Fractal Image Compression by the Classification in the Wavelet Transform Domain

Daiki ENDO Tsuyoshi HIYANE Kiyoaki ATSUTA Shozo KONDO
Faculty of Engineering, Tokai University
e-mail : kondo@keyaki.cc.u-tokai.ac.jp

Abstract

In the fractal image compression the domain-range comparison step of the encoding is very computationally intensive. Therefore in order to minimize the number of domains compared with a range a classification scheme is used. In this paper we propose a new theory for classification of domain-range blocks. The classification uses Non-decimated Separable Discrete Wavelet Transform. The encoding using the proposed classification is compared with that by Y. Fisher [1],which uses the average and the variance as features of images and classifies domain-range blocks into 72 classes. The Y. Fisher's classification uses the variance which represents only a messy degree of image intensities. The new classification proposed in this paper represents more effective features of images and classifies domain-range blocks into 432 classes by using the average and the power. With this classification, we are able to encode faster and realize high quality of the image in fractal image compression.

1. Introduction

In the fractal image compression the domain-range comparison step of the encoding is very computationally intensive. So in order to minimize the number of domains compared with a range a classification scheme is used. The classification scheme is very effective to reduce computation time for the comparison. Before the encoding, all the domains in the domain library are classified; this avoids reclassification of domains. During the encoding, a potential range is classified, and only domains with the same (or near) classification are compared with the range. This significantly reduces the number of domain-range comparisons. By "near" classifications we mean squares that would have been classified differently if their pixel values were slightly different. The idea of using a classification scheme was developed independently in [2] and [3].

Many classification schemes are possible; for example, Jacquin used a scheme that classified a sub-image into flat, edge, and texture regions, and Y.Fisher used a scheme that classified a sub-image into 72 classes by calculating the average and the variance of sub-image. In this paper a new classification of domain-range blocks is proposed using Non-decimated Separable Discrete Wavelet Transform (NSDWT for short). The encoding using the proposed classification is compared with that by Y.Fisher.

2. The Classification

2.1. Y. Fisher's classification

In the Y. Fisher's classification, a square sub-image is divided into the upper left, upper right, lower left, and lower right quadrants, numbered sequentially. On each quadrant values proportional to the average and the variance is computed: Let $r_1^i, \ldots, r_n^i$ (i =1,2,3,4) be the pixel values in the quadrant i , the average A_i and the variance V_i are defined as follows.

$$A_i = \frac{1}{n}\sum_{j=1}^{n} r_j^i , \qquad (1)$$

$$V_i = \sum_{j=1}^{n} (r_j^i - A_i)^2 . \qquad (2)$$

It is always possible to orient the sub-image so that the A_i are ordered in one of the following three ways:

Major Class 1 : $A1 \geqq A2 \geqq A3 \geqq A4$
Major Class 2 : $A1 \geqq A2 \geqq A4 \geqq A3$
Major Class 3 : $A1 \geqq A4 \geqq A2 \geqq A3$

These correspond to the brightness levels shown in Figure 1. Once the rotation of the square has been fixed, each of the 3 major classes has 24 subclasses consisting of the 24 orderings of the V_i . Thus, there are 72 classes in all.

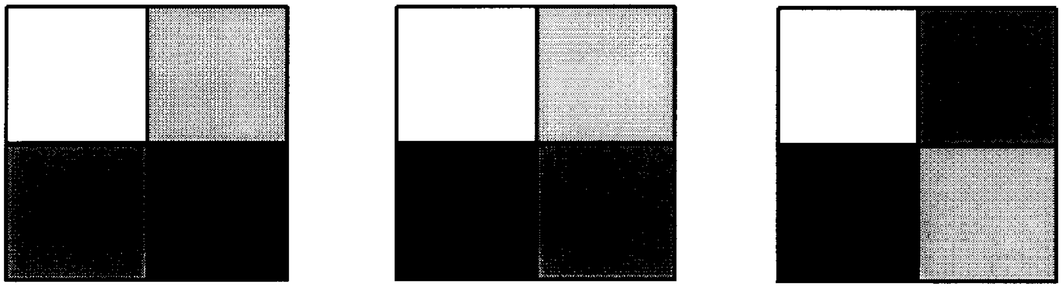

Figure 1. A square image can always be oriented so that the brightness, or average values, of its quadrants fall into one of these three canonical positions.

