

## On truth-conditions for *if* (but not quite only *if*)\*

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### 1 The Folklore

Folklore has it that there is no good way of assigning truth-conditions to ordinary, indicative *if*'s in such a way that they are bounded from above by strict implication, from below by material implication, and *if*'s like

- (1) If Carl is away, then if Lenny is away then Sector 7G is empty

are rendered equivalent to

- (2) If Carl is away and Lenny is away, then Sector 7G is empty.

For — according to the lore — any attempt at pulling off this feat will reduce them to material implication, and that is decidedly not a good way of assigning truth-conditions to ordinary, indicative *if*'s. That is too bad since it seems like they *are* bounded from above by strict implication (they are true when their antecedents straight-out entail their consequents), *are* bounded from below by material implication (they can't be true when their antecedents are true but consequents false), and *do* go in for the kind of equivalence between (1) and (2).

Even before making the folklore argument less rough — and we will — we can see that something has to go. But choices seem hard to come by. Perhaps, as some say, we should give up on thinking that (1) and (2) are true in just the same spots. Variably strict stories about *if*, built atop an ordering of possibilities,

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do just that (Stalnaker, 1975). Or perhaps, as others say, it's our prejudice against the material conditional that needs reforming. True, that would give us more entailments than we want, the paradoxes of material implication among them. But maybe we can explain those away by appeal to the implicatures, conversational or conventional, of ordinary indicative *if*'s (Grice, 1967; Lewis, 1976; Jackson, 1987). Or perhaps, as still others say, the reason we can't find truth-conditions fitting the bill is that *if*'s don't have truth-conditions at all — they serve only to express conditional attitudes (Gibbard, 1981; Edgington, 1995; Bennett, 2003).

With choices like these it would be best not to choose at all. Something *does* have to go, all right, but I say it is the folklore itself that gets the boot. It goes wrong by assuming a wrong picture about what an assignment of truth-conditions must be like. I will demonstrate that twice over.

## 2 The Argument

But first let's make the choices clearer by making the trouble the folklore promises clearer.

Fix a propositional language  $L$  to serve as an intermediate language — bits of natural language get assigned meanings by associating them with sentences of  $L$ , which themselves are interpreted. Assume that  $L$  is equipped with a set of atoms, the connectives  $\neg$ ,  $\wedge$ , and the binary connective (*if*  $\cdot$ )  $(\cdot)$ .<sup>1</sup> Unless we say otherwise,  $p, q, r, \dots$  range over the non-iffy fragment of  $L$ , and  $P, Q, R, \dots$  over arbitrary sentences of  $L$ .

One way of proceeding — Gibbard's (1981) way — begins with a complex *if* like

$$(3) \quad (\textit{if } p \supset q)((\textit{if } p)(q))$$

The argument is that — given the constraints that *if* is bounded from above by strict implication, from below by material implication, and that the likes of (1) entail and are entailed by the likes of (2) — this is everywhere true and that it entails the corresponding material conditional  $(p \supset q) \supset ((\textit{if } p)(q))$ . Anything entailed by something everywhere true must be everywhere true, and a material

<sup>1</sup>Some — e.g., Kratzer (1986) — deny that *if* is a connective in the first place, and take it instead to be a device whose only job is to mark the restriction of some other operator. I say that *if*'s restricting job is a job done best by taking it to be a connective. But that is an argument for another day — see Gillies (2007b).

conditional everywhere true is just one whose antecedent entails its consequent. Whence it follows  $p \supset q$  entails the ordinary indicative  $(if\ p)(q)$ .<sup>2</sup>

But this way of putting things assumes more than is required by assuming that *if* can be scoped under arbitrary connectives. Although I see why a defender of truth-conditions for *if* often *does* say that, I see no reason why she *must*. So we should put the argument in a way that does not assume it.<sup>3</sup>

Assume that — for any non-iffy  $p, q$ , and  $r$  — the following all hold:

**(Upper Bound):** if  $p$  entails  $q$  then, for any world  $i$ ,  $(if\ p)(q)$  is true at  $i$

**(Lower Bound):** if  $p \wedge \neg q$  is true at  $i$  then  $(if\ p)(q)$  can't be

**(Import):**  $(if\ p \wedge q)(r)$  entails  $(if\ p)((if\ q)(r))$

Then the folklore has it that the ordinary, indicative *if* is just the material conditional after all. Precisely: for any world  $i$ ,  $(if\ p)(q)$  is true at  $i$  iff either  $p$  is false at  $i$  or  $q$  is true at  $i$ .

*Proof.* The left-to-right direction is not in dispute. The other direction is secured by two rather dull entailments. Suppose  $p$  is false at  $i$ . Since  $(\neg p \wedge p)$  entails  $q$  — and since by (U) *if* is bounded from above strict implication — then for any world whatever

$$(4) \quad (if\ \neg p \wedge p)(q)$$

is true at that world, hence at  $i$ . So, by (I), the complex *if*

$$(5) \quad (if\ \neg p)((if\ p)(q))$$

must also be true at  $i$ . Since  $\neg p$  is true at  $i$ , the embedded conditional  $(if\ p)(q)$  cannot be false at  $i$  — else *if* would not be bounded from below by material implication, violating (L). And so it must be true at  $i$ .

Suppose  $q$  is true at  $i$ . Since  $(q \wedge p)$  entails  $q$  — and since by (U) *if* is bounded from above strict implication — then for any world whatever

$$(6) \quad (if\ q \wedge p)(q)$$

<sup>2</sup>See Gibbard (1981, pp. 234–235). This particular way of telling the story has been retold often — see, e.g., Bennett (2003); Edgington (1995, 2006); Kratzer (1986).

