Journal of Graph Algorithms and Applications
http://jgaa.info/ vol. 0, no. 0, pp. 0–0 (0)

More Flexible Radial Layout

Ulrik Brandes[1] *Christian Pich*[2]

[1]Department of Computer & Information Science, University of Konstanz
[2]Chair of Systems Design, ETH Zürich

Abstract

We describe an algorithm for radial layout of undirected graphs, in which nodes are constrained to concentric circles centered at the origin. Such constraints are typical, e.g., in the layout of social networks, when structural centrality is mapped to geometric centrality or when the primary intention of the layout is the display of the vicinity of a distinguished node. Our approach is based on an extension of stress minimization with a weighting scheme that gradually imposes radial constraints on the intermediate layout during the majorization process, and thus is an attempt to preserve as much information about the graph structure as possible.

1 Introduction

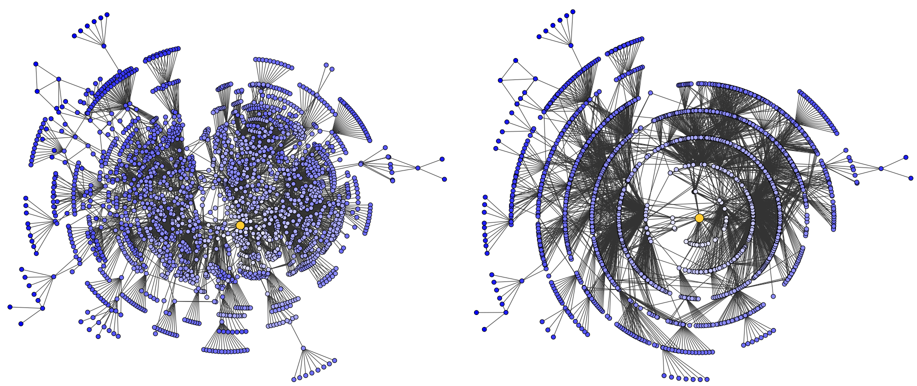
In radial graph layout the nodes are constrained to lie on a set of concentric circles; for some or all nodes in the graph a radius is given, which typically encodes non-structural information, or the results of a preceding analysis. Historical examples of such drawings date back to the 1940s [23], and the special case in which all nodes are required to lie on the same circle is a often referred to as circular layout.

We are interested in designing a method to determine layouts that meet the following two, possibly contradicting, criteria:

- *Representation of distances:* The Euclidean distance between two nodes in the drawing should correspond to their graph-theoretical distance.
- *Radial constraints:* Nodes are associated with the radius of a circle centered at the origin, and are constrained to be placed on the circumference of this circle.

While the first criterion is a general readability objective in undirected graph layout, the constraints in the second criterion are specific to the application at hand.

An example is the exploration of hierarchies with discrete (nominal-scale) layers [8]; in [25] large such hierarchies are laid out radially as a tree, followed by an incremental force-based placement. This approach was later modified



(a) unconstrained (b) with radial constraints

Figure 1: A social network (courtesy of Carola Lipp; 2075 nodes, 4769 edges), consisting of two known clusters. The darkness of nodes is proportional to their distances from the distinguished focal node, which also defines the radii used in the constrained layout. Note that distances are represented more clearly, while the two clusters are apparent, still.

for dynamic real-time exploration of a filesharing network in [26], where users interactively select a node to be moved into the center, triggering an update of the immediate surrounding of that node. A different approach is to adapt the Sugiyama framework, originally designed for layout in parallel layers, to radial layers [1].

In the case of fixed radii defined to represent some continuous (interval-scale) node valuation, unary constraints are imposed on the drawing. This scenario is introduced in [5] to map any (structural) centrality index to visual centrality. Layouts are determined from a combination of simulated annealing, which is very flexible and allows for penalty costs, e.g., for edge crossings, and force-directed placement. Because of its high computational cost, this method does not scale even to moderately sized graphs, though. For applications in social network analysis, it was therefore replaced by a combinatorial approach based on circular layout [2].

Our present approach addresses the task more uniformly by formulating both of the above criteria as objective functions measuring how far a layout is from meeting them. While the first objective is captured by a common function known as weighted stress [16], we try to accomplish the second goal using stress-like terms measuring the representation error with respect to the radial constraints, and attempt to minimize a linear combination of the two objectives.

Quite recently, other extensions of the stress term have been used for drawing graphs with explicitly formulated aesthetic criteria, such as the uniform scattering of the nodes in a graph over a unit disk [20], penalizing node overlaps [15], or preserving a given topology [13].

All these approaches modify the target *distances* themselves in one form or another, while the approach presented here is based on engineering the *weights* used in the stress minimization model. The weights are coefficients of error terms involved in the quality criteria to be minimized. If chosen carefully, these weights can be used to influence the configuration resulting from optimizing the modified stress function; see Fig. 1 for an example. We are not aware of previous work in graph drawing which systematically adjusts weights to adapt an objective function to meet layout criteria.

2 Preliminaries

Let $G = (V, E)$ be a simple undirected graph, i.e., $E \subseteq \binom{V}{2}$. We will denote the cardinalities of the node and edge sets by $n = |V|$ and $m = |E|$, respectively; it is sometimes convenient to index nodes by numbers, $V = \{v_1, \dots, v_n\}$. The graph-theoretical distance between two nodes u, v is the number of edges on a shortest path between u and v and is denoted $d_{u,v}$ or, when there is no danger of confusion, d_{uv}. The matrix $D = (d_{uv})_{uv} \in \mathbb{R}^{n \times n}$ contains the distances between every two nodes in G; the diameter of G is the maximum distance between any two nodes in G, $\mathrm{diam}(G) = \max_{u,v \in V} d_{uv}$. All graphs are assumed to be connected; otherwise, connected components are considered individually.