2.2. NSDWT

We explain the NSDWT [4] which uses a non-orthogonal wavelet. For the 1-D case the decomposition process is shown in Figure 2. (a). We can decompose the input x(n) into the low frequency component $y_1(n)$ and the high frequency component $z_1(n)$ by the low-pass filter $h_0(n)$ and the high-pass filter $g_0(n)$, respectively. That is, $y_1(n)$ and $z_1(n)$ are defined as follows

$$y_1(n) = \sum_k h_0(k)x(n-k), \tag{3}$$

$$z_1(n) = \sum_k g_0(k)x(n-k). \tag{4}$$

The reconstruction process is shown in Figure 2. (b). The low frequency component is filtered by $h^*_0(n)$ and the high frequency component is filtered by $g^*_0(n)$. Then we can reconstruct x(n) by the sum of the two filtered components.. In the NSDWT, the decomposed $y_1(n)$ and $z_1(n)$ have the same size as x(n). Therefore, $y_1(n)$ and $z_1(n)$ contain redundant information. Table 1 shows the coefficients of the filters $h_0(n)$ and $g_0(n)$. We define the synthesizing filters $h^*_0(n)$ and $g^*_0(n)$ as follows

$$h^*_0(n) = h_0(-n), \tag{5}$$

$$g^*_0(n) = g_0(-n). \tag{6}$$

The filter $h_0(n)$ is symmetric with respect to the zero and the filter $g_0(n)$ is skew-symmetric with respect to 1/2. We denote by $h_p(n)$ and $g_p(n)$ the discrete filters obtained by putting 2p-1 zeros between each coefficients of respectively the $h_0(n)$ and $g_0(n)$.

For the 2-D case we use the low pass filter $h_0(n)$,the high pass filter $g_0(n)$ and $l_0(n)$. The filters are applied to the image in both the horizontal and vertical directions separately as shown Figure 2. (b).

Table 1. The Coefficients of the filters

n	$h_0(n)$	$g_0(n)$	$l_0(n)$
0	0.0	0.0	-0.00003
1	0.0	-0.00008	0.00727
2	0.0625	-0.01643	0.03118
3	0.2500	-0.10872	0.06623
4	0.3750	-0.59261	0.79113
5	0.2500	0.59261	0.06623
6	0.0625	0.10872	0.03118
7	0.0	0.01643	0.00727
8	0.0	0.00008	0.00003

we define $l^*_0(n)$ as follows

$$l_0{}^*(n) = l_0(-n). \tag{7}$$

To decompose an input image into the low frequency component $h_0(n)$ is applied to the image in both the horizontal and vertical directions. The vertical (horizontal)high frequency component can be obtained by applying $g_0(n)$ ($l_0(n)$) to the image in the horizontal direction and $l_0(n)$ ($g_0(n)$) in the vertical direction as shown in Figure 2. (c). The decomposition (reconstruction) process can be continued using $h_p(n)$, $g_p(n)$ and $l_p(n)$ ($h^*_p(n)$, $g^*_p(n)$ and $l^*_p(n)$).

2.3. A New Classification using the NSDWT

The Y. Fisher's classification uses the variance which represents only a messy degree of image intensities. We propose a new classification using the NSDWT which represents more effective features of an image.

An original image is decomposed into the low frequency component Y(m,n), the vertical high frequency component $Z_1(m,n)$ and the horizontal high frequency component $Z_2(m,n)$ by the NSDWT shown in Figure 2. (c).

