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Parametrization of cyclic motion and transversal sections

Cyclic motion is at the core of many sports such as running, swimming, or cycling. The study of corresponding kinematic variables is fundamental for the evaluation of training routines and the assessment of performance. Conventional kinematic analysis of human cyclic locomotion derives characteristic features from a few single cycles (e.g., average peak, average difference between peaks, as in Harris and Smith, 1996). However, most current methods ignore the full dynamics of the motion that may reveal important additional insight into patterns of motor control (Schablowski-Trautmann & Gerner, 2006).

An approach to the analysis of multi-dimensional kinematic variables is to view them in their state space and to intersect the cyclic orbit with a co-dimension 1 hyperplane, called transversal section (Vieten, Sehle & Jensen, 2013). This corresponds to so-called Poincaré sections, a tool for the analysis of dynamical systems nearby a periodic solution. Features can be extracted from the intersection points on a single transversal section in order to characterize the overall motion. E.g., the variance may indicate the degree of regularity of cyclic motion. It may also be of interest to compute a representative average cycle and to study how the kinematic variables change with the phase angle of the cyclic motion.

The timing of muscular, neurological, and respiratory systems varies according to the environmental, bio-mechanical, and morphological constraints. Therefore, individual cycles in cyclic motion differ in duration and local speed. It follows that the calculation of an average cycle of a set of cycles requires a phase alignment.

Our contributions are: (1) An algorithm for calculating an average periodic cycle based on dynamic time warping (DTW) and modifications of DTW barycentric averaging (DBA). Then cycles are aligned with the average cycle yielding their parametrization by phase. (2) A definition of the quality of cycle intersections with Poincaré sections, providing a criterion for the right choice of the section.

1 Methods

Data and assumptions. Let $x_M[n]$, $n = 0,1,\dots$ be a *d*-dimensional time series in $\mathbb{R}^d$, uniformly sampled from continuous motion in state space and including measurement noise. For applications in sport science we assume that the motion is band limited at about 12 Hz, the sampling rate is above the Nyquest rate of 24 Hz and the motion is cyclic with a fundamental frequency of about 1 Hz, as, e.g., in typical human gait. The cycles should be readily segmented, e.g., by placing segment boundaries at pronounced extremal values of one of the *d* time series components.
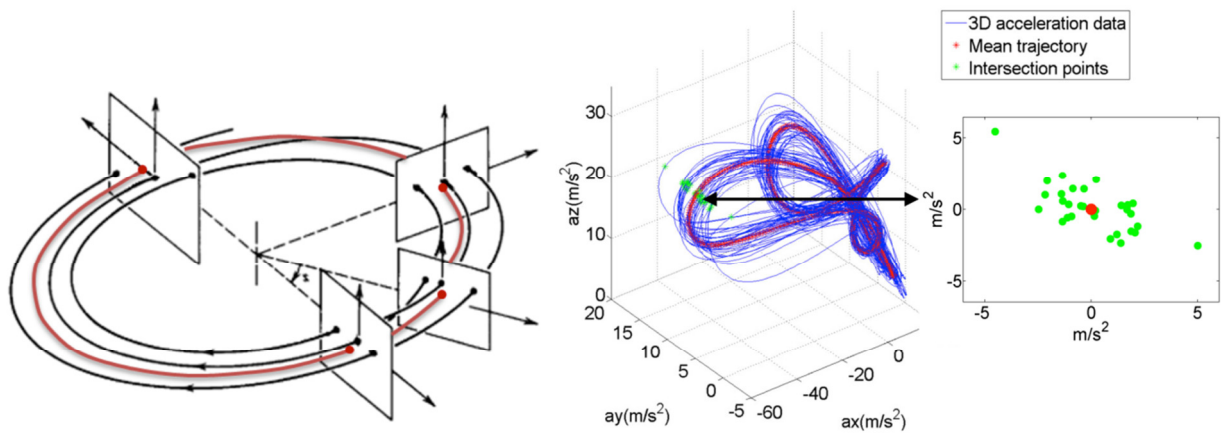
Fig. 1. Poincaré sections. Left: Schematic plot of parameterized transversal (Poincaré) sections belonging to a periodic trajectory (red, adopted from Moon, 1992, and modified). Middle: 3D plot of a cyclic trajectory of acceleration data (blue), the periodic mean trajectory (red), and a transversal section (green points). Right: 2D close-up view of the transversal section.