Two-dimensional node positions are denoted by $p(v) = (x_v, y_v)$. The Euclidean distance between two nodes in a layout p is defined as $\|p(u) - p(v)\| = \left((x_u - x_v)^2 + (y_u - y_v)^2\right)^{1/2}$.

3 Stress, Weights, and Constraints

3.1 Stress

The foundation of our method is multidimensional scaling (MDS) [3, 9]. Originating in psychometrics and the social sciences, MDS has been established and widely used for graph drawing since its popularization by Kamada and Kawai [19]. While there is a wide range of variants and extensions, we here concentrate on the *stress minimization* approach [16].

Given a set of target distances among a set of n objects, the overall goal is to place these objects in a low-dimensional Euclidean space in such a way that the resulting distances fit the desired ones as well. In the graph drawing literature, the desired distances are usually graph-theoretical (shortest-path) distances d_{uv}, and the goal is to find two-dimensional positions $p(v)$ for all nodes $v \in V$ with

$$\|p(u) - p(v)\| \approx d_{uv}$$

attained as closely as possible for all pairs u, v. When the configuration is not required to satisfy any further constraints, the objective function, called (weighted) *stress*, is the sum of squared residuals

$$\sigma(p) = \sum_{u,v} w_{uv} \left(d_{uv} - \|p(u) - p(v)\| \right)^2 \tag{1}$$

over all the $n(n-1)/2$ pairs of nodes, where $w_{uv} \geq 0$ is a weight for the contribution of the particular error term $(d_{uv} - \|p(u) - p(v)\|)^2$ associated with the pair u, v.

There is wide consensus that configurations with a small stress value tend to be structurally informative, and aesthetically pleasing. The state-of-the-art approach to finding such layouts is *stress majorization* [10, 16]; starting from an initial configuration, it generates an improving sequence of layouts. When no coordinates are at hand, the iterative process may be initialized at random, but more favorable and robust strategies are available. The experiments of [7] indicate that approximate classical scaling [6] is the method of choice.

During stress majorization, new positions $\hat{p}(u) = (\hat{x}_u, \hat{y}_u)$ for every node $u \in V$ can be computed from the current positions using the update rules

$$\hat{x}_u \leftarrow \frac{\sum_{v \neq u} w_{uv} \left(x_v + d_{uv} \cdot (x_u - x_v) \cdot b_{uv} \right)}{\sum_{v \neq u} w_{uv}} \tag{2}$$

$$\hat{y}_u \leftarrow \frac{\sum_{v \neq u} w_{uv} \left(y_v + d_{uv} \cdot (y_u - y_v) \cdot b_{uv} \right)}{\sum_{v \neq u} w_{uv}} \tag{3}$$

where

$$b_{uv} = \begin{cases} \frac{1}{\|p(u) - p(v)\|} & \text{if } \|p(u) - p(v)\| > 0, \\ 0 & \text{otherwise.} \end{cases}$$

This update is repeated until the relative change in the entire configuration is below a predefined threshold value, a predefined number of steps, or some other criterion. The sequence of layouts generated in this way can be shown to have non-increasing stress and to converge towards a local minimum [11].

3.2 Weights for Constraints

In early applications of MDS, each pair u, v of objects was assigned the same unit weight corresponding to $w_{uv} = 1$ in (1). When a target distance is unknown for some pair, it is simply ignored by using a zero weight for its contribution to the stress.

The standard weighted scheme for graph drawing uses $w_{uv} = d_{uv}^{-2}$. It was introduced as *elastic scaling* by McGee [21], and is equal to the one used by Kamada and Kawai [19]. Its superiority is due to an emphasis of small distances over large ones. This is because the fit of local distances is visually important, but also because it means that instead of fitting absolute values by minimizing *absolute* residual error terms

$$(d_{uv} - \|p(u) - p(v)\|)^2 \ ,$$

the objective is reformulated in *relative* error terms

$$(1 - \|p(u) - p(v)\|/d_{uv})^2 \ .$$

Summing these over all pairs gives

$$\sum_{u,v} \left(1 - \frac{\|p(u) - p(v)\|}{d_{uv}}\right)^2 = \sum_{u,v} \frac{1}{d_{uv}^2} \left(d_{uv} - \|p(u) - p(v)\|\right)^2.$$

A reason for the favorable aesthetic properties of low-stress layouts is that no node is preferred over others because minimization of the objective function is an attempt to achieve a balance in the fit of the desired distances. In most scenarios this is appropriate and tends to give the drawing a pleasing appearance.

In some cases, it may be desirable to put more emphasis on some nodes, while other nodes are regarded less important, for instance by centering the view on a node and visualizing this node's neighborhood more prominently. This can be done by introducing suitable constraints on the configuration. When these constraints can be formulated in terms of target distances, choosing the weights in a suitable way allows to impose them on the resulting layout without changing the layout algorithm.

What follows is a general framework for constrained graph drawing in scenarios in which constraints can be expressed in terms of target distances. While the range of possible applications is much wider, our contribution will concentrate on the radial layout scenario. To avoid confusion, objective function (1) will be referred to as *distance stress*, denoted by $\sigma_W(p)$. The subscript indicates that the stress defined using weight matrix $W = (w_{uv})_{uv} \in \mathbb{R}^{n \times n}$. This stress model is extended by a second set of weights $Z = (z_{uv})_{uv}$ used for the *constraint stress* defined by

$$\sigma_Z(p) = \sum_{u,v} z_{uv} \left(d_{uv} - \|p(u) - p(v)\|\right)^2 \ . \tag{4}$$

Its minimization is an attempt to fit the same distances and hence aims at representing the same information, but highlights different aspects.