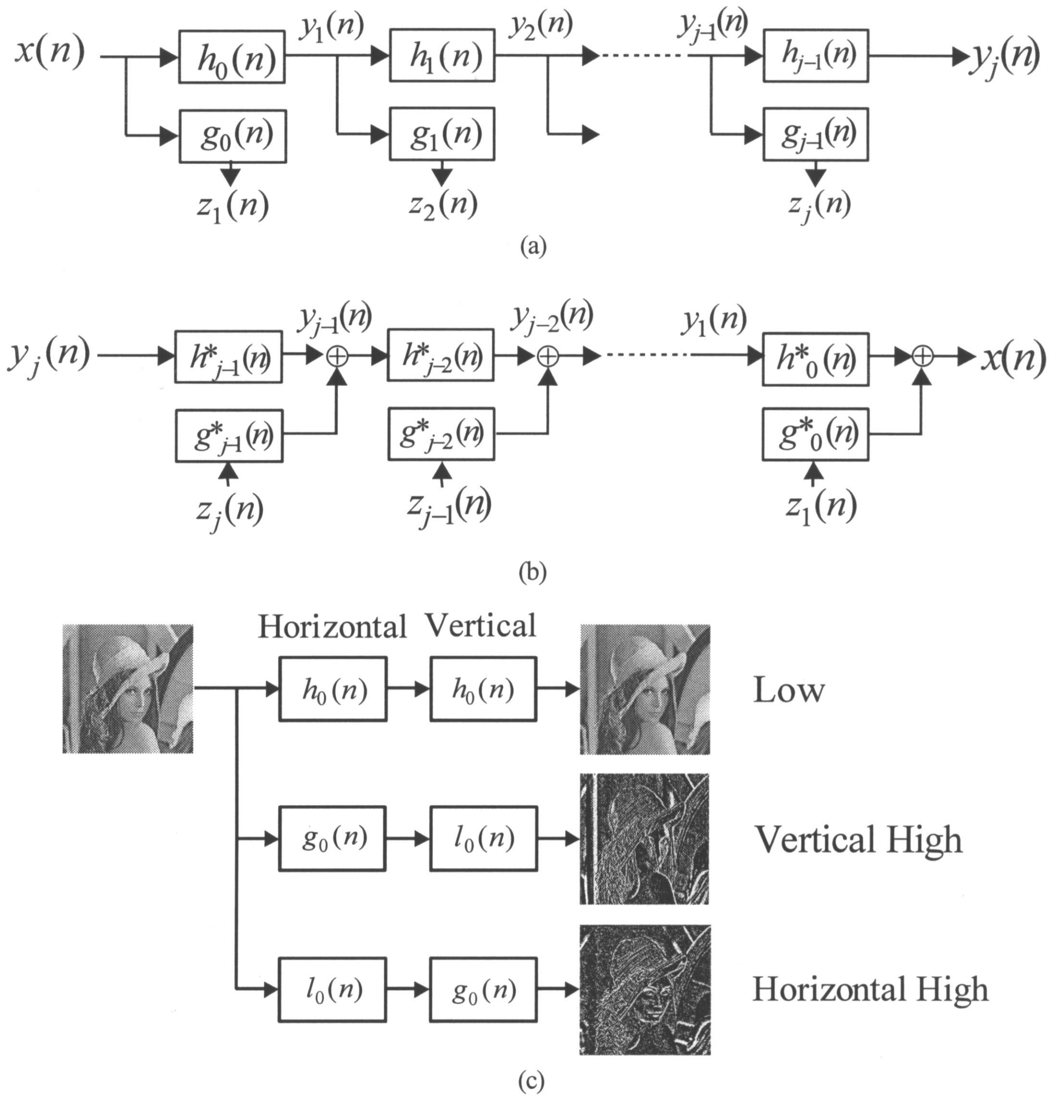

Figure 2. NSDWT (a) The decomposition process in 1-D case. (b) The reconstruction process in 1-D case. (c) The decomposition process in 2-D case by "Lena".

Classification of the low frequency component is the same as the Y. Fisher's described above. Then the low frequency component Y(m,n) is classified into 3 major classes. Each of the two high frequency components is classified into 12 subclasses consisting of the 12 configurations of the P_i 's maximum and the P_i 's minimum where P_i are the power of the quadrants defined as follows.

$$P_i = \sum_{j=1}^{n} r_j^{i\,2} \quad (\mathrm{i}=1,2,3,4). \qquad (8)$$

In the high frequency component, the V_i seems to be the P_i, because the A_i is almost zero. Therefore we use the power as the features of an image. Thus, there are $3\times12\times12=432$ classes in all.

3. Experiments

The encoding using the proposed classification is compared with that using the Y.Fisher's classification. We use "Lena", "Girl", "Milkdrop" and "Mandrill" (512×512[8bit/pixel] respectively) as experimental images. We measure PSNR, Bit Rate, and the encoding time with the Quadtree Partition. 12 bits were used to quantize the scaling coefficient and the offset. We use PC with Pentium II 300MHz.

