Fast Fractal Encoding in Frequency Domain

O.C. Au*, M.L. Liou**, L.K. Ma
Department of Electrical and Electronic Engineering
Hong Kong University of Science and Technology
Email: *ecau@ee.ust.hk, **eeliou@ee.ust.hk

ABSTRACT

Fractal image compression applies the self-similarity property of image. Many researches have been done to study the properties of fractal coding in image domain. In this paper, however, we try to explore the features of fractal coding in frequency domain. We firstly overview the properties of fractal coding in image domain, then we derive the corresponding formula of scaling factor and offset of affine transform in DCT domain. Applying the energy compaction property of DCT, we propose a fast fractal encoding algorithm by using only a small number of low frequency DCT coefficients in measuring the similarity between range block and domain block. We further propose a possible fast hybrid fractal encoding algorithm which combines existing fast search methods, statistical normalization and frequency domain comparison.

1 INTRODUCTION

There has been much interest in applying fractal in encoding images due to the potential of very large compression ratio [1]. Fractal is a class of geometric shapes of which the complexity and details of structures at different levels of magnification are the same. Each small partition of fractal object can be considered as a reduced-scale replica of the whole object. This self-similarity property allows an iterated function system (IFS) to be formed. To adjust better to real images, the partitioned IFS (PIFS) is a variation of IFS in which each small partition of the image is self-similar to another portion of the image. This paper is focused on PIFS.

In PIFS, an image is divided into non-overlapping range blocks. For each range block, a domain block pool is defined. Exhaustive search is performed in search of the best domain block together with the corresponding optimal affine transformation that minimize distortion measures such as mean square error (MSE), which is an image domain measure. It is this exhaustive search which makes fractal encoding extremely computational expensive. If the domain block search can be simplified, the computation requirement of fractal encoding can be much reduced.

In this paper, we look at fractal from a different angle. We attempt to explore the similarity properties of the range blocks and domain blocks in the frequency domain. A range block and a domain block with high degree of similarity in image domain should have high degree of similarity in the frequency domain. So frequency domain similarity measures may be as good and effective as image domain similarity measures. In this paper, we derive the optimal scaling factor and offset in terms of minimum MSE in the frequency domain. While all pixels in the domain block are equally important, different frequency coefficients carry different importance. This allows us to design a fast domain block searching algorithm without significant performance degradation.

2 OPTIMAL SCALING FACTOR AND OFFSET IN IMAGE DOMAIN

The formula for the scaling factor s and offset o that minimizes the mean square error (MSE) between the domain blocks and range blocks in the image domain are as follows [1].

$$s = \frac{N^2 \sum_{ij} d_{ij} r_{ij} - \sum_{ij} d_{ij} \sum_{ij} r_{ij}}{N^2 \sum_{ij} d_{ij}^2 - (\sum_{ij} d_{ij})^2} \tag{1}$$

$$o = \frac{1}{N^2}(\sum_{ij} r_{ij} - s \sum_{ij} d_{ij}) \tag{2}$$

$$MSE = \frac{1}{N^2} \sum_{ij} (r_{ij} - s d_{ij} - o)^2 \tag{3}$$

where N is the length of the domain and range blocks, d_{ij} and r_{ij} are the ij^{th} pixel in the domain block d and range block r respectively.

Since the s, o and MSE need to be computed between each possible pair of domain block and range block, the computation requirement is tremendous making it impractical to implement a fractal encoder.

There are many ways being proposed to reducing the computation requirement of fractal encoding, including block classification and statistical normalization[2]. In this paper, we propose to perform fast fractal encoding by performing the MSE computation in frequency domain and using, not all, but a reduced number of coefficients.

2.1 FAST FRACTAL ENCODING IN FREQUENCY DOMAIN

One way to speed up fractal encoding is to reduce the number of pixels to be considered in each comparison. We test briefly the possibility of reducing the size of range block and domain block by direct down sampling in image domain. If a block is down-sampled from 8x8 to 4x4, the speed-up factor is equal to 4. We observe that both subjective visual quality and the Peak-Signal-to-Noise-Ratio ($PSNR$) will not drop significantly. If a block is down-sampled from 8x8 to 2x2, the speed-up factor will increase to 16. However, simulation results show that both $PSNR$ and the visual quality degrade significantly. Computation reduction by down-sampling in image domain does not appear to work.