3.3 Interpolated Weights

A straightforward approach to satisfy constraints associated with an additional weight matrix Z is to minimize (4) directly, say, after minimization of distance stress σ_W. This tends, however, to result in trivial solutions. Consider for instance radial constraints forcing each node $v \in V$ to be at distance r_v from the center. Clearly, we may end up in a layout with $x_v = r_v, y_v = 0$ from any initial configuration.

Instead, distance and constraint stress should be reduced simultaneously. An effective approach is to combine them into a joint majorization process, operating on a linear combination of the stress measures $\sigma_W(p)$ and $\sigma_Z(p)$ changing gradually in favor of the constraints.

Initially, nodes are allowed to move freely without considering constraints at all, by minimizing just $\sigma_W(p)$. Then, constraints are granted more and more control over the layout by dynamically changing coefficients in this combination, shifting the bias from one criterion to the other [4]. The relative influence of distance and radial components is determined by the coefficients in the linear combination

$$\sigma_{(1-t)\cdot W+t\cdot Z} = (1-t)\cdot\sigma_W(p) + t\cdot\sigma_Z(p)\ . \tag{5}$$

This is easily incorporated into the stress majorization process by changing update rules (2) and (3) to

$$\begin{aligned}
\hat{x}_u &\leftarrow \frac{\sum_{v\neq u}\big((1-t)\cdot w_{uv} + t\cdot z_{uv}\big)\cdot\big(x_v + d_{uv}\cdot(x_u - x_v)\cdot b_{uv}\big)}{\sum_{v\neq u}\big((1-t)\cdot w_{uv} + t\cdot z_{uv}\big)}\ ,\\
\hat{y}_u &\leftarrow \frac{\sum_{v\neq u}\big((1-t)\cdot w_{uv} + t\cdot z_{uv}\big)\cdot\big(y_v + d_{uv}\cdot(y_u - y_v)\cdot b_{uv}\big)}{\sum_{v\neq u}\big((1-t)\cdot w_{uv} + t\cdot z_{uv}\big)}\ .
\end{aligned}$$

In the majorization process, radial constraints are enforced neither directly nor immediately, so that the main visual features of the initial configuration can be preserved. The bias is shifted from the distance component towards the radial component by gradually increasing t from 0 to 1. When the number of iteration steps k is fixed, linear interpolation yields values $t = 0, \frac{1}{k}, \frac{2}{k}, \ldots, \frac{k-1}{k}, 1$. Otherwise, the iterative process may simply be repeated with a sequence of values for k converging to 1 from below until the layout is sufficiently stable. Using either variant, in each step, a slightly different objective function is sought to be minimized, and the current iterate preconditions the next step, thus smoothing the sequence of iterates.

The multidimensional scaling literature sometimes distinguishes different forms of constraints [17]. With *soft (weak)* constraints, solutions are allowed to deviate from the given constraints, and this deviation is penalized by additional stress. With *hard (strong)* constraints, only solutions which satisfy the constraints exactly are feasible.

In this terminology, an unconstrained MDS problem can be thought of as a special case of a weakly constrained problem, in which the deviation penalty is zero. In our case, arriving at $t = 1$ in (5) turns the weakly constrained problem into a strongly constrained one, provided that the set of constraints can be satisfied, i.e., a solution with zero constraint stress exists. In all other cases, it should be noted that, even though the distance component vanishes when $t \rightarrow 1$, minimizing $\sigma_{(1-t)\cdot W+t\cdot Z}(p)$ is *not* the same as minimizing $\sigma_Z(p)$ because of the running preconditioning described above.

4 Radial Layout

To illustrate the utilization of radial constraints for interest-based graph layout, we discuss three different scenarios in this section.

4.1 Focusing on a Node

In a *neighborhood diagram*, the focus is put on a node by distorting its surroundings. Here we constrain all others to be located at a Euclidean distances from the distinguished node that corresponds to their graph-theoretical distances from it, i.e., the distance-k neighborhood is mapped to the k circle centered on that node (which can be regarded as the geometric k-neighborhood).

To implement this design, the constraint weight matrix takes those pairs of nodes into account that the focal node, say v_i, is involved in, with all other weights reduced to zero. Matrices D and W are defined as above, and the constraint weight matrix $Z = (z_{uv})_{uv}$ has non-zero entries only in the i-th row and column

$$Z = \begin{bmatrix} 0 & \cdots & 0 & w_{v_1 v_i} & 0 & \cdots & 0 \\ \vdots & \ddots & 0 & \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & 0 & w_{v_{i-1} v_i} & 0 & \cdots & 0 \\ w_{v_i v_1} & \cdots & w_{v_i v_{i-1}} & 0 & w_{v_i v_{i+1}} & \cdots & w_{v_i v_n} \\ 0 & \cdots & 0 & w_{v_{i+1} v_i} & 0 & \cdots & 0 \\ \vdots & \ddots & 0 & \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & 0 & w_{v_n v_i} & 0 & \cdots & 0 \end{bmatrix} .$$