Filtering and segmentation. To reduce the noise and to smoothen the time series $x_M[n]$ we apply a low pass filter with a cutoff frequency of 12 Hz. Then we segment the data obtaining K cycles $x_{M,k}[n]$, $n = 0,\dots,N_k$ - 1, $k = 0,\dots,K$ - 1. The cycles may hold differing numbers of samples, N_k, and so we uniformly resample these sequences with the same number of samples, N. This yields a set of K cycles, each with N samples, $x_k[n] = 0,1,\dots,N$ - 1, $k = 0,\dots,K$ - 1.

Approach to phase registration. The first goal of this contribution is to align all K cycles so that they are parameterized by the phase from $[0,2\pi]$. See Figure 1. An average periodic cycle, $x_a[n]$, $n = 0,\dots,N$ - 1, is constructed from all K cycles. This periodic signal defines the phase as $2\pi n/N$ for the n-th sample $x_a[n]$ of the average cycle. Each individual cycle is aligned to the mean cycle by DTW so that if $x_k[m]$ registers with $x_a[n]$, then the phase for $x_k[m]$ is defined as $2\pi n/N$. The averaging is accomplished by a modification of the DBA algorithm, that is also based on DTW.

Dynamic time warping (DTW). DTW optimally aligns two data sequences $x[n]$ and $y[n]$, $n = 0,\dots,N$ - 1. An alignment is given by two monotonically increasing index sequences $s_0,\dots, s_M$ and $t_0,\dots, t_M$ with $s_0 = t_0 = 0$, $s_M = t_M = N$, and $0 \leq s_{k+1} - s_k \leq 1$, $0 \leq t_{k+1} - t_k \leq 1$ for $k=0,\dots,M$ - 1, implying that the samples $x[s_k]$, $y[t_k]$, $k = 0,\dots,M$ are pairwise aligned. A time warping incurs a cost $\sum_k c(s_k,t_k)$, where the local costs are usually $c(i,j) = (x[i] - y[j])^2$ in case of scalar time series. For d-dimensional time series $c(i,j) = \|x[i] - y[j]\|^2$ can be taken. Dynamic programming provides the optimal time warping with minimal total cost, see Müller (2007) for details.

DTW barycentric averaging (DBA). For a set of time series a characteristic 'average' time series may be desired. DBA is a local optimization algorithm similar to clustering methods like k-means (Petitjean, Ketterlin & Gancarski, 2011) minimizing the total cost given by the sum of all costs for the DTW alignments with the average series. In each iteration of the algorithm all sample points of the average series are updated by the arithmetic mean of all corresponding samples of the given series.

Modifications of DBA. Applying the standard cost function in the DTW algorithm in the context of the DBA method yields sharp peaks of the average cycle that are not
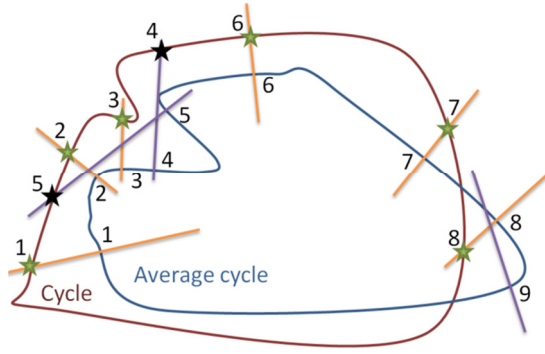
Fig. 2. For noisy real-world data Poincaré sections may not be well-defined: Nine points, labeled 1 to 9, on the average cycle are taken as origins of planar Poincaré sections. The corresponding intersections may not be in sequence (point 5 is before point 2) and the section for point 9 does not intersect with the given cycle at all.

characteristic of the underlying motion but can be regarded as DBA artifacts. The cause for this problem is that in the averaging step of DBA, many samples of a given cycle may be aligned with the same sample of the average cycle. This strongly pulls the sample of the average cycle towards the given cycle, yielding the spurious peaks. Our solution is to introduce weights for the averaging such that each cycle is accounted for by the same weight.