<sup>3</sup>A similar version of this way of putting things can be found in Veltman (1985). If indicatives support a deduction theorem —  $r$  and  $p$  conspire to entail  $q$  iff  $r$  alone entails  $(if\ p)(q)$  — then we are in trouble enough, for then the equivalence between (1) and (2), and hence (I), follows straightaway.

is true at that world, hence at  $i$ . So, by (I), the complex *if*

$$(7) \quad (\textit{if } q)((\textit{if } p)(q))$$

must also be true at  $i$ . Since  $q$  is true at  $i$ , the embedded conditional  $(\textit{if } p)(q)$  cannot be false at  $i$ —else *if* would not be bounded from below by material implication, violating (L). And so it must be true at  $i$ .  $\square$

It thus appears that there is no good way of assigning truth-conditions to ordinary, indicative *if*s— not, at least, if we constrain them in the ways we have. But appearances can mislead.

I favor a view according to which ordinary, indicative *if*s are strict conditionals over the set of relevant possibilities in a context. Roughly put: they say that all the antecedent possibilities compatible with the context are also consequent possibilities. But my aim here is not to defend this hypothesis.<sup>4</sup> Instead I want to tell enough of the strict conditional story to show that the folklore needs revising. I will tell two versions of that story.

### 3 First Version

The first version begins in familiar territory. Sentences get truth values at an index—a world—with respect to a context. A context determines the set of possibilities compatible with the relevant information in that context. These contexts are idle when it comes to interpreting sentences of  $L$  with no *if*s since they are, *ex hypothesi*, context invariant: a negation is true at a world (in a context) iff the negated claim isn't true at that world (in that context); a conjunction is true just in case its conjuncts are. But, so the strict conditional story goes, the iffy bits depend on a set of possibilities compatible with the relevant information. It is the job of a context to determine such a set. Thus—assuming that *if* is indeed a strict conditional over such a set—the meaning of *if* is context dependent. But it is not unruly.

I have not said very much about what contexts are, and I do not want to start now. But I will insist on what I did say: they determine sets of relevant possibilities over which *if* is a strict conditional. Since that is the only contribution of contexts we'll care about, we can just as well identify contexts with the

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<sup>4</sup>Some of that is done elsewhere—see, e.g., Gillies (2004, 2007b). I favor a similar line on counterfactuals, though—naturally enough—the domains over which the two sorts of conditionals are strict tend to differ (Gillies, 2007a). There are, to be sure, other strict conditional stories for indicatives on the market, though none that tell the story in just the way I do.

functions that determine such sets: they are functions from indices to sets of indices. Such functions, I will assume, are *well-behaved*:

**Constraint (WELL-BEHAVEDNESS).**  $c$  is a context only if for any worlds  $i$  and  $j$ :

1.  $i \in c(i)$
2. if  $j \in c(i)$  then  $c(i) \subseteq c(j)$

Thus, no matter the context: (1) the facts at  $i$  are always relevant, though perhaps not decisive, to the truth of a conditional at  $i$ ; and (2) if a possibility is live in a context, then that is a settled matter. Together the constraints straightaway imply that the set of relevant worlds in a context is *closed*: if  $j \in c(i)$ , then  $c(i) = c(j)$ . We can gloss  $c(i)$  as the set of possibilities compatible with the context  $c$  is meant to represent. That such sets are closed means that possibilities relevant in a context do not vary between worlds compatible with it.

The schoolyard version of the Ramsey test says that an indicative conditional is true in a context just when adding the information carried by its antecedent to that context leaves us in a situation in which the consequent is true. That is very nearly the strict conditional story I want to tell.

Since truth depends on both an index and a context, adding the information carried by the antecedent affects two changes and accordingly the *if*-clause has two jobs to do. First: it restricts the set of indices throughout which we check for the consequent's truth. We look to see whether, throughout the set of indices at which the antecedent is true, the consequent is also true. Second: it contributes to the relevant context for figuring out whether — at an index in that set — the consequent is true. If the consequent is context-invariant, then this second job is trivially done. But not otherwise.

Making both jobs explicit: an indicative  $(if P)(Q)$  is true — at  $i$ , with respect to  $c$  — iff all the worlds in  $c(i)$  at which  $P$  is true with respect to  $c$  are all worlds at which  $Q$  is true with respect to  $c$ -plus-the-information-that- $P$ . Let  $c + P$  be the subordinate or derived context got by adding the information carried by  $P$  to  $c$ . That is how the truth of an indicative is sensitive to both jobs assigned by the schoolyard version of the Ramsey test: that we look to only  $P$ -possibilities in  $c(i)$  is how the first job gets done; that we shift the context from  $c$  to  $c + P$  for evaluating  $Q$  is how the second job gets done.

Given our current ambitions, we can assume that adding information to  $c$  is proper:  $(c + P)$  carries the information carried by  $P$  and contains at least as much information as  $c(i)$ . And the simplest policy that enforces that takes, for

any  $i$ ,  $(c + P)(i)$  to be  $c(i)$  restricted by the proposition expressed by  $P$  in  $c$ . Officially:

**Definition 1** (STRICT CONDITIONAL, v.1.0).

1.  $c + P = \lambda i.c(i) \cap \llbracket P \rrbracket^c$
2.  $\llbracket (\text{if } P)(Q) \rrbracket^{c,i} = 1$  iff  $c(i) \cap \llbracket P \rrbracket^c \subseteq \llbracket Q \rrbracket^{c+P}$

This is enough to disrupt the legend.

*Proof.* (U): Suppose  $p$  entails  $q$ . Then all the  $p$ -worlds are  $q$ -worlds. But then—no matter the world  $i$  or context  $c$ — $c(i) \cap \llbracket p \rrbracket \subseteq \llbracket q \rrbracket$ , and so  $(\text{if } p)(q)$  is bound to be true at  $i$  in  $c$ . (L): Suppose that  $(p \wedge \neg q)$  is true at  $i$ . Then—no matter the context— $i$  is one of the worlds relevant to an *if* at  $i$ . And so there is a  $(p \wedge \neg q)$ -world in  $c(i)$ , and thus  $(\text{if } p)(q)$  is bound to be false at  $i$ . (I): Suppose—for arbitrary  $c$  and  $i$ —that  $\llbracket (\text{if } p \wedge q)(r) \rrbracket^{c,i} = 1$ . Then all of the  $(p \wedge q)$ -worlds in  $c(i)$  are worlds at which  $r$  is true—with respect to  $c$ . To see that all the  $p$ -worlds in  $c(i)$  are worlds at which  $(\text{if } q)(r)$  is true—with respect to  $c + p$ —consider any  $p$ -world in  $c(i)$ . Call it  $j$ , and note that

