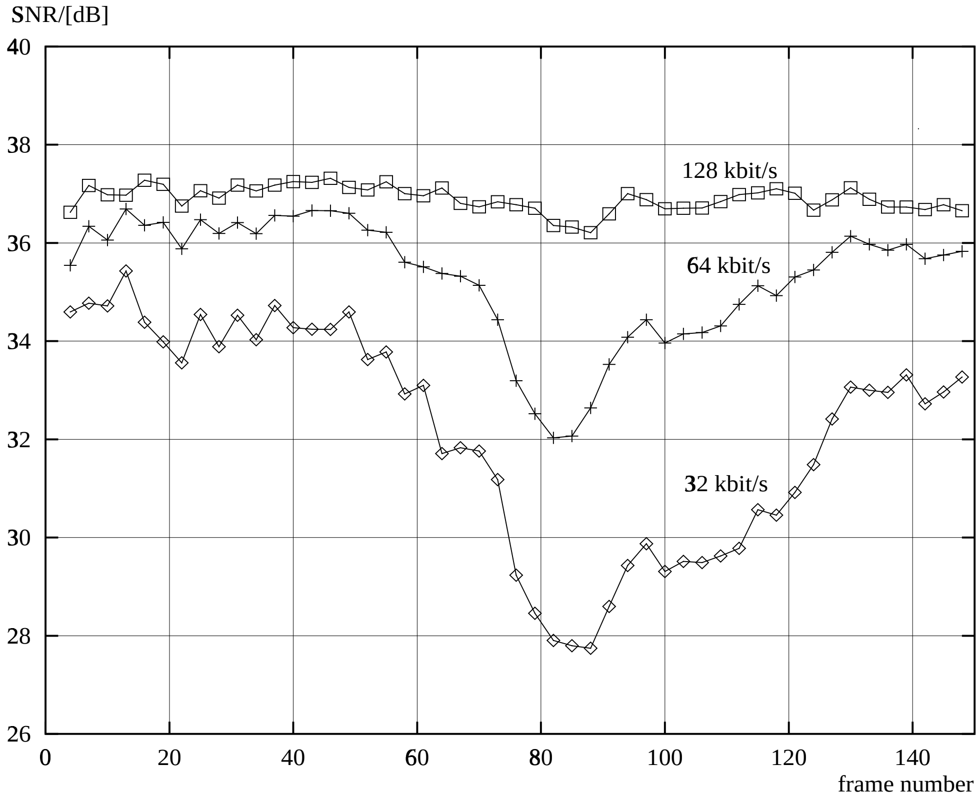

Figure 5: Coding results for CIF test sequence *Miss America* for three different channel data rates.

These results indicate that our proposal is capable of coding typical videophone sequences at data rates of about 64 kbit/s with reasonable quality. However it must be mentioned that no motion estimation and entropy coding of the parameters is employed. By doing this, which is currently under investigation, the reconstruction quality is expected to go beyond the actual standards.

5. CONCLUSIONS

In this paper we described the implementation of a video coding system using a fractal encoding scheme for still images. Based upon the hybrid coding concept the foreground image for which the prediction failed is encoded and transmitted to the receiver. The rest represents static background and needs not to be transmitted. To improve reconstruction quality we developed a segmentation algorithm which performs the partitioning of the prediction error image before the encoding process. In order to account for the characteristics of the encoding scheme a priori knowledge is taken into consideration. This is done by estimating the expected coding error from the block sizes and/or by the spatial grey level distribution of the block. We presented some coding results for a simple implementation of our scheme which are very promising. Especially implementation of motion estimation and entropy coding of the transform parameters are expected to provide further improvement.

Please attach figure (6) here

Figure 6: Coding results for test image *Miss America*.
Upper left: original frame #148; upper right: encoded frame #148 with 128 kbit/s.
Lower left: encoded frame #148 with 64 kbit/s; lower right: encoded frame #148 with 32 kbit/s.

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