Moreover, consecutive samples aligned with the same sample of the average cycle lead to parts of piecewise constant phase, not appropriate for physical motion. Thus, we include a regularization term in the cost function to penalize constant phase,

$$c(i,j) = (1-\lambda)\frac{||x_a[i] - x_k[j]||^2}{E} + \lambda\frac{(i-j)^2}{T}$$

where E and T are normalization constants, such that for $\lambda = 0.5$ on average the two terms contribute about the same costs in the DTW algorithm. We used $E = \frac{1}{N}\sum_{n=0}^{N-1}||x_a[n]||^2$ and $T = (N/4)^2$. The parameter $\lambda \in [0,1]$ determines the tradeoff between the classical DBA ($\lambda = 0$) and plain arithmetic averaging ($\lambda = 1$).

Phase registration. After the computation of the mean periodic cycle, all cycles, given by time series $x_k[j]$, can be registered by DTW with the mean cycle so that each sample $x_k[j]$ is attributed by the phase of the corresponding point $x_a[j]$ on the mean cycle. For each phase value $\Phi = 2\pi i/N$, $i = 0,1,\ldots,N-1$ let the set A_Φ denote the set of all points from the entire motion trajectory that are assigned to the phase Φ. By design of the DTW algorithm the sets A_Φ need not be disjoint and may contain several points of a single cycle $x_k[n]$. Such cases are artifacts resulting in physically unrealistic phase discontinuities and time intervals of constant phase, respectively.

Poincaré sections. Consider the transversal section at phase $2\pi i/N$ given by inter-sections of the motion trajectory with the normal plane at the sample $x_a[i]$ of the average cycle, i.e., the linear subspace that is anchored at $x_a[i]$ and orthogonal to the tangent of the average cycle at $x_a[i]$. We imagine that the Poincaré section is then pushed forward to the next sample $x_a[i+1]$ with updates of the corresponding intersections. However, it is not granted, that (a) intersections for the next Poincaré section may correspond to points of cycles that stem from the future instead of the
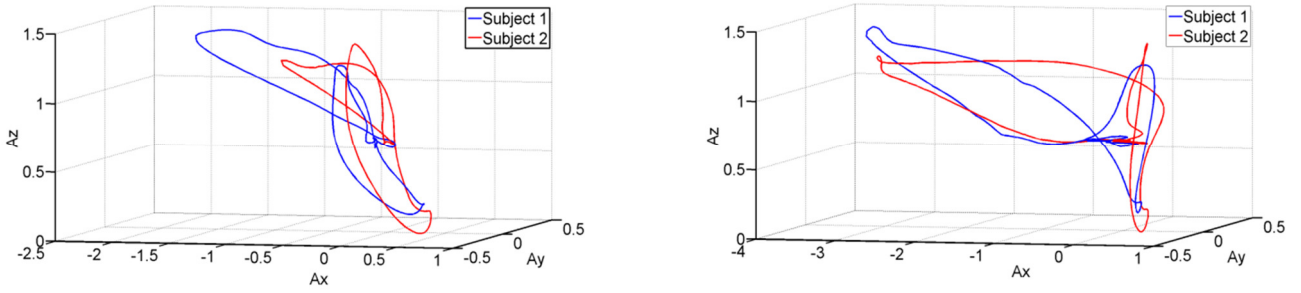
Fig. 3. Average periodic trajectories for two participants for walking (left) and running (right). The graph shows that it is possible to discriminate different persons as well as different activities from the average cycles.
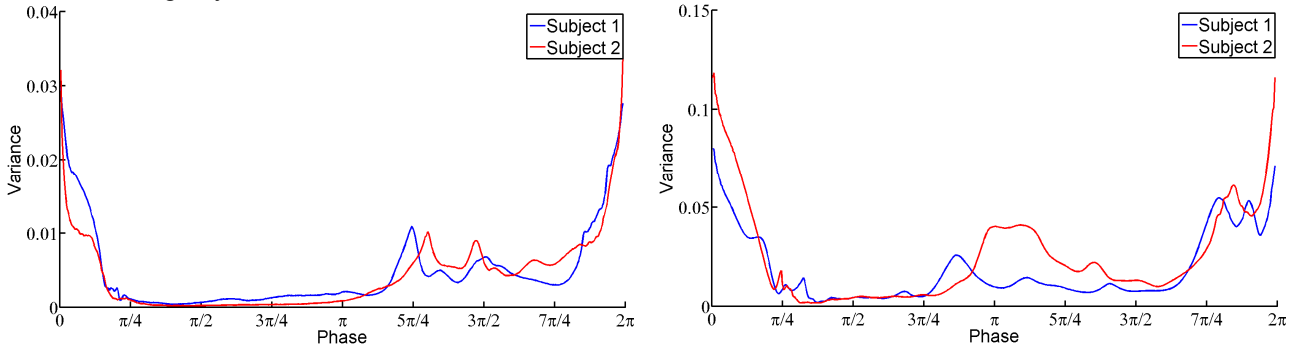