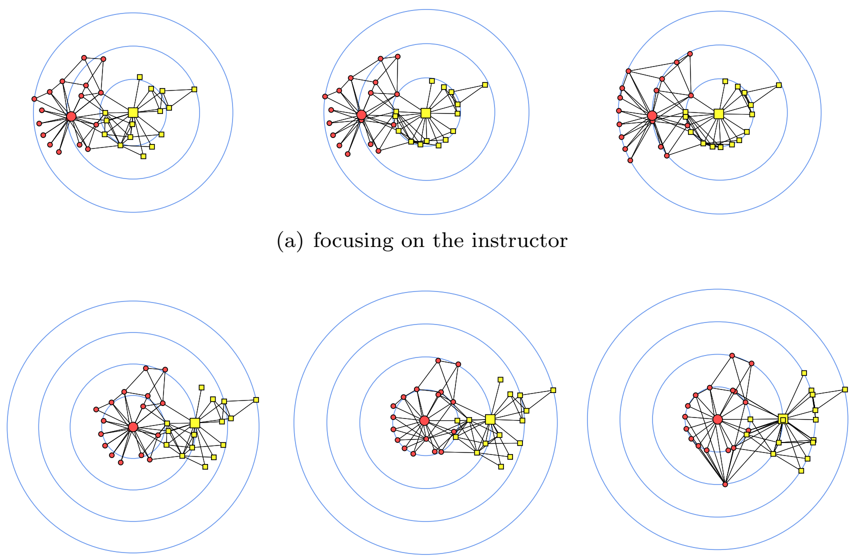
These are derived from the distances to the focal node, so that interpolating from W to Z gradually increases the focal node's relative impact on the configuration.

For dynamic visualization scenarios, an inherently smooth transition between layouts with different foci can be obtained by simply using the intermediate layouts given by the steps in the majorization process. In the transition from one focus to the other, it is advantageous not to interpolate directly between the two corresponding constraint weight matrices, but to take a detour via the original weight matrix having entries d_{uv}^{-2}, so as to re-introduce all the shortest-path distances to remove artifacts potentially introduced after focusing on the first node.

As an example, we consider a famous social network studied by Zachary and, subsequently, many others [27]. It describes friendship relations among 34 members in the karate club of a U.S. university in the 1970s. Over the course of a two-year study, the network breaks apart into two clubs because of disagreements between the administrator and the instructor, with the latter leaving the club and taking about half of the members with him. Following [22], this data set has been used frequently as a benchmark for the performance of various clustering approaches.

Fig. 2 shows how the same initial layout, which is computed by minimizing stress without constraints, is gradually modified into radial layouts, one focusing on the instructor and the other on the administrator.

The insight gained from Fig 2 is two-fold. Technically, it is visible that large parts of the overall shape of the layout are preserved well during the gradual relative increase of constraint stress. Substantively, we can see immediately that the decision to leave the club is in one-to-one correspondence with the presence



(a) focusing on the instructor

(b) focusing on the administrator

Figure 2: Radial layouts of Zachary's karate club network ($n = 34, m = 77$), by weight interpolation, for $t \in \{0, 0.9, 1\}$. Members leaving with the instructor are shown as yellow squares, members staying with the administrator as red circles.

in the neighborhood of the instructor or administrator. The only exceptions are the two members in each group that have direct ties with both of them. The two rightmost drawings clearly tell the whole story and also show that practically *any* reasonable clustering approach should be able to recover the division into those leaving and staying from the structure of friendships. Hence, this network is actually a very poor benchmark for the assessment of clustering approaches.

4.2 Centrality Drawings

A special property of the constraints in the previous section is that their target distances correspond directly to a column in distance matrix D. In *centrality drawings*, the requirement is that radii are given as part of the input, and therefore in general do not correspond to the distance from an existing focal node. It is, however, easy to augment the distance matrix accordingly.

Assume that nodes are numbered $v_1, \ldots, v_n$ and that the radii are given as additional input in a vector $r = [r_1, \ldots, r_n]^T \in \mathbb{R}^n$, with $r_i \geq 0$ for all $i = 1, \ldots, n$. Since radial constraints can be specified in terms of distances from the origin, we express then as

$$\|p(v_i)\| = r_i \ .$$

This way the origin can be incorporated as a dummy node v_{n+1} with artificial target distances $d_{v_i,v_{n+1}} = d_{v_{n+1},v_i} = r_i$, and the stress majorization procedure is applied to a layout problem of $n+1$ objects. Such a dummy is used, e.g., in [4] to enforce a circular configuration by using the same radius for all objects. Distance and weight matrices are set up for (5) as

$$D = \begin{bmatrix} d_{v_1v_1} & \cdots & d_{v_1v_n} & r_1 \\ \vdots & \ddots & \vdots & \vdots \\ d_{v_nv_1} & \cdots & d_{v_nv_n} & r_n \\ r_1 & \cdots & r_n & 0 \end{bmatrix} ,$$

$$W = \begin{bmatrix} d^{-2}_{v_1v_1} & \cdots & d^{-2}_{v_1v_n} & 0 \\ \vdots & \ddots & \vdots & \vdots \\ d^{-2}_{v_nv_1} & \cdots & d^{-2}_{v_nv_n} & 0 \\ 0 & \cdots & 0 & 0 \end{bmatrix} ,$$

$$Z = \begin{bmatrix} 0 & \cdots & 0 & r_1^{-2} \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & r_n^{-2} \\ r_1^{-2} & \cdots & r_n^{-2} & 0 \end{bmatrix} .$$

Let $c = (c_v)_{v \in V}$ be a centrality measure on the nodes of a graph $G = (V, E)$. Radii for nodes $v_i \in V = \{v_1, \ldots, v_n\}$ can be specified as

$$r_i = \frac{\mathrm{diam}(G)}{2} \cdot \left(1 - \frac{c_{v_i} - \min_{u \in V} c_u}{\max_{u \in V} c_u - \min_{u \in V} c_u + c(G)} \right) ,$$

where the factor of $\mathrm{diam}(G)/2$ serves to bring them to the same scale as the distances based on shortest paths used in the distance stress. The parameter $c(G)$ is a small offset that larger than zero if there are several nodes of maximum centrality [5]. As an alternative, non-linear scaling of centralities can be to emphasize the structure in different centrality intervals. For instance, the central (peripheral) areas are enlarged by applying a concave (convex) function magnifying regions of smaller (larger) centrality scores.