$$\begin{aligned} \llbracket (\text{if } q)(r) \rrbracket^{c+p,j} = 1 &\text{ iff } (c + p)(j) \cap \llbracket q \rrbracket \subseteq \llbracket r \rrbracket \\ &\text{ iff } (c(j) \cap \llbracket p \rrbracket) \cap \llbracket q \rrbracket \subseteq \llbracket r \rrbracket \\ &\text{ iff } (c(i) \cap \llbracket p \rrbracket) \cap \llbracket q \rrbracket \subseteq \llbracket r \rrbracket \\ &\text{ iff } \llbracket (\text{if } p \wedge q)(r) \rrbracket^{c,i} = 1 \end{aligned}$$

(Each step is a straight application of Definition 1, except for the third which appeals to the well-behavedness of  $c$ .) Whence it follows that if  $(\text{if } p \wedge q)(r)$  is true—at  $i$ , in  $c$ —then so must be  $(\text{if } p)((\text{if } q)(r))$ . Indeed, they are equivalent.

And yet: *if* is not the material conditional. For let  $c(i)$  contain two worlds,  $i$  and  $j$ , the first a  $(p \wedge q)$ -world and the second a  $(p \wedge \neg q)$ -world. Then at  $i$  in  $c$ :  $p \supset q$  is true, but  $(\text{if } p)(q)$  is false.  $\square$

#### 4 Intermezzo

There are two noteworthy features of this strict conditional story. First: it treats *if* as doubly context dependent. And second: it treats *if* as a doubly shifty operator. Take each in turn.

It is a well-worn fact that, at least for some sentences, truth values at an index depend also on features of context. A sentence of our language  $L$  is

*locally* context dependent just in case it is possible that its truth value at a given index  $i$  varies across contexts. There is nothing especially noteworthy in that. But perhaps there are sentences whose truth values depend not only on matters of context but also have their truth values in a context uniformly: whatever truth value they get — at a world, in a context — they get in all worlds compatible with that context. Any sentence so dependent would either be true at any admissible world with respect to the context or true at none with respect to that context. Such sentences are *globally* dependent on features of context.

According to the first way of telling the strict conditional story, *if* is context dependent, both locally and globally. It is locally context dependent since an *if* at  $i$  can be false in one context  $c$  but true in another  $c'$  because the first does, and the second does not, have counterexamplifying possibilities compatible with it. And globally: an ordinary, indicative  $(if\ p)(q)$  is true — at  $i$  in  $c$  — just in case all of the  $p$ -worlds in  $c(i)$  are  $q$ -worlds;  $c$  is closed, so if  $j$  is in  $c(i)$  then  $c(j) = c(i)$ ; and hence  $(if\ p)(q)$  is true at  $i$  in  $c$  iff it is also be true at  $j$  in  $c$ . That makes global context dependence in  $L$  due to the fact that *if* is a global modality, restricted to the context  $c$ .<sup>5</sup>

There is an argument — championed by Edgington (1995, 2006) — that no non-truth-functional assignment of truth-values to *if* can be right because the barest information supporting the material conditional seems also and always to be sufficient for the ordinary, indicative *if*. What makes a non-truth-functional story non-truth-functional is that the truth value of  $(if\ p)(q)$  — at  $i$ , in  $c$  — is not fixed by the truth values of  $p$  and  $q$  at  $i$ . And so if  $p$  is false and  $q$  is true at  $i$  then perhaps the conditional is true at  $i$  and perhaps it isn't. Both are possible. But then the barest information supporting the material conditional — i.e., just the information that  $(\neg p \vee q)$  — would not also and always be enough to support  $(if\ p)(q)$ . The variability in truth value of an *if* with a false antecedent and true consequent that is required by non-truth-functionality is just what is ruled out by the supporting facts. No non-truth-functional theory can get this right.

Not quite! This is true: if  $p$  is false and  $q$  is true at  $i$  then perhaps the conditional is true at  $i$  and perhaps it isn't. But this variability may well be a variation in truth value across contexts, not within any one context. And variability like that does nothing to preclude the supporting facts. For any

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<sup>5</sup>Given a standard relational structure  $\langle W, R \rangle$ , with set of indices  $W$  and accessibilities between them as recorded in  $R$ , a *global modality* is an operator  $A$  such that  $\models_i Ap$  iff  $\models_j Ap$  for any  $i, j \in W$ . An example: suppose we introduce an operator  $E$  such that  $\models_i Ep$  iff for some  $j \in W \models_j p$ . According to this version of the strict conditional story, *if* is a global modality whose range has been restricted to the possibilities compatible with  $c$ .

given  $c$  it is open to have  $(if\ p)(q)$  get the same truth value at any two worlds compatible with  $c$ , and so at any two at which  $p$  is false. That is just what the strict conditional story as I have told it does. It delivers a truth value uniformly throughout the set of worlds compatible with  $c$  at  $i$ : iff  $c(i)$  has any  $(p \wedge \neg q)$ -worlds is the indicative false throughout. But things may be different in different contexts: a context minimally characterizing that  $(\neg p \vee q)$  has no  $(p \wedge \neg q)$ -worlds — and so the indicative is true therein. The argument misfires because it overlooks the possibility that the non-truth-functional truth conditions of *if*'s might be both locally and globally dependent on features of context.

It is a well-worn fact that the truth values — at an index, in a context — of some sentences depend on the truth values of their embedded constituents at different indices. Such sentences are *index-shifty*. Thus tense operators:

(8) Jimbo washed the dishes

is true at an index (in a context) just in case *Jimbo washes the dishes* is true at a (recent-ish) earlier index. And modal operators:

(9) Jimbo has to wash the dishes

is true at an index (in a context) just in case *Jimbo washes the dishes* is true at all the indices compatible with the house rules.

