# Improved Fractal Image Coding

**Kai Uwe Barthel, Thomas Voyé and Peter Noll**
Institut für Fernmeldetechnik, Technische Universität Berlin
Einsteinufer 25, D-1000 Berlin 10, Germany

## ABSTRACT

*In this paper we propose an optimized fractal image coder. With a better codebook design, vector quantization of the luminance transformation parameters, an adaptive search scheme, and entropy coding of the geometrical transformation parameters we obtain an improved coding performance as compared to existing fractal image coders. We show how the optimal luminance transformation can be determined. Our experimental results show that the Lena image can be coded at the rate of 0.13 bpp to 0.4 bpp to yield PSNR-values of 30 dB to 35 dB.*

## INTRODUCTION

The principle of fractal image coding is based on the theory of iterated transformations developed by Barnsley [1]. Fractal image coding consists in finding a construction rule that produces a fractal image which approximates the original image. Redundancy reduction is achieved by describing the original image through smaller copies or parts of the same image.

Typical digital images are not self-transformable in the sense that a system of transformations when iteratively applied to any initial image, yields an image close to the image to be encoded. One solution to this problem is a block oriented coding scheme proposed by Jacquin [2,3], and was recently improved by Jacobs et al. [4]. The image to be encoded is partitioned into non-overlapping, square blocks (*range blocks*). The task of a fractal coder is to find a larger block of the same image (*domain block*) for every range block in such a way that a contractive transformation $\tau$ of the domain block is a good approximation of the range block. $\tau$ is a combination of a *geometrical transformation* $\gamma$ and a *luminance transformation* $\lambda$.

The geometrical transformation consists of a spatial contraction and a position shift. The luminance transformation changes the contrast and the brightness of a block. The union of all possible combinations of transformations $\gamma \circ \lambda$ is called *virtual codebook*. This codebook needs not to be transmitted to the decoder, because a reconstruction can be generated iteratively together with the decoded image.

We show that there is exactly one optimal pair of geometrical and luminance transformation that can be found as follows: For all codebook blocks to be examined the optimal luminance transformation is evaluated. The codebook block with the smallest approximation error represents the best $\lambda$-$\gamma$-pair.

At the decoder, the 'fractal code', iteratively applied to *any* initial image, generates the reconstructed image.

## DESCRIPTION OF THE CODER

The basic principle is similar to Jacquin`s coder, the important improvements are explained referring to the structure as shown in Fig. 1. We use a three level hierarchy. The range block sizes are 16x16, 8x8 and 4x4 pixels. A range block can be split into four smaller range blocks . The decision about splitting the larger range blocks depends on the approximation error in the range blocks of the next hierarchical level. If this error exceeds a given level, an additional transformation is determined for the smaller range blocks.

### Codebook design

The search for the geometrical transformation can be seen as a search in a codebook that contains the set of all spatial contracted domain blocks. Instead of constructing a pixel of a contracted block from the average of four pixels in the domain block [2], we perform low-pass filtering followed by sub-sampling. The codebook blocks are then free of aliasing errors, and a smaller distortion is achieved with the same codebook size.