Examples of centrality drawings for Zachary's karate club network are shown in Figure 3. The left column is based on *closeness centrality* [24]

$$c_v = \frac{1}{\sum_{t \in V} d_{vt}} ,$$

which is simply the inverse average distance from a vertex to all others. The right column contains drawings based on *betweenness centrality* [14]

$$c_v = \sum_{s \neq v \neq t \in V} \delta(s, t|v) ,$$

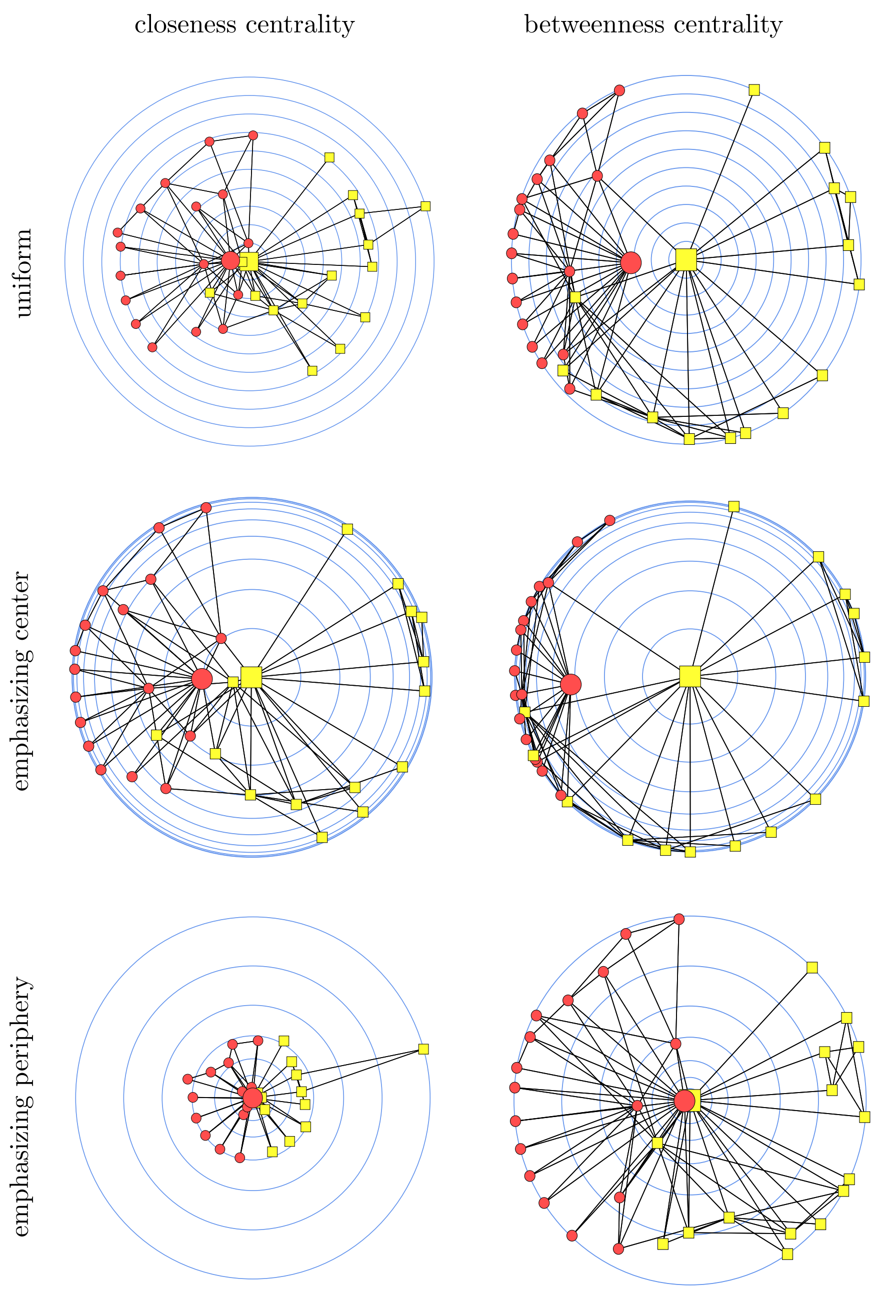


Figure 3: Centrality layouts of the karate club social network, using two common centrality measures to define the radii of nodes. Center and periphery are emphasized using transformed radii $r'_i = 1 - (1 - r_i)^3$ and $r'_i = r_i^3$ ($0 \leq r_i \leq 1$ and $0 \leq r'_i \leq 1$, respectively.

Algorithm 1: Layout with general radial constraints

Input: connected undirected graph $G = (V, E)$,
radii $r_v \in \mathbb{R}_{>0}$ for all $v \in V$, number of iterations $k \in \mathbb{N}$