We might imagine another kind of shiftiness: the truth values — at an index, in a context — of some sentences might depend on the truth values of their embedded constituents at ever so slightly different contexts. Such sentences would be *context-shifty*.

According to the first way of telling the strict conditional story, *if* is certainly index-shifty. Whether

(10) If Carl is at the party, then Lenny is at the party

is true — at  $i$ , in  $c$  — depends on whether *Lenny is at the party* is true at various other worlds: those antecedent-worlds compatible with the context. But it is also context-shifty since the consequent is evaluated in a subordinate or derived context: *Lenny is at the party* is evaluated not in  $c$  but in  $c$ -plus-the-information-that-Carl-is-at-the-party. This is a shift that makes no difference if — as in this case — the consequent has no context sensitive bits in it.

But only *if*. That is how it can be that *if* is a strict conditional that validates

the equivalence between  $(if\ p \wedge q)(r)$  and  $(if\ p)((if\ q)(r))$ . Taking *if* to be a merely index-shifty strict conditional, given well-behavedness, could not do that. Mere index-shiftiness would require only that all of the antecedent-worlds compatible with the context be worlds at which the consequent is true — true, that is, with respect to the context as it was when the conditional was issued. Such a merely index-shifty story thus assigns truth-conditions as follows:

$$(11) \quad \llbracket (if\ P)(Q) \rrbracket^{c,i} = 1 \text{ iff } c(i) \cap \llbracket P \rrbracket^c \subseteq \llbracket Q \rrbracket^c$$

No such story about *if* renders the likes of (1) equivalent to the likes of (2).

Example: Take any  $p, q$ , and  $r$ . Suppose it is settled in  $c$  that either all of them are true, or just one is. Then  $(if\ p \wedge q)(r)$  is true at a world compatible with  $c$  — say  $i$  — since all of the  $(p \wedge q)$ -worlds compatible with  $c$  are indeed  $r$ -worlds. But assuming mere index-shiftiness,  $(if\ p)((if\ q)(r))$  is not true at  $i$ , in  $c$ . There is a  $(q \wedge \neg r)$ -world compatible with the context. Whence it follows that at no world not yet ruled out is  $(if\ q)(r)$  true — true, that is, with respect to  $c$ . *A fortiori* at no  $p$ -world not yet ruled out is  $(if\ q)(r)$  true with respect to  $c$ .

Mere index-shiftiness would be enough if the schoolyard version of the Ramsey test only assigned one job to *if*-clauses, the job of restricting the domain throughout which we check for the consequent's truth. Truth depends on both index and context, though, and so the *if*-clause has its second job to do. So more than mere index-shiftiness is required. That is why indicatives are also context-shifty.

## 5 Second Version

The first version of the strict conditional story began in familiar territory and proceeded by assigning to the doubly shifty *if*'s doubly context dependent propositions. That makes the truth of an *if* — at a world, in a context — sensitive to both that context and a subordinate context got by hypothetically adding some information to it.

But we might begin elsewhere.<sup>6</sup> Rather than assigning truth values to sentences of  $L$  at indices (in contexts) let's assign them truth values in contexts, full-stop. Before  $c$  was a function from worlds to sets of worlds compatible with the relevant information in  $c$ . Now let's simply identify a context  $s$  with

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<sup>6</sup>See Gillies (2004).

the set of worlds compatible with it. (As before, assume that the world at which an *if* is issued is always among the worlds compatible with the context; that is, the facts are always relevant, though perhaps not decisive, to the truth of an indicative.) The semantic values of sentences of  $L$  are their *context change potentials*: how they change the contexts in which they are successfully issued. A sentence is true in a context, we will say, just in case applying its semantic value to the context idles. That is: iff an assertive utterance of it in that context adds no information to it — for then the information carried by the sentence is already present. To say all of this properly we will have to say what it means for a sentence to add its information to a context.

Taking semantic values to be context change potentials, rather than propositions or truth-values or some other apparently more pedestrian thing, is not exotic. It is simply to take the meanings of declarative sentences to be like the meanings of recipes and programs. A program  $\pi$  means what it does, and what it does depends on the state in which it is executed. So what  $\pi$  means — its denotation  $\llbracket \pi \rrbracket$  — is the set of pairs of states such that executing  $\pi$  in the first member of the pair terminates in the second. On this way of telling the strict conditional story, assertive utterances — including assertive utterances of *if*s — are programs for changing contexts.

Some programs have as their point non-null effects: `set the value of  $x$  to 1`. This should affect a change, the setting of  $x$  to 1. Non-iffy constructions have as their point the non-null effect of making the context reflect the information they carry. Thus the changes they induce are straightforward: atomic sentences remove worlds from the context in which the atoms are not true, negation is relative complementation, and conjunction is functional composition. But not all programs are like this. Some programs have as their point null effects: `check whether the value of  $x$  is 1`. Such a program — a *test* program — returns either its input state (if the value of  $x$  is 1) or fails (otherwise). And *if*s, according to this version of the strict conditional story, are like these test programs. An *if* tests to see whether, in a context in which it is issued, the information carried by the antecedent brings the consequent in its wake. If so, the context passes the test posed by the conditional; otherwise, not.

