Cause and Chance

This paper addresses some seemingly unrelated problems in the theory of objective chance. According to Popper, Giere, and others, objective chances are to be understood as *propensities*, where propensities are certain causal dispositions. This raises three questions:

1. What kinds of (sets of) causes give rise to chances? For example, if I smoke, does that confer upon me a chance of developing lung cancer?

2. As Humphreys has noted, there are ‘backward’ conditional probabilities that are not plausibly understood as propensities. If a smoker has a probability of developing lung cancer, then a lung cancer patient has a probability of being a smoker. But if at least some probabilities must be understood in terms other than propensities, why not understand all objective probabilities in this way.

3. Where does the mathematical structure of chance come from? Popper and Giere sometimes describe propensities as physical magnitudes akin to charge. But it is surely an empirical matter, e.g. that charge comes in discrete units, that there are positive and negative charges, etc. But it does not seem to be an empirical matter, e.g. that probabilities take values between 0 and 1, that probabilities are additive, etc.

Let’s start with the third problem. One promising line of solution to the second problem is to make the mathematical structure of chance parasitic upon the mathematical structure of rational credence (or degree of belief). There are a number of arguments that purport to show that rational credences ought to obey the probability calculus (such as Dutch Book arguments, calibration arguments, and Joyce’s ‘non-pragmatic’ argument). A number of authors, especially Mellor, Skyrms, and Lewis, have argued for a systematic link between chances and rational credences. The most explicit of these is Lewis’s ‘principal principle’ (PP). This principle states that:

\[ Cr(A \mid Ch(A) = p \& E) = p \]

where ‘\(Cr\)’ is an agent’s rational credence, A is any proposition, Ch is objective chance, and E is any proposition that is ‘admissible’ with respect to A. PP makes two assertions: (i) that beliefs or suppositions about the chance of A tends to set the agent’s credence in A equal to the value of the chance believed or supposed; and that the chance of A screens off A from all admissible propositions. But PP raises its own questions:

3a. How is it connected to the idea that chances are causal propensities?

3b. Which propositions are admissible? While Lewis provides a great deal of informal discussion, and several examples and special cases, no rigorous criteria have been offered.
These two problems are inter-related. For example, it seems plausible that finite frequencies obey a PP-like principle. Given only the information that Pat is a Briton, and that 15.7% of Britons are aged 65 or older, a rational agent ought to assign credence .157 that Pat is aged 65 or older. But being British does not seem to create a causal disposition or propensity to be 65 or older. Moreover, for this PP-like principle, all sorts of propositions will count as inadmissible: Pat’s gender, where in Britain Pat lives, whether Pat plays bridge or listens to Coldplay, etc.

I propose that all of these problems can be addressed using the apparatus of Causal Bayes Nets (CBNs). A CBN consists of an directed acyclic graph and a probability function. The nodes on the graph represents variables that stand in causal relations to one another, and a directed edge (or ‘arrow’) from X to Y represents that X has a causal influence on Y that is not mediated by any of the other variables in the graph. The set of parents of a variable X (PA(X)) is the set of variables with an arrow pointing directly into X. Descendants are then defined in the obvious way (although it will be convenient to add that any variable X is considered a descendant of itself). The probability function satisfies the Causal Markov Condition (CMC):

\[ P(X | PA(X) & Y) = P(X | PA(X)) \]

so long as Y is not a descendant of X; i.e., the parents of X screen X off from all of its non-descendants. There is strong evidence that actual causal systems obey CMC so long as the set of variables included in the graph is sufficiently rich (in a way that can be made precise).

I propose that conditional probabilities of the form \( P(X | PA(X)) \) have all of the properties of chances. This account resolves all of the problems raised above.

1. It tells us that the set of direct causes of X suffice to confer a chance upon X.

2. It follows from CMC that the joint probability distribution over all variables can be factored into conditional probabilities of the form \( P(X | PA(X)) \). Specifically, \( P(X_1,\ldots,X_n) = \Pi_i P(X_i | PA(X_i)) \). Thus all probabilities can be constructed out chances.

3. This proposal establishes a connection between the causal structure of chances and Lewis’s PP. Suppose that a rational agent believes (or supposes) that a given CBN accurately describes some causal system, and sets her credences in accordance with the probability in that CBN. Then the conditional probabilities of the form \( P(X | PA(X)) \), together with the actual values of PA(X) will function as an objective chance function for that agent. That is, the agent’s credence in X, given \( P(X | PA(X)) \) & PA(X), will equal \( P(X | PA(X)) \). Moreover, this credence will remain the same if the agent conditionalizes on any further information about the values of variables, so long as these do not include X or any descendants of X. This allows us to give a purely causal characterization of admissibility: information about Y is admissible with respect to X just as long as Y is not a causal descendant of X.
Finally, it is possible to generalize this approach to allow other sets of causes of X to confer chances upon X, thus capturing the idea that the chance of X can evolve over time. CMC entails a graphical criterion for screening-off called D-separation. Using D-separation, we can identify sets of causes that suffice to screen off X from other variables in the right way to count as objective chance.