Output: coordinates $p(v)$ with $\|p(v)\| = r_v$ for all $v \in V$

$D \leftarrow$ matrix of shortest path distances d_{uv}
$W \leftarrow$ matrix of inverse squared distances d_{uv}^{-2}
$p \leftarrow$ layout with low distance stress $\sigma_W(p)$
for $t = 0, \frac{1}{k}, \frac{2}{k}, \ldots, \frac{k-1}{k}, 1$ **do**
  for $v \in V$ **do**

$$x_v \leftarrow \frac{\sum_{u \in V \setminus \{v\}} (1-t) \cdot w_{uv}\big(x_u + d_{uv} \cdot (x_v - x_u) \cdot b_{uv}\big) + t \cdot r_v^{-2}(r_v x_v a_v)}{(1-t) \sum_{u \in V \setminus \{v\}} w_{uv} + t \cdot r_v^{-2}}$$

$$y_v \leftarrow \frac{\sum_{u \in V \setminus \{v\}} (1-t) \cdot w_{uv}\big(y_u + d_{uv} \cdot (y_v - y_u) \cdot b_{uv}\big) + t \cdot r_v^{-2}(r_v y_v a_v)}{(1-t) \sum_{u \in V \setminus \{v\}} w_{uv} + t \cdot r_v^{-2}}$$

where $\delta(s, t|v)$ is the dependency of $s, t \in V$ on $v \in V$, which is defined as the fraction of shortest (s, t)-paths that contain v as an inner vertex. Not surprisingly, both the instructor and the administrator are central according to any measures. It is interesting to note, however, that this is due to the fact that they integrate largely separate neighborhoods. The layouts reveal that closeness values have a higher resolution in the center, whereas betweenness has more variance in the periphery. These diagrams should not be seen as part of a serious exploration, though, but as mere illustrations of possible use cases.

Simplified pseudo-code for general radial constraints is given in Algorithm 1, where quantities a_v are defined as

$$a_v = \begin{cases} \frac{1}{\|p(v)\|} & \text{if } \|p(v)\| > 0, \\ 0 & \text{otherwise .} \end{cases}$$

4.3 Travel Time Maps

Schematic maps have become an essential guide for travelers in public transportation systems. Such maps commonly depict lines, stations, zones, and connections to other traffic systems. Since the primary use of such maps is for travel planning, usability and readability are more important criteria than the accurate representation of actual geographic positions. In the graph drawing literature, this drawing style is called *metro map layout* (see, e.g., [18] for a force-directed approach).

The seminal design is Harry Beck's map of London Underground, commonly known as the Tube. It has been and still is being reworked and improved, and it has inspired similar maps for systems of public transportation all over the world. While schematic maps are widely perceived as very useful, a potential drawback is that they tend to distort a user's perception of distance, thus potentially compromising decisions made in the travel planning process, e.g., because stations are displayed as more proximate than they actually are.

If the starting and ending stations of a planned journey are known, radial constraints can be used to highlight the time needed for traveling between them by focusing only on the starting station as described above. Alternatively, shortest paths between the two stations can be highlighted by putting the focus on both of them at the same time.

Again, $D, W \in \mathbb{R}^{n\times n}$ are defined as the matrices of shortest-path distances and their inverse squares, respectively. The constraint weight matrix is set to

$$Z = \begin{bmatrix} 0 & \cdots & 0 & w_{v_1,v_{n-1}} & w_{v_1,v_n} \\ \vdots & \ddots & 0 & \vdots & \vdots \\ 0 & \cdots & 0 & w_{v_{n-2}v_{n-1}} & w_{v_{n-2}v_n} \\ w_{v_{n-1},v_1} & \cdots & w_{v_{n-1},v_{n-2}} & 0 & w_{v_{n-1}v_n} \\ w_{v_n,v_1} & \cdots & w_{v_n,v_{n-2}} & w_{v_nv_{n-1}} & 0 \end{bmatrix}, \qquad (6)$$

where v_{n-1} and v_n are assumed to be the focal nodes. When interpolating from the original weight matrix W to the constraint weight matrix Z, distances to (and between) the two focused nodes become increasingly influential.

As an example we use a connection graph of the Tube with approximate station locations and travel times.[1] Radial layouts are given in Fig. 4, where stations are placed at a distance from the center proportional to their estimated minimum travel times from two sample stations. Since travel times are only approximately related to shortest-path distance, these examples are more closely related to centrality drawings than to neighborhood diagrams.

Their combination is shown in Fig 5. Even though this map is only an experimental illustration of a scenario with two foci, it does convey a sense of alternate direct routes and detours.

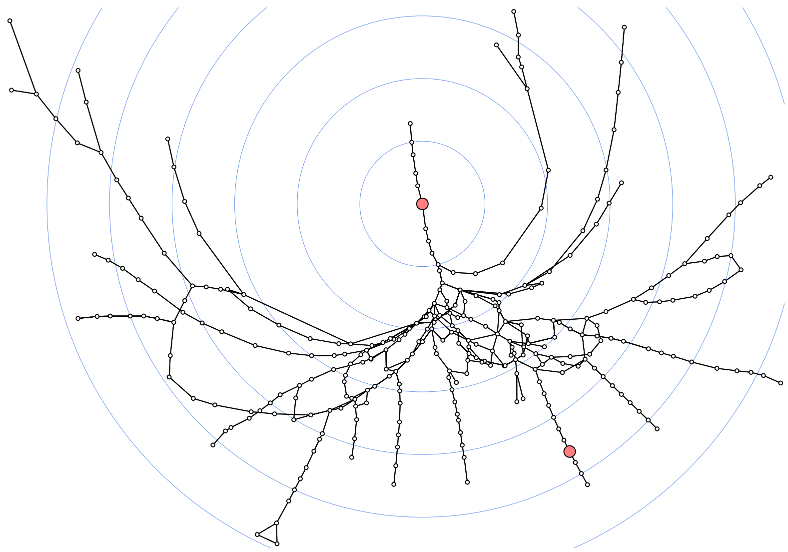
5 Discussion

We argued that radial constraints fit well into the framework of multidimensional scaling by stress majorization with a penalty function.