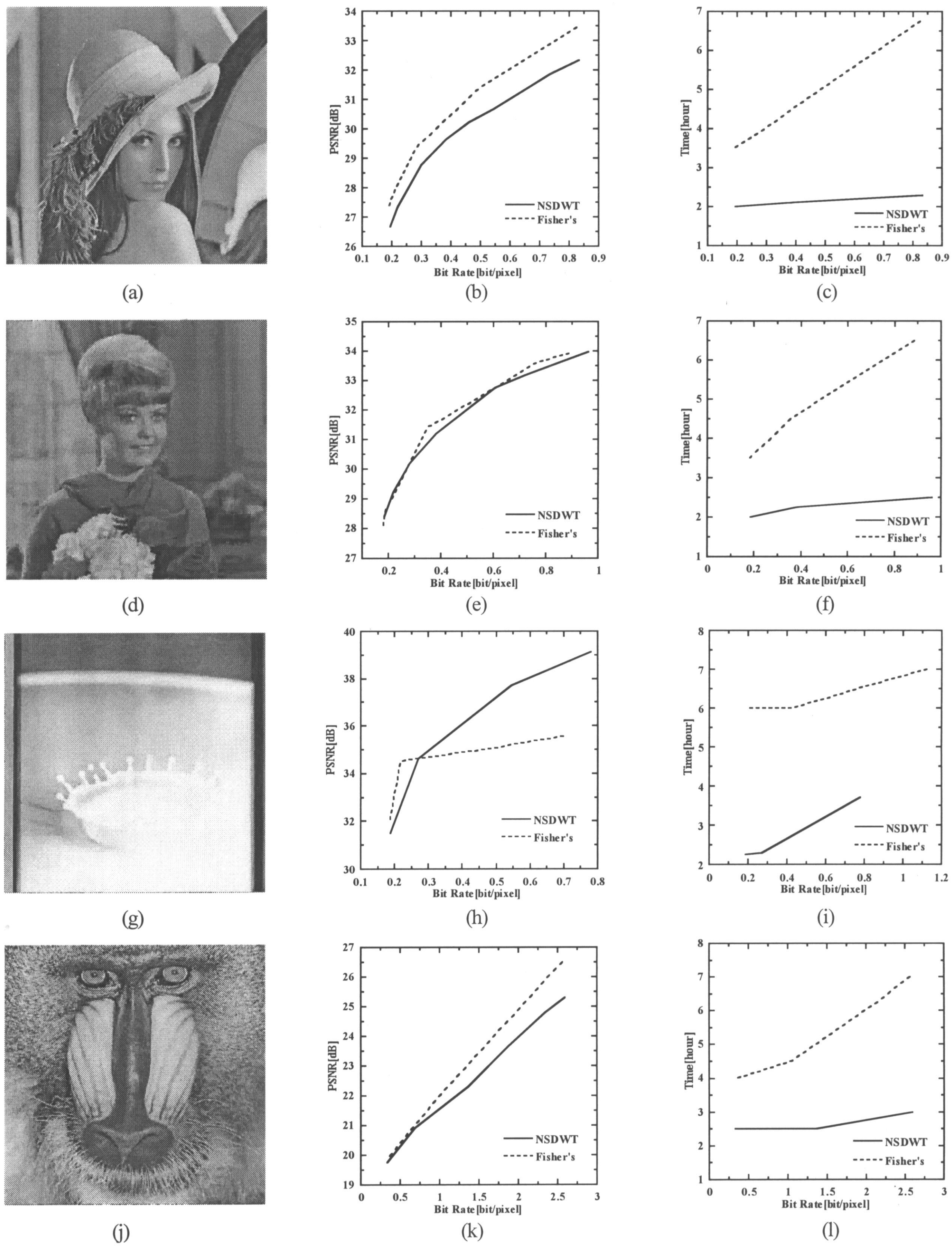

Figure 3. Experimental Results (a) The decoded image "Lena" (28.77[dB], 0.29[bit/pixel]). (d) The decoded image "Girl" (29.21[dB], 0.21[bit/pixel]). (g) The decoded image "Milkdrop" (34.61[dB], 0.26[bit/pixel]). (j) The decoded image "Mandrill" (22.31[dB], 1.36[bit/pixel]). (b), (e), (h) and (k) are the PSNR versus compression ratio curve of each images. (c), (f), (i) and (l) are the encoding time versus compression ratio curve of each images.

4. Experimental Results

The decode images which are encoded by the proposed classification, the PSNR versus compression ratio curve and the encoding time versus compression ratio curve are shown in Figure 3. By this comparison we can see that the encoding quality of the Y.Fisher's classification is a little better than the one using the NSDWT for "Lena". For "Girl", the encoding quality of the classification using the NSDWT is nearly equal to the Y.Fisher's in the low bit rate. For "Milkdrop", the encoding quality of the classification using the NSDWT is better than the Y.Fisher's in the high bit rate especially. For "Mandrill", the encoding quality of the Y.Fisher's classification is a little better than the one using the NSDWT, but the encoding quality of the classification using the NSDWT is nearly equal to the Y.Fisher's in the low bit rate. PSNR is low relatively because of containing a large amount of the high frequency component. As to the encoding time, we can see that the encoding time rate of the classification using the NSDWT is 1/2 ~ 1/3 of the Y.Fisher's.

5. Conclusions

We have discussed about the classification scheme of fractal image compression. Although the encoding quality of the classification using the NSDWT depends on the kinds of the images, we know that the encoding quality of the classification using the NSDWT is nearly equal to the Y.Fisher's. Especially as to the image which is containing a large quantity of the low frequency component, such as "Milkdrop", we can see that the encoding quality of the classification using the NSDWT is better than the Y.Fisher's. And the encoding time performance of the classification using the NSDWT is better than the Y.Fisher's. Thus the classification scheme using the NSDWT is more effective to reduce the encoding time than the Y.Fisher's .

6. Future Works

We have used only the 1^{st} high frequency components of the NSDWT for the classification. However the 1^{st} high frequency component contain a relatively large amount of the white noise, therefore the 1^{st} high frequency components of the NSDWT is influenced by the white noise very much. Thus we can expect that classification by the 2^{nd} high frequency components are better than that by the 1^{st} high frequency components.

7. References

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