Take the straightforward updating behavior of the non-iffy bits as a constraint:

**Constraint** (NON-IFFY UPDATES). For any  $s$ , atom  $p$ , and arbitrary  $P, Q$ :

1.  $s[p] = \{i \in s : i(p) = 1\}$

2.  $s[\neg P] = s \setminus s[P]$
3.  $s[P \wedge Q] = s[P][Q]$

Suppose truth at a context is just saturation:  $P$  is true in  $s$  just in case  $s[P] = s$ . It then follows from this constraint that for the fragment of  $L$  with no *if*s whatever there is no difference between truth at a context and truth at an index with respect to a context.<sup>7</sup>

Our guide for what *if* means is still the schoolyard version of the Ramsey test. Issuing an indicative (*if*  $P$ )( $Q$ ) in  $s$  tests whether adding the information carried by  $P$  to  $s$  is sufficient for  $Q$ . That is: it tests whether  $Q$  is true in the subordinate context  $s[P]$ . Officially:

**Definition 2** (STRICT CONDITIONAL, v.2.0).

1.  $P$  is true in  $s$  iff  $s[P] = s$
2.  $s[(if\ P)(Q)] = \{i \in s : Q \text{ is true in } s[P]\}$

This either returns all of  $s$  or none of it, depending on whether the test posed is passed. Thus an indicative is true in a context  $s$  iff its consequent is true in the derived or subordinate context got by adding the information carried by its antecedent to  $s$ . It follows straightaway that an ordinary indicative (*if*  $P$ )( $Q$ ) is true in a context just in case all the worlds surviving an update with  $P$  also survive an update with  $Q$ , and so it is a strict conditional analysis properly so-called.<sup>8</sup> And, as before, this is enough to disrupt the legend.

*Proof.* Take any non-iffy  $p, q$ , and  $r$ . (U) If  $p$ -worlds are all  $q$ -worlds, then since — no matter the context  $s$  —  $p$  is true in  $s[p]$ , it follows that  $q$  is true in  $s[p]$ . Hence  $s[(if\ p)(q)] = s$  and the conditional is true in  $s$ . Since it is true in any context whatever, it is true at every singleton context and so at every world. (L) If  $p$  is true at  $i$ , and  $q$  false at it, then at no context with which  $i$  is compatible is the conditional true — indeed, its negation is. (I) The conditional

<sup>7</sup>To see this, take truth-at-a-world for atomic sentences as primitive (possible worlds have the job of determining truth values for atomic sentences) — if atomic sentence  $p$  is true-at- $i$ , write  $i(p) = 1$ . We can then talk about truth-at-a-world as a special case of truth-at-a-context, if we like: truth-at- $i$  is just truth at the singleton context  $\{i\}$ . Now, suppose  $P$  has no *if* in it. Then  $W[P]$  is just the set of  $P$ -worlds and — for any context  $s$  whatever —  $s[P]$  is just  $s \cap \llbracket P \rrbracket$ . Thus,  $P$  is true in  $s$  iff  $s \cap \llbracket P \rrbracket = s$  — which is just to say iff  $s \subseteq \llbracket P \rrbracket$ . Reducing this to truth-at-a-world talk, we then have:  $P$  is true-at- $i$  iff  $i \in \llbracket P \rrbracket$ .

<sup>8</sup>Equivalently: add a diamond to our language, and interpret it as a possibility test in this way:

$$\cdot s[\diamond P] = \{i \in s : s[P] \neq \emptyset\}$$

Now, let  $\square$  abbreviate  $\neg \diamond \neg$ . Then, given Definition 2, it follows that (*if*  $P$ )( $Q$ ) induces just the changes to a context that  $\square(P \supset Q)$  does.

so analyzed supports a deduction theorem: if, and only if,  $Q$  is true in  $s[P]$  is the conditional  $(if P)(Q)$  true in  $s$ . Now suppose that  $(if p \wedge q)(r)$  is true in  $s$ . Hence  $r$  is true in  $s[p \wedge q]$ , and so in  $s[p][q]$ . And so  $(if q)(r)$  is true in  $s[p]$ , and  $(if p)((if q)(r))$  in  $s$ .

And yet: *if* is not the material conditional. For they induce different changes to contexts, and thus have different semantic values, and so behave differently under other operators. Example:  $\neg(p \supset q)$  is true in  $s$  iff, for each  $i$  in  $s$ , it is true in  $\{i\}$ . Not so for the indicative. Let  $s$  contain just two worlds —  $i$  and  $j$  — the first a  $(p \wedge q)$ -world, the second a  $(p \wedge \neg q)$ -world. Then  $\neg(if p)(q)$  is true in  $s$  but not at  $\{i\}$ .  $\square$

## 6 BOGOF

The first version of the strict conditional analysis takes *if* to be a doubly shifty operator and assigns doubly context-sensitive propositions to it. These two features conspire, letting *if* behave in some respects like the material conditional while still acting as a strict conditional over the set of possibilities compatible with the context.

The second version of the strict conditional story also takes *if* to be context dependent, and doubly so. First: because whether or not a context passes the test posed by an indicative depends on what possibilities are still live in that context. Change what possibilities are live, and you may change the fate of the *if*. Second: because the context relevant for the evaluation of a conditional's consequent is a subordinate or derived context got from the original context by hypothetically updating it. The second version of the strict conditional story also insists — as did the first version — that the truth value of an indicative in a context depends on global facts about that context. An indicative induces an all or nothing test on a context in which it is issued, and that test is not a continuous function of the possibilities that make that context up.<sup>9</sup> That is why  $\neg(if p)(q)$  and  $\neg(p \supset q)$  have different truth conditions.

These two features conspire, letting *if* behave in some respects like a material conditional while still acting as a strict conditional over the set of possibilities compatible with the context. For suppose that, as a matter of fact,  $p$  is false at  $i$  and  $q$  is true at  $i$ . There are contexts containing  $i$  at which  $(if p)(q)$  is true and there are contexts containing  $i$  at which it is false. Both

<sup>9</sup>That is: it is not in general so that  $s[(if P)(Q)]$  is the same as  $\bigcup_{i \in s} \{i\} [(if P)(Q)]$ .

are possible. That is the mark of a non-truth-functional conditional. And yet: any context minimally characterizing  $(\neg p \vee q)$ , and so any such context that contains  $i$ , must be a context at which  $(if\ p)(q)$  is true. That is the mark of material implication.

Although the two versions of the strict conditional analysis start in very different territory, they come to much the same thing. That is because both stories provide ways for *if*-clauses to do both jobs assigned to them by the schoolyard version of the Ramsey test and the ways they provide come to the same thing. They rely on the same sort of shifty behavior and global context dependence. This sameness can be seen in three ways: the two versions agree on what indicatives are true, on how they change contexts they are issued in, and on entailments involving them.