Here we propose to measure the similarity (or MSE distortion) of the range blocks and domain blocks in the frequency domain rather than the image domain. A range block and a domain block with high similarity in image domain should have high similarity in frequency domain as well. In this paper, we concentrate on the discrete cosine transform (DCT) which is commonly used in the JPEG, MPEG and ITU-T H.263 standards for image and video compression as well as video conferencing. DCT is well known to have excellent energy compaction property for natural images, with energy concentrating mainly in the low frequency coefficients.

In image domain, all pixels are equally important in the blocks. Due to the energy compaction property of DCT, the low frequency DCT coefficients contain most of the energy of the blocks and thus are more important than the high frequency coefficients. It is thus possible to use only the low frequency DCT coefficients, rather than all, to measure the similarity between blocks with essentially the same result.

Here we propose to measure the similarity of blocks using only the low frequency coefficients. To make this possible, we will need to derive the optimal s, o and the corresponding expression for MSE.

2.2 Optimal s and o in DCT domain

Let the affine transform from the domain block d to the range block r be

$$sd + o1_n = r \tag{4}$$

where d, r and 1_n are nxn matrices and 1_n has all elements being unity. Taking DCT on both side, we obtain

$$sD + oDCT(1_n) = R \tag{5}$$

where R, D and $DCT(1_n)$ are the DCT of r, d and 1_n respectively. In real images, equality does not hold usually in the affine transform. We thus minimize the MSE distortion

$$MSE = \sum_{i,j}(sD_{ij} + oDCT(1_n)_{ij} - R_{ij})^2 \tag{6}$$

in DCT domain. By applying partial differentiation to MSE with respect to s and o and setting the resulting expressions to zero, the optimal s and o can be found to be

$$s = \frac{\sum_{(i,j)\neq(0,0)} R_{ij}D_{ij}}{\sum_{(i,j)\neq(0,0)} D_{ij}^2} \tag{7}$$

$$o = \frac{R_{0,0} - sD_{0,0}}{DCT(1_n)_{00}} \tag{8}$$

Since DCT is unitary, the MSE in image domain is equal to the MSE in DCT domain. Thus, the derived optimal s and o in DCT domain is the same as their counterparts in image domain.

Due to the energy compaction property of DCT, most energy is concentrated in the low frequency DCT coefficients. We thus propose to approximate Eq. 6 and 7 by performing the summation only for the low frequency DCT coefficients. The reduced number of DCT coefficients can lower the computation requirement of the mean square error and the optimal s.

Often eight simple isometries are considered in fractal coding. The DCT of the eight isometric transform of the blocks are related to the DCT of the original block in a simple way[3], with negligible additional computation requirement. Besides, no extra memory is needed to store the DCT of different isometries.

2.3 Reordering for efficient encoding

Although the proposed method can reduce the computation complexity of the block comparison, additional computation is needed to perform the DCT. Actually, the amount of computation for the DCT may be even more than the block comparison.

Taking advantage of the Calculate-One-Use-Many property in fractal encoding, this problem can be solved by a simple reordering at the expense of some additional memory requirement. The situation is similar to the techniques of statistical normalization, where the normalization process need only be done once, but the normalized range block and domain block will be used many times. Normally, for each range block, it is compared with all possible domain block to find the best matched affine transformed domain block. In the proposed reordering, the order is reversed.

Here is proposed reordering. The DCT of each range block is computed and stored prior to the search. In the search, for each domain block, the DCT is computed and checked against the precomputed DCT of each range block. If the MSE between the domain block and a range block is less than that between the range block and the previously examined domain blocks, the current domain block is considered the latest best match for the range block with the corresponding optimal s and o.

Here we only need to compute the DCT coefficients of range blocks and domain blocks once, and these DCT coefficients will be re-used many times. Hence the computation of DCT becomes insignificant when compared with the whole encoding process.

To reduce computation further, we only need to compute those low frequency DCT coefficients that will be used to compute the MSE, s and o for all the eight isometries.

2.4 Selection of DCT coefficients

In this section, we address the issue of selecting the DCT coefficients. In previous sections, we propose to use the low frequency DCT components for comparison to take advantage of the energy compaction property of DCT. However we may have a better performance if we dynamically select coefficients during the block comparison. In natural images, the low frequency DCT coefficients often contain most of the energy and thus are more important than the high frequency coefficients. However it is not always true, especially when only a small number of AC coefficients are retained. We may miss some important medium or high frequency coefficients with relatively high energy.