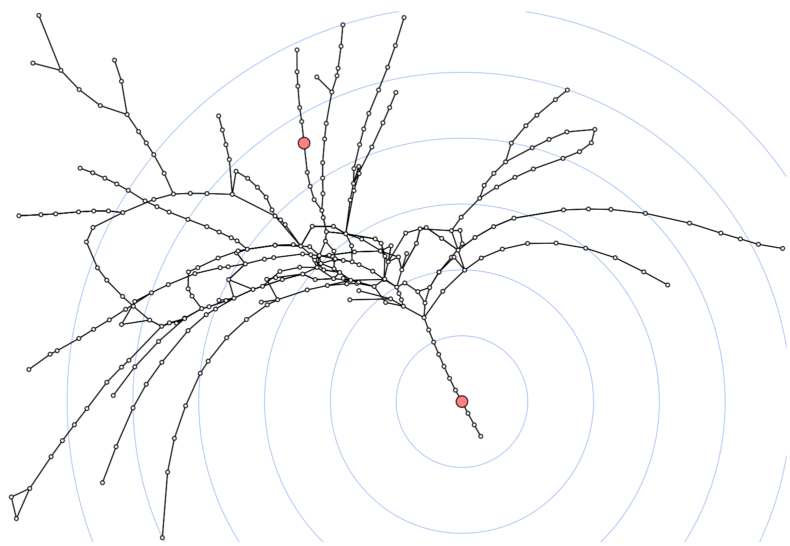
An obvious advantage is the simplicity of our approach, because radii can be expressed in terms of target distances and thus require only minor modifications of available implementations for stress minimization.

Since the method can be initialized with any layout and constraints are introduced only gradually, we are likely to end up in a feasible solution close to the initial one. While sensitivity to initialization is usually a disadvantage of iterative layout methods, it is very welcome in the present scenario, because it instills hope that some properties of a high-quality unconstrained layout can

[1] Made available by Tom Carden at `http://www.tom-carden.co.uk/p5/tube_map_travel_times/applet/`.

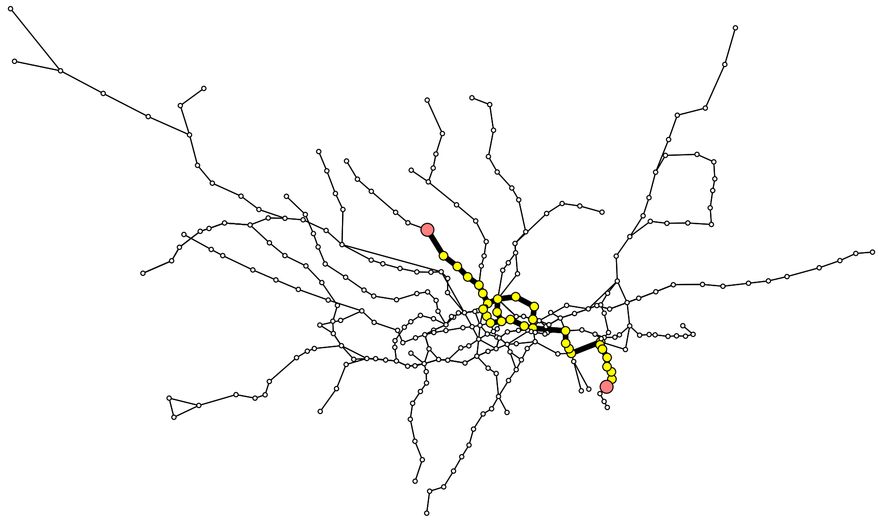


(a) travel time from Golders Green

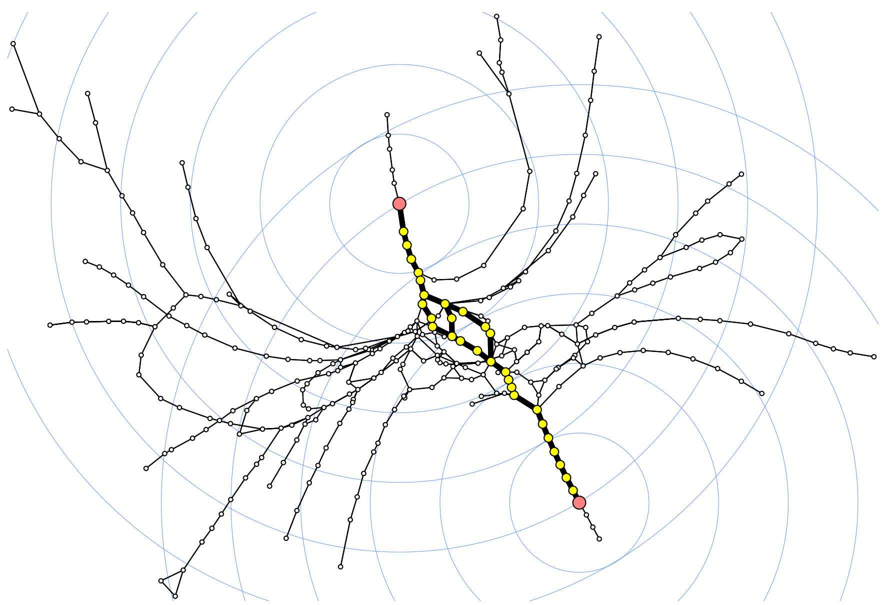


(b) travel time from Greenwich

Figure 4: Radial layouts of the London Tube graph using estimated travel times. The concentric circles indicate travel times in multiples of 10 minutes. The stations are constrained to be at distance equal to their minimum travel times.



(a) geographically accurate



(b) dual-focus radial layout with circles in 10min intervals

Figure 5: Tube graph with fastest routes between Golders Green and Greenwich highlighted using thicker edges.

be preserved in the solution obtained. Together with the greater degree of freedom during most of the process, it is possible that stress majorization with penalty functions is not only simpler, but also more effective than gradient-projection methods [12] which maintain a feasible solution throughout. An in-depth comparison is therefore an important direction for future research.