Let  $i$  be any world compatible with context  $c$  (in the sense relevant for the first version of the strict conditional story), and  $s$  be a context (in the sense relevant for the second version of the strict conditional story). Now, suppose that  $c(i) = s$ . And take an indicative  $(if\ P)(Q)$ . Then if  $j$  is in  $s$  — equivalently, if  $j$  is compatible with  $c$  — then the subordinate contexts that both analyses implicate coincide:  $(c + P)(j) = s[P]$ . And the two stories exploit these derived contexts in the same strict-conditional way, using the subordinate context as relevant for checking the truth of  $Q$ . Whence it follows that the indicative  $(if\ P)(Q)$  is true at  $i$  in  $c$  according to the first version iff, according to the second version,  $Q$  is true in  $s[P]$  and so iff  $(if\ P)(Q)$  is true in  $s$ , full-stop. That is the first way of seeing that the two versions come to the same thing.

To put things the other way around: begin with the first analysis, and suppose that the characteristic effect brought on by a successful assertion of  $P$  in a context  $c$  is to increment the information in that context with the proposition expressed by  $P$  in  $c$ . Suppose incrementing goes by intersecting. Then — no small coincidence! — the characteristic effect brought on by a successful assertion of an indicative according to the first analysis is just the context change potential assigned to it by the second analysis.

Consider an indicative  $(if\ P)(Q)$  at  $i$ , in  $c$ . Whatever truth value it has at  $i$  in  $c$ , it has at every  $j \in c(i)$ . Whence it follows that

$$(12) \quad \llbracket (if\ P)(Q) \rrbracket^c = \begin{cases} c(i) & \text{if } c(i) \cap \llbracket P \rrbracket^c \subseteq \llbracket Q \rrbracket^{c+P} \\ \emptyset & \text{otherwise} \end{cases}$$

And adding that proposition to the worlds compatible with the context will

return either the whole lot of them or none at all. If every world in  $c(i) \cap \llbracket P \rrbracket$  is a world that  $\llbracket Q \rrbracket^{c+P}$  maps to true, then the result of incrementing  $c(i)$  with  $\llbracket (if\ P)(Q) \rrbracket^c$  is  $c(i)$ ; otherwise it is  $\emptyset$ . (This holds good for any  $j \in c(i)$ .) That is: the incrementing is idle if all the antecedent-possibilities are worlds where the consequent is true with respect to the derived context  $c + P$ ; otherwise, incrementing fails. That is precisely how, according to the second version of the strict conditional analysis, indicatives change the contexts they are issued in. That is the second way of seeing that the two versions of the strict conditional story come to the same thing.

But wait — don't you smell a rat? Consider the first version of the strict conditional story. I said that it meets the demands of (L) and thereby treats indicatives as at least as strong as material conditionals. True enough it meets the demands of (L), but that requirement says less than what we might have wanted. It requires that, for non-iffy  $p$ 's and  $q$ 's, the truth of  $(p \wedge \neg q)$  rules out the truth of  $(if\ p)(q)$ . But suppose — as the folklore argument requires — we have an embedded indicative in the consequent instead. Then (L), restricted as it is to non-iffy  $p$ 's and  $q$ 's, is silent.

Still we might wonder whether the prohibition (L) imposes is generally right: does the truth of  $(P \wedge \neg Q)$ , no matter whether  $P$  and  $Q$  contain iff bits, rule out the truth of  $(if\ P)(Q)$ ? Not according to the first version of the strict conditional story.

For suppose that  $p$  is false at  $i$  but that  $c(i)$  has some  $(p \wedge \neg q)$ -worlds. Then  $(if\ p)(q)$  is false at  $i$  in  $c$  — there are  $(p \wedge \neg q)$ -worlds compatible with the context. But since no worlds are both  $p$ -worlds and  $\neg p$ -worlds,  $(if\ \neg p)((if\ p)(q))$  is true at  $i$  in  $c$ . So we have an *if* true at  $i$  in  $c$ , with a true antecedent and false consequent (at  $i$  in  $c$ ).

A less contrived case: take any  $p, q$ , and  $r$ . Suppose that it is settled in  $c$  that either all of them are true or just one is. Now suppose that, as it happens,  $p$  is true at  $i$ . The complex  $(if\ p)((if\ q)(r))$  is true at  $i$  in  $c$ . But the embedded  $(if\ q)(r)$  is not true at  $i$  in  $c$ : there is a  $(q \wedge \neg r)$ -world compatible with  $c$ . So we have an *if* true at  $i$  in  $c$ , with a true antecedent and false consequent (at  $i$  in  $c$ ).

Things only seem to get worse from here: for if indicatives are not, in general, bounded from below by the corresponding material conditionals then they also do not, in general, go in for modus ponens. That would be an embarrassment than which none greater can be imagined.

I agree that it would be an embarrassment for the first version of the strict

conditional story *if*, according to it, an *if* together with its antecedent did not bring the truth of its consequent in their wake. But what does that mean, *bring the truth of its consequent in their wake*? Let  $c$  be any context and  $i$  a world compatible with it. Consider any complex *if* with antecedent  $P$  and consequent  $Q$ . And suppose that both  $P$  and  $(if\ P)(Q)$  are true at  $i$  in  $c$ . Then according to the first way of telling the strict conditional story  $Q$  is also true at  $i$  in  $((c + P) + (if\ P)(Q))$ . Since the complex *if* is true at  $i$  in  $c$ , adding  $(if\ P)(Q)$  to  $(c + P)$  idles. So  $Q$  is true at  $i$  in  $(c + P)$ .

Therefore the truth of an *if* plus its antecedent at a world in a context brings with it the truth of its consequent at that world in the context as it would be after adding the information carried by the antecedent to it. That is as good a reason as any to say that the *if* together with its antecedent entails its consequent, and that is as good a reason as any to say that *if* does in general go in for modus ponens after all.