Hence we propose the following approach: instead of comparing a number of the lowest frequency components, we can compare the same number of the DCT coefficients with the highest energy in the range block. We will call these the "active DCT coefficients".

The improvement can be significant when the number of AC coefficients is small. This method can improve the $PSNR$ while keeping the computational requirements essentially the same. In image domain, energy is roughly equally distributed to all pixels, thus it is difficult to extract the most important (with energy) features of the blocks. In frequency domain, this is possible due to the energy compaction property of DCT.

2.5 Computation reduction and Discussions

The proposed fast fractal encoding in frequency domain is simulated using the 512x512 Lenna image. Fast fractal encoding using spatial subsampling and full exhaustive search fractal encoding are also performed for comparison.

When 8 rather than 64 pixels ($Fig.1$) or coefficients are used, a computation reduction factor of approximately 8 is achieved. When image domain subsampling is used, a drop in PSNR of 3.10dB, as compared with the full search fractal encoding image, is observed which is quite significant ($Fig.2$). When 8 fixed lowest DCT coefficients are used in the proposed scheme, the drop in PSNR is reduced to 1.72dB which is still significant ($Fig.3$). However when 8 active DCT coefficients are used in the proposed scheme, the drop in PSNR is further reduced to 0.96dB ($Fig.4$). This suggests that 8 active DCT coefficients, though small in number when compared with 64, can capture the essential characteristics of the image.

When 4 rather than 64 pixels or coefficients are used, a computation reduction factor of 16 is achieved. A PSNR drop of close to 4 dB is observed in the image domain subsampling. The PSNR drop is reduced to 3.2dB when the lowest 4 DCT coefficients are used. It is further reduced to 2.5dB when 4 active DCT coefficients are used. This experiment suggests that four pixel or coefficients are insufficient in capturing the essential characteristics of the image. But if only four coefficients can be used, the active DCT coefficients are significantly better than the fixed DCT coefficients.

Here a computation reduction factor of 8 can be obtained with good performance. This proposed fast encoding technique with 8 active DCT coefficients can potentially be combined with other algorithms to really speed up the fractal encoding to make it practical.

3 Possible Fast Hybrid Fractal Encoding

Using the proposed frequency domain search, a fast hybrid technique can be created to reduce computational complexity of fractal encoding. The technique includes three main parts. They are 1) existing fast search method, 2) statistical normalization[2] and 3) Frequency Domain Comparison. Existing fast search method such as block classification or local search can be employed to reduce the number of block comparisons. Within each block comparison, statistical normalization is used to eliminate the computation of the optimal s and o. The proposed frequency domain comparison is then applied to reduce the number of elements used in the computation of MSE.

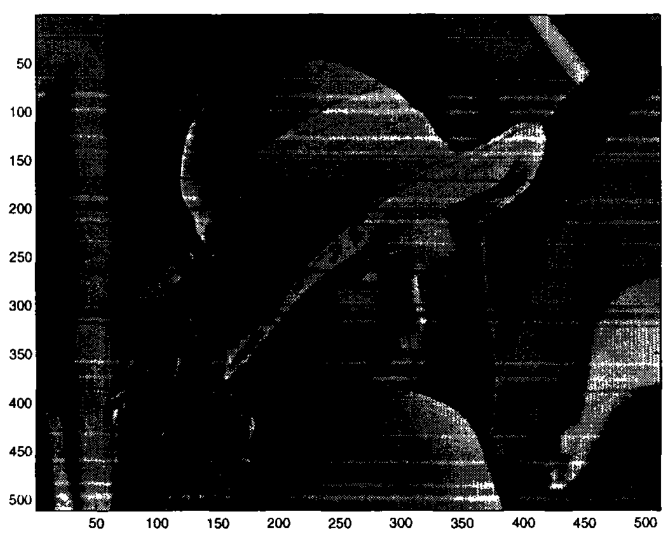

Figure 1: Decoded image "Lenna" with 64 pixels, $BR = 0.485bpp$, $PSNR = 30.77dB$.