Other possible extensions of this work include its use in approximate constraint satisfaction (by stopping the iterations before constraints become dominant) or animation. Moreover, the approach generalizes to other constraints expressible as targets distances and an thus be used to enforce, e.g., desired groupings or variation in the contribution of substructures to the overall layout.

References

[1] C. Bachmaier. A radial adaption of the Sugiyama framework for visualizing hierarchical information. *IEEE Transactions on Visualization and Computer Graphics*, 13(3):583–594, 2007.

[2] M. Baur and U. Brandes. Crossing reduction in circular layouts. In J. Hromkovič, M. Nagl, and B. Westfechtel, editors, *Proceedings of the 30th International Workshop on Graph-Theoretical Concepts in Computer Science (WG'04)*, volume 3353 of *Springer LNCS*, pages 332–343, 2004.

[3] I. Borg and P. Groenen. *Modern Multidimensional Scaling*. Springer, 2005.

[4] I. Borg and J. Lingoes. A model and algorithm for multidimensional scaling with external constraints on the distances. *Psychometrika*, 45(1):25–38, 1980.

[5] U. Brandes, P. Kenis, and D. Wagner. Communicating centrality in policy network drawings. *IEEE Transactions on Visualization and Computer Graphics*, 9(2):241–253, 2003.

[6] U. Brandes and C. Pich. Eigensolver methods for progressive multidimensional scaling of large data. In M. Kaufmann and D. Wagner, editors, *Proceedings of the 14th International Symposium on Graph Drawing (GD'06)*, volume 4372 of *Springer LNCS*, pages 42–53, 2007.

[7] U. Brandes and C. Pich. An experimental study on distance-based graph drawing. In *Proceedings of the 16th International Symposium on Graph Drawing (GD'08)*, volume 5417 of *Springer LNCS*, pages 218–229, 2009.

[8] M.-J. Carpano. Automatic display of hierarchized graphs for computer-aided decision analysis. *IEEE Transactions on Systems, Man and Cybernetics*, 10(11):705–715, 1980.

[9] T. F. Cox and M. A. A. Cox. *Multidimensional Scaling*. CRC/Chapman and Hall, 2001.

[10] J. de Leeuw. Applications of convex analysis to multidimensional scaling. In J. R. Barra, F. Brodeau, G. Romier, and B. van Cutsem, editors, *Recent Developments in Statistics*, pages 133–145. Amsterdam: North-Holland, 1977.

[11] J. de Leeuw. Convergence of the majorization method for multidimensional scaling. *Journal of Classification*, 5(2):163–180, 1988.

[12] T. Dwyer, Y. Koren, and K. Marriott. Constrained graph layout by stress majorization and gradient projection. *Discrete Applied Mathematics*, 309:1895–1908, 2008.

[13] T. Dwyer, K. Marriott, and M. Wybrow. Topology preserving constrained graph layout. In *Proceedings of the 16th International Symposium in Graph Drawing (GD'08)*, volume 5417 of *Springer LNCS*, pages 230–241, 2009.

[14] L. C. Freeman. A set of measures of centrality based on betweenness. *Sociometry*, 40:35–41, 1977.

[15] E. R. Gansner and Y. Hu. Efficient node overlap removal using a proximity stress model. In *Proceedings of the 16th International Symposium in Graph Drawing (GD'08)*, volume 5417 of *Springer LNCS*, pages 206–217, 2009.

[16] E. R. Gansner, Y. Koren, and S. North. Graph drawing by stress majorization. In *Proceedings of the 11th International Symposium in Graph Drawing (GD'03)*, volume 2912 of *Springer LNCS*, pages 239–250, 2004.

[17] W. J. Heiser and J. Meulman. Constrained multidimensional scaling, including confirmation. *Applied Psychological Measurement*, 7(4):381–404, 1983.

[18] S.-H. Hong, D. Merrick, and H. A. D. do Nascimiento. The metro map layout problem. In *Proceedings of the 2004 Australasian symposium on Information Visualisation*, ACM International Conference Proceeding Series, pages 91–100, 2004.

[19] T. Kamada and S. Kawai. An algorithm for drawing general undirected graphs. *Information Processing Letters*, 31:7–15, 1989.

[20] Y. Koren and A. Çivril. The binary stress model for graph drawing. In *Proceedings of the 16th International Symposium in Graph Drawing (GD'08)*, volume 5417 of *Springer LNCS*, pages 193–205, 2009.

[21] V. E. McGee. The multidimensional scaling of "elastic" distances. *The British Journal of Mathematical and Statistical Psychology*, 19:181–196, 1966.

[22] M. E. J. Newman and M. Girvan. Finding and evaluating community structure in networks. *Physical Review E*, 69:026113, 2004.

[23] M. L. Northway. A method for depicting social relationships obtained by sociometric testing. *Sociometrics*, 3:144–150, 1940.

[24] G. Sabidussi. The centrality index of a graph. *Psychometrika*, 31:581–603, 1966.

[25] G. J. Wills. NicheWorks – interactive visualization of very large graphs. In *Proceedings of the 5th International Symposium in Graph Drawing (GD'97)*, pages 403–414, 1997.

[26] K.-P. Yee, D. Fisher, R. Dhamija, and M. Hearst. Animated exploration of dynamic graphs with radial layout. In *Proceedings of the IEEE Symposium on Information Visualization 2001 (InfoVis '01)*, pages 43–50, 2001.

[27] W. W. Zachary. An information flow model for conflict and fission in small groups. *Journal of Anthropological Research*, 33:452–473, 1977.