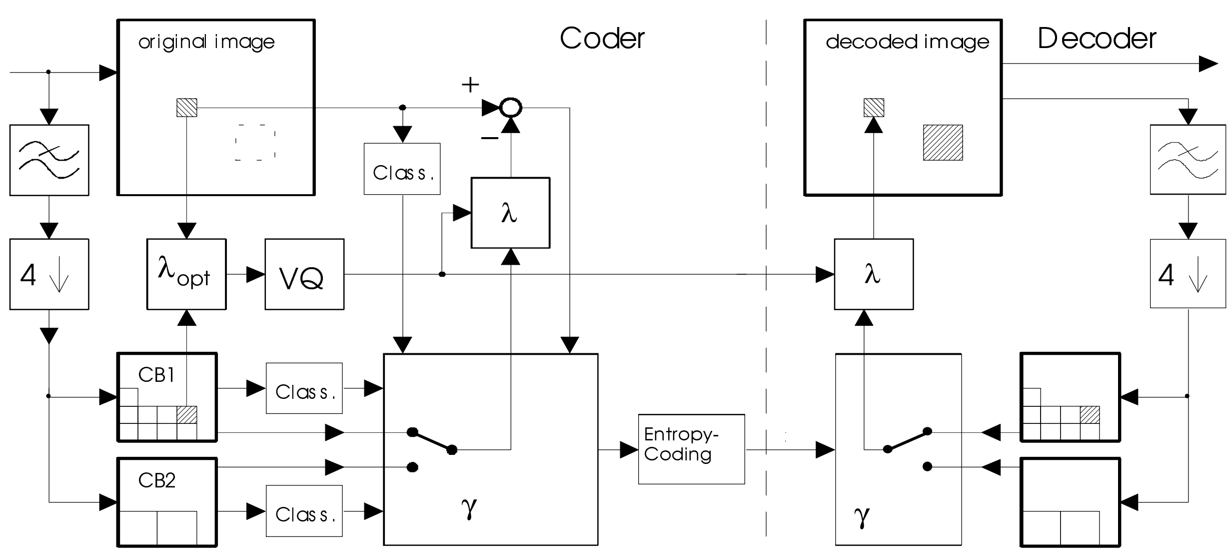


Fig. 1. - Coder and decoder structure of a fractal block coder (here shown with a two level hierarchy)

### Classification

To adapt the complexity of the geometrical transformation to the "complexity" of the range blocks, range and codebook blocks are classified. We use a variance-classification to subdivide the blocks into the classes *Shade, ShadeMid, Midrange* and *MidEdge*.

All range blocks with shade property are approximated by their mean. For all other range blocks the fractal coding consists in finding a codebook block that, together with the luminance transformation, produces the best approximation. All codebook blocks with the same properties as the range block are examined. For range blocks with edge properties, reflections and rotations (isometries) are examined as well. When the best block is found, it is classified in an unambiguous way (Midrange resp. Edge).

### Optimal luminance transformation / Approximation error calculation

The luminance transformation $\lambda$ serves to adjust the contrast ($a$) and brightness ($b$) of a codebook block. There is a pair of parameters ($a_{opt}$, $b_{opt}$) that minimizes the squared error between range block $\boldsymbol{f}$ and the transformed codebook block $\boldsymbol{g}$ of any class of size NxN. The luminance transformation has to be found such that

$$e^2\left(\mathbf{f},\lambda(a,b,\mathbf{g})\right)=\frac{1}{N^2}\sum_{m=0}^{N-1}\sum_{n=0}^{N-1}\left[f_{m,n}-\left(a\cdot g_{m,n}+b\right)\right]^2=\min!$$

To find the minimum of the error function the partial derivatives of $e^2(a,b)$ must be set to zero.

$$a_{opt}=\frac{C_{gf}(0,0)}{\sigma_g^2}\qquad\qquad b_{opt}=\mu_f-a_{opt}\cdot\mu_g$$

$C_{gf}$ denotes the covariance of g and $f$, $\sigma_g^2$ the variance of g.

The resulting error for optimally transformed blocks depends only on the variance of the range block and the correlation coefficient $\rho_{gf}$ of both blocks.

$$e^2\left(\mathbf{f},\lambda_{opt}(\mathbf{g})\right)=\sigma_f^2\left(1-\rho_{gf}^2\right)\quad\text{with}\quad\rho_{gf}=\frac{C_{gf}(0,0)}{\sigma_g\sigma_f}$$

If the $a$-$b$-pairs are quantized or limited, we get an additional error that depends on the deviation from the optimal pair ($\Delta a = a - a_{opt}$ , $\Delta b = b - b_{opt}$)

$$e^2\left(\boldsymbol{f},\lambda_{quant}(\boldsymbol{g})\right)=e_{opt}^2+e_{quant}^2$$
$$=\sigma_f^2\left(1-\rho_{gf}^2\right)+\left(\Delta a\cdot\mu_g+\Delta b\right)^2+\Delta a^2\cdot\sigma_g^2$$

### Vector quantization of the luminance transformation

The parameters of the luminance transformation are continuous and have to be quantized. We use vector quantization because the $a$ and $b$ values are correlated. Furthermore, the topology of the optimal $a$-$b$-pairs is very similar for many pictures.