The picture generalizes. Entailment is not mere preservation of truth-at-point. That would be fine if truth values never were sensitive to context-shiftiness. But we know better. Truth is sensitive to incremental shifts in information. So entailment must also keep track of shifts in contexts as we go along, adding the information carried by premises to the evolving context. Precisely:

**Definition 3** (ENTAILMENT, v.1).  $P_1, \dots, P_n$  entails  $Q$  iff for any  $i$  and  $c$ :

$$\begin{aligned} \text{if } \llbracket P_1 \rrbracket^{c,i} = 1 \text{ and } \dots \text{ and } \llbracket P_n \rrbracket^{(c+\dots+P_{n-1}),i} = 1 \\ \text{then } \llbracket Q \rrbracket^{((c+\dots+P_{n-1})+P_n),i} = 1 \end{aligned}$$

Entailment is still preservation of truth at  $i$ , but we keep track of context shifts by adding the information of the premises thereby incrementing just which context is in play. All of this is very reminiscent of Stalnaker's (1975) pragmatic surrogate for entailment — *reasonable inference* — except that this is the real thing. And we will find no counterexamples to modus ponens with it. What modus ponens requires is  $Q$ 's being entailed by  $P$  and  $(if\ P)(Q)$ ; this strict conditional story meets that demand if we understand — as we ought — “entails” in the incrementally sensitive way carved out by Definition 3.

This departs from flat-footed entailment, but only conservatively and only where it ought. Flat-footed entailment is preservation of truth at a point, ignoring any context-shiftiness:  $P_1, \dots, P_n$  flat-footedly entail  $Q$  iff no matter the choice of  $i$  and  $c$ , if  $\llbracket P_1 \rrbracket^{c,i} = 1$  and  $\dots$  and  $\llbracket P_n \rrbracket^{c,i} = 1$  then  $\llbracket Q \rrbracket^{c,i} = 1$ . If  $Q$

is context-invariant, then the difference between the incrementally sensitive picture of entailment and its flat-footed counterpart is a difference that makes no difference. Simple case: for any non-iffy  $p$  and  $q$ ,  $p$  and  $(if\ p)(q)$  flat-footedly entail  $q$  iff they entail  $q$  in our incrementally sensitive way. That is why the would-be trouble for modus ponens — that would be trouble only if we opted for flat-footed entailment — involved an embedded *if* in the consequent. Entailment ought not be flat-footed, and so there is no trouble here for modus ponens.

That is all right for the first version of the strict conditional story. But we might have started in different territory, as does the second version of the strict conditional story. That analysis does not assign truth values to sentences at indices with respect to contexts but assigns them truth values in contexts, full-stop. Iff a sentence would not change a context  $s$  at all were we to add its information to  $s$  is that sentence true in  $s$ . If we like, we can then stick to the characterization of entailment as preservation of truth:

**Definition 4** (ENTAILMENT, v.2.0).  $P_1, \dots, P_n$  entails  $Q$  iff for any  $s$ :

if  $P_1, \dots, P_n$  are all true in  $s$  then so is  $Q$

And with entailment so understood an indicative  $(if\ P)(Q)$  — even one with embedded *ifs* figuring in it — together with  $P$  do entail  $Q$ . For if the conditional is true in  $s$ , then  $s[(if\ P)(Q)] = s$ . Thus  $Q$  is true in the subordinate context  $s[P]$ . But *ex hypothesi*  $P$  is also true in  $s$ . Whence it follows that  $s[P] = s$  and thus that  $Q$  is also true in  $s$ .

So we could rest content with the idea that entailment is preservation of truth, and leave it at that. That would be to opt, in the lingo of dynamic semantics, for a *test-to-test* characterization of entailment.<sup>10</sup> Better, I think, to treat entailment not as preservation of truth but as preservation of information. That way we make entailment sensitive to the incremental addition of information as we go along. Suppose that — no matter the context — adding the information carried by  $P$  to it always results in a context in which  $Q$  is true. That is as good a reason as any to say that the former entails the latter. More generally:

**Definition 5** (ENTAILMENT, v.2.1).  $P_1, \dots, P_n$  entails  $Q$  iff for any  $s$ :

if  $s[P_1] \dots [P_n] = s'$  then  $Q$  is true in  $s'$

<sup>10</sup>For menus of options for dynamic consequence relations see [Veltman \(1996\)](#); [van Benthem \(1996\)](#).

This is to opt, in the lingo of dynamic semantics, for an *update-to-test* characterization of entailment. It is again straightforward to show — with entailment so understood — that  $(if\ P)(Q)$  and  $P$  entail  $Q$ . For if  $(if\ P)(Q)$  is true in  $s$ , then  $Q$  is true in  $s[P]$ . (If  $(if\ P)(Q)$  is not true in  $s$ , then the entailment is vacuous.) But  $s[(if\ P)(Q)]$  just is  $s$ . Hence  $Q$  is true in  $s[(if\ P)(Q)][P]$ .

Opting for the incrementally sensitive relations of entailment carved out by Definition 3 (for the first version of the strict conditional story) and Definition 5 (for the second) closes the gap between the two analyses. Suppose  $s$  and  $c$  characterize the same context, that (where  $i$  is compatible with  $c$ ) it is the case that  $c(i) = s$ . And consider whether the sequence of premises  $P_1, \dots, P_n$  entails  $Q$ . There is nothing to choose between taking the first version of the strict conditional story together with Definition 3 and taking the second version together with Definition 5. Each package deal delivers the same goods. That is the third way of seeing that the two versions come to the same thing.

The two versions then agree on when indicatives are true, how they change contexts in which they are successfully issued, and on the entailments involving them. We thus really have one story here, not two.

## 7 Dolci

And it is a story worth telling. Ordinary indicative conditionals *do* seem bounded from above by strict implication, *do* seem bounded from below by material implication, and *do* seem to go in for the kind of import/export equivalence between (1) and (2). The folklore says we have to either explain some of this away, or deny that *if* is a propositional operator, or swallow hard and live with the burden of the horseshoe story. Some have gotten used to those choices and have talked themselves into thinking the choices aren't so bad. But not me. (And not them, either. That's why they try to convince us their favored path isn't so bad; if it really were painless, we wouldn't need convincing.)