Fig. 4. Time warped and aligned cycles may be used for within-stride analysis. Here the multidimensional variance is shown as a function of phase. Persons and activities are as in Figure 3.

past, and that (b) such intersections always exist, see Figure 2. To address these problems we propose to use the DTW algorithm to compute the Poincaré sections.

To this end, we replace $x_k[j]$ by $p(i,j)$ the cost function $c(i,j)$, where $p(i,j)$ denotes the intersection of the polygonal line from $x_k[j-1]$ to $x_k[j]$ and on to $x_k[j+1]$ with the normal plane at $x_a[i]$ if an intersection exists. In principle, the DTW algorithm enforces the desired monotonicity of alignment, and here, it limits the first problem (a) to small deviations. If there is no intersection, $p(i,j)$ is taken to be the point on the segment closest to the plane, i.e., one of the three segment end points. Thereby, although an intersection could not be found, at least a nearby sample point is taken into account, ameliorating the second problem (b).

Quality of Poincaré sections. When analyzing dynamics of movement by means of Poincaré sections the question arises, which section to take, as any one of the sections belonging to phase parameters in $[0,2\pi]$ may be used. In theory, for deterministic systems, i.e., systems governed by differential equations, all sections are equivalent, conveying the same, complete information. However, in practice measurements are noisy and outlier cycles may lack intersections with some of the Poincaré sections as pointed out above. We suggest to use signal-to-noise ratio (SNR) to quantify the quality of a section. Let A_Φ denote the set of points in the Poincaré section at phase Φ. Some of these points may be outside of the planar section. The variance of the signal at phase Φ is captured by the covariance matrix. The signal energy in the Poincaré section is given by the sum of the eigenvalues of the matrix. We divide this energy by the average power of the noise at the points, estimated by the squared Euclidean distances between the measurements and the
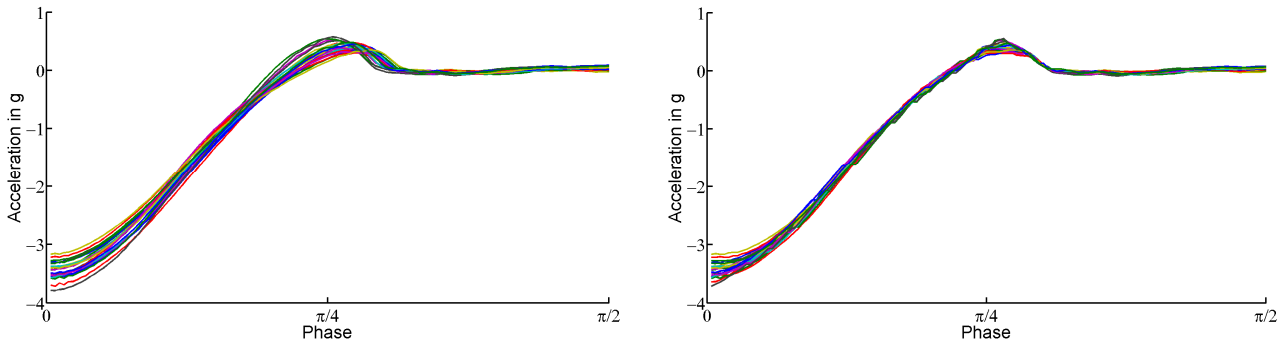
Fig. 5. Effect of our averaging technique: On the left simple arithmetic averaging is performed after scaling differing cycle times to the same value (2π). On the right the aligned, time warped cycles are shown.