### Geometrical transformation / adaptive search

When searching for a codebook block, that is similar to the range block, we use a special search scheme. The search path has the form of a spiral and starts with the codebook block that corresponds to the domain block directly above the range block (Fig. 2.).

The number of codebook blocks to be examined is variable. For range blocks with edge property, we search through a larger number of codebook blocks than for midrange blocks. The search for a transformation is aborted if the approximation error is below a threshold value at the first search position or at the end of a minimal search area. To be able to encode the search indices efficiently, we introduce additional classes for the minimal search regions and the first midrange block (Midrange1).

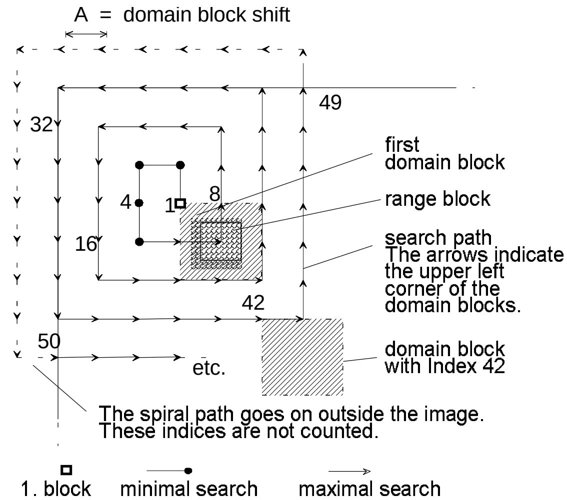


Fig. 2. - spiral search path

### Coding of the transformation parameters

The class of a range block and its partition are entropy coded. For all classes except Shade and Midrange1 a geometrical index has to be transmitted. In the case of edge blocks three additional bits indicate one of eight isometries. For shade blocks, the information of the luminance transformation consists of the index of the quantized mean. In all other cases, it is the index of the luminance vector quantization.

## SIMULATION RESULTS AND CONCLUSION

We have proposed an optimized fractal image coder performing comparably to state-of-the-art vector quantizers. Our coder produces images which are aesthetically and qualitatively better than images coded by JPEG and Jacquin`s and Jacobs` fractal coder. The edge reproduction of our coder is excellent. High frequency regions with low variance tend to be smoothed out. Some blockiness is visible at very low bit rates. Table 1 shows the performance compared to JPEG resp. Jacobs` results. Further improvements are expected with an optimized codebook design for the luminance transformation. A combination with other image coding techniques and a luminance transformation, that is better able to approximate larger blocks, should bring further progress.

| | JPEG | our coder | Jacobs | our coder |
|---|---|---|---|---|
| rate (bpp) | 0.37 | 0.37 | 0.22 | 0.15 |
| PSNR (dB) | 33.46 | 34.67 | 30.71 | 30.71 |

Table 1. coder performance ("Lena" image size 512x512)


[1] M. F. Barnsley, *Fractals Everywhere*. New York: Academic Press, 1988.

[2] A. Jacquin, *Image Coding Based on a Fractal Theory of Iterated Contractive Image Transforms*. IEEE Transactions on Image Processing Vol. 1. No. 1 1992

[3] A. Jacquin, *Image Coding Based on a Fractal Theory of Iterated Contractive Image Transforms*. SPIE Vol. 1360 Visual Communications and Image Processing ´90

[4] E. W. Jacobs, Y. Fisher and R. D. Boss, *Image Processing : A study of iterated transform method, Signal Processing* 29 (1992) 251-263