Still, I admit that it is tough to shake the feeling that *no* strict conditional story can be plausible no matter what choices it promises to save us from because *all* such stories inherit the properties that characterize strict implication. Strict conditionals go in for characteristic patterns of entailment and some of those patterns seem at odds with the corresponding behavior of *if*.<sup>11</sup>

Antecedent strengthening is one such pattern. Take an egregious example:

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<sup>11</sup>See Scott (1971) for a characterization result for strict implication.

- (13) If there is sugar in the coffee, it tastes sweet  
 ??So: if there is sugar and diesel oil in the coffee, it tastes sweet

The charge is that strict conditional stories must render this an entailment, and therefore that they all predict that whenever the first *if* is true so must be the second. And that rules all such stories out of court from the start. To be sure: if all the *p*-worlds throughout some set *X* are *q*-worlds, then every subset of the *p*-worlds in *X* are also *q*-worlds. And that does seem to be pretty bad news for any strict conditional story.

The charge is legitimate, but does not stick. That is because the trouble can be explained away. I will sum up by sketching two strategies for doing that.

The first strategy is to accept the entailment, and explain away the trouble on pragmatic grounds. This is a path taken by Veltman (1985) for his particular strict conditional story, but applies across the board. All we need to do is assume (i) that *if*'s implicate that their antecedents are possible, and (ii) that entailments, to be happy, must remain entailments when we add the implicatures of their conclusions as explicit premises — in particular, adding those implicatures shouldn't make us rethink the truth of the premises. Thus: *if there is sugar and diesel oil in the coffee ...* implicates that there might be; for (13) to be troubling, its premise must be compelling even once we add that implicature outright as an additional premise.

But there is no trace of plausibility in this argument:

- (14) There might be sugar and diesel oil in the coffee  
 ??If there is sugar in the coffee, it tastes sweet  
 So: if there is sugar and diesel oil in the coffee, it tastes sweet

Adding the implicature destroys the acceptability of the original conditional premise: for any context in which *there might be sugar and diesel oil in the coffee* is true will — assuming the standard quantificational story about epistemic modals — contain some sugar-and-diesel-oil-in-the-coffee-worlds. None of those are worlds where the coffee tastes sweet, and so any such context will have a counterexamplifying world to the simple indicative *if there is sugar in the coffee, it tastes sweet*. So the entailment in (13) is ruled out — its intuitive oddness explained — for straightforward pragmatic reasons.

Saddling us, or seeming to so saddle us, with entailments like (13) is only part of the problem. Antecedent strengthening is one source of potential embarrassment, but contraposition is another. Here is an instance of the

alleged further embarrassment:

- (15) If it rains, it won't pour  
 ??So, if it pours, it won't rain.

But, just as with the alleged embarrassment of antecedent strengthening, adding the implicature of the conclusion — that it might pour — destroys the acceptability of the premise. It's just not true that if it rains it won't pour given that it might pour.

We could just as easily tell our strict conditional stories in ways that do not classify instances of antecedent strengthening like (13) and instances of contraposition like (15) as entailments in the first place. Rather than explaining away the entailments on pragmatic grounds — saying that the entailments are real, but not happy — we can block the would-be entailments outright. That is *ceteris paribus* preferable anyway.

And it is easy. All we need to do is assume: (i) that *if*'s presuppose that their antecedents are possible; and (ii) that a sentence can be true in a context only if its presuppositions are met. Both assumptions are defensible.<sup>12</sup> Thus an indicative (*if*  $P$ )( $Q$ ), in a context, presupposes that  $P$  is compatible with that context.

To say this properly, we need to amend the strict conditional stories as I have told them. But the amendment is both uniform and obvious: everything goes just as we said before provided the context satisfies the presupposition, but we say that things go undefined otherwise. That is something we can put as a side constraint. So amend the analyses in Definition 1 and Definition 2 accordingly:

**Constraint** (DEFINEDNESS).

1.  $\llbracket (\textit{if } P)(Q) \rrbracket^{c,i}$  is defined only if  $c(i) \cap \llbracket P \rrbracket^c \neq \emptyset$
2.  $s[\llbracket (\textit{if } P)(Q) \rrbracket]$  is defined only if  $s[P] = s' \neq \emptyset$

Thus, in the simple case where  $P$  is non-iffy:  $\llbracket (\textit{if } P)(Q) \rrbracket^{c,i}$  is defined only if there is a  $P$ -world in  $c(i)$  and  $s[\llbracket (\textit{if } P)(Q) \rrbracket]$  is defined only if there is a  $P$ -world in  $s$ . Each is defined only if, with respect to the relevant set of worlds,  $P$  is possible.

Even without taking a stand on what happens when such presuppositions are not met, we can see how the charge against strict conditional stories based

<sup>12</sup>The second assumption is a platitude; and I am not alone in making the first assumption — see, e.g., Stalnaker (1975); von Stechow (1998); Gillies (2007a,b).

on the alleged embarrassments of antecedent strengthening and contraposition misses its mark. The would-be entailment in (13) misfires — it is no entailment at all. That is because any context in which the premise *if there is sugar in the coffee, it tastes sweet* is true is a context in which there are no live sugar-and-deisel-oil worlds. But that is what the conclusion presupposes. So any such context must be a context in which the conclusion's presupposition is not met and *a fortiori* a context in which the conclusion cannot be true.

So too for the would-be entailment in (15). Any context in which the premise *if it rains, it won't pour* is true must be a context in which it might rain and all such rainy possibilities are non-pouring possibilities. But in no such context can the contrapositive be defined — it presupposes that there are some pouring possibilities among the rainy possibilities — and *a fortiori* in no such context can the contrapositive be true. Indeed this particular conditional won't ever be true (pouring entails raining, after all); the best it can hope for is undefinedness.

All of this holds good for both ways of telling the strict conditional story for ordinary indicative *ifs*. So the charge, whether in the guise of antecedent strengthening or contraposition, does not stick. Far from it: from the vantage point of the strict conditional stories it is easy to see why indicatives don't in general go in for those characteristic patterns that would otherwise be so embarrassing.

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