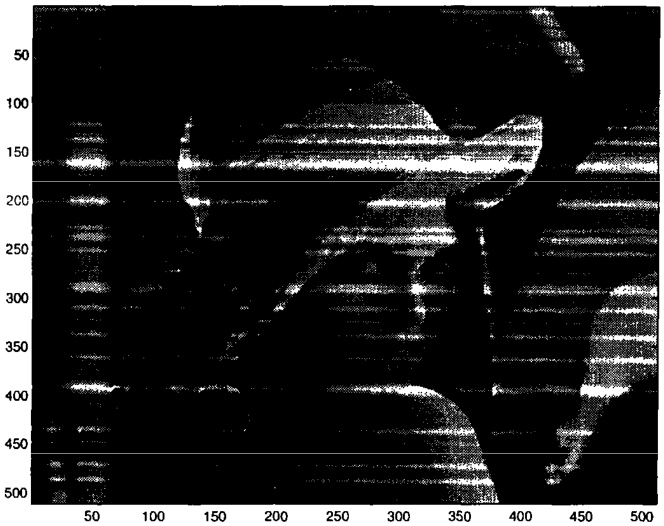

Figure 2: Decoded image "Lenna" with 8 mean pixels, $BR = 0.502bpp, PSNR = 27.67dB$.

4 Conclusion

In this paper, we propose to reduce fractal encoder complexity by performing the block comparison in DCT domain rather than image domain. We propose to achieve computation reduction by using only a small number of important DCT coefficients. We also derive the optimal scaling factor and offset in DCT domain. Simulation results suggest that the proposed fast algorithm can achieve significant computation reduction at the expense of only slightly reduced visual quality.

References

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[3] Y. Zhao and B. Yuan, "A Hybrid Image Compression Scheme combining Block-Based Fractal Coding and DCT", *Image Communication*, Vol. 8, No. 2, pp.73-78, Mar. 1996.

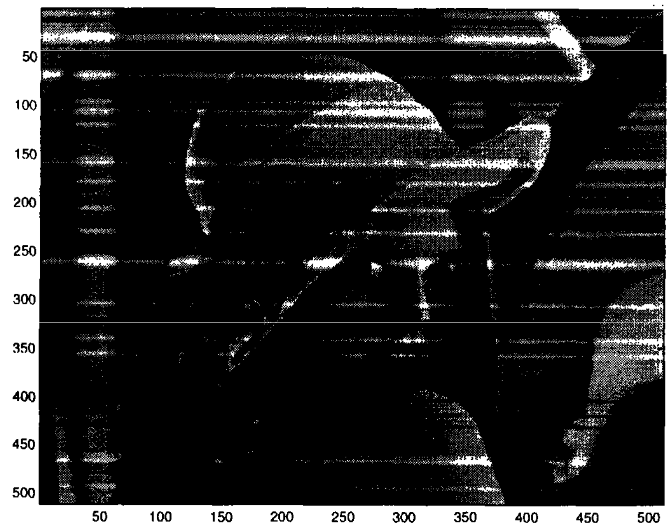

Figure 3: Decoded image "Lenna" with 8 lowest DCT coefficients, $BR = 0.502bpp, PSNR = 29.05dB$.

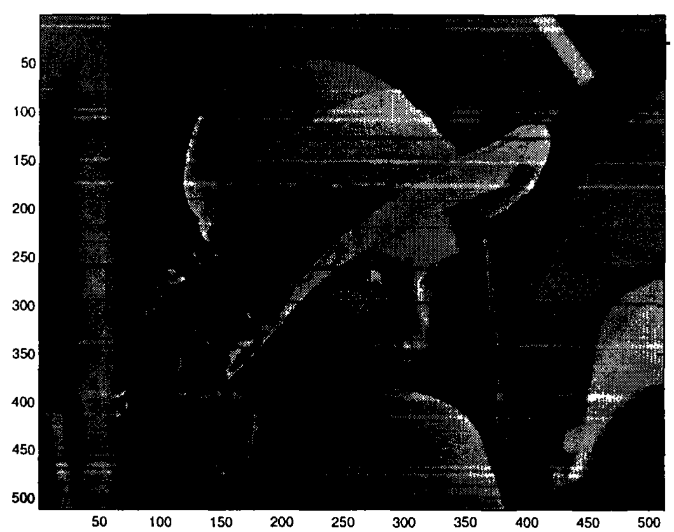

Figure 4: Decoded image "Lenna" with 8 active DCT coefficients, $BR = 0.501bpp, PSNR = 29.81dB$.