smoothed data. An SNR = 0 dB indicates that the noise is of the same power as that of the signal. This implies that motion analysis, based on Poincaré sections should be such that the corresponding SNR is maximal, and at least greater than zero.

Another requirement for valid analyses of Poincaré sections should be that the section set A_ϕ contains only intersections of the motion trajectory with the corresponding normal plane to the average trajectory. Therefore, we suggest to monitor the percentage of points in A_ϕ that are true intersections and to select only sections with 100% intersections (see Figure 6).

2 Experiments, data processing, and results

Eight healthy adults (7 male, 1 female, age 23–31 years old, weight 46–68 kg) did 12 tests of 3 minutes each on a treadmill (walking, jogging, running). We varied the treadmill slope, speed, and whether or not 1.5 kg weights were attached to the ankles. One minute of rest was given after 9 minutes of tests. The motion data was acquired by a 3D accelerometer (RehaWatch, Hasomed), attached to the right shoe. This yielded time series A_x, A_y, A_z of acceleration (in units of g, acceleration by gravity) at a sampling rate of 600 Hz.

We reduced the noise by a Savitzky-Golay filter, i.e., by least squares regression on a sliding window. We used quadratic polynomials and a window of 91 samples, yielding a filter with a cutoff frequency of 12 Hz. For the SNR calculation, the difference between the original and the filtered signals was taken as noise. The gait cycles could clearly be segmented by extremal values in A_x, designating the instant of heel strike in each cycle. Each cycle was Fourier transformed and then upsampled by trigonometric interpolation to 512 samples per cycle. The average cycle was calculated for each set of cycles, i.e., for each activity of each participant, with $\lambda = 0.5$.

We applied our methods for averaging of cyclic motion, phase registration, and Poincaré sections. Figures 3 and 4 show the average cycles and the variance as a function of phase for two activities and two persons. Figure 5 shows the effectiveness of the time warping for a small set of cycles. The warped series are much closer together.
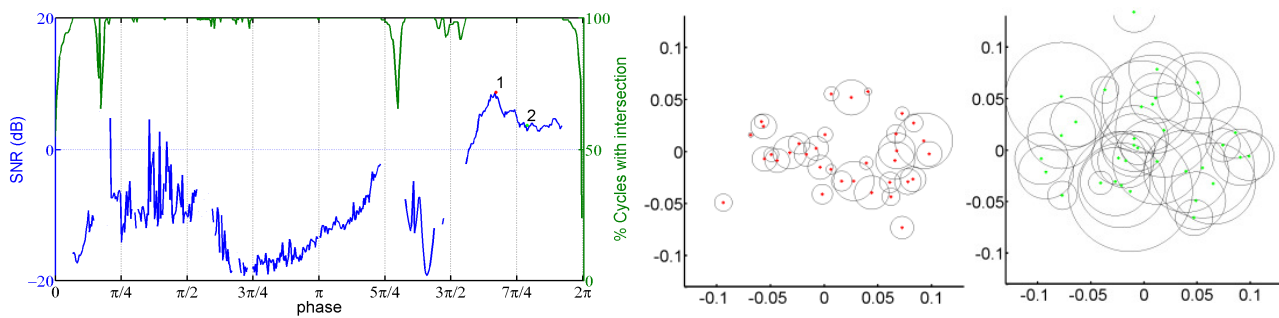
Fig. 6. An analysis of the quality of Poincaré sections computed using the modified DBA and registration algorithm. The signal-to-noise ratio is shown for those Poincaré sections that intersect with all of the cycles (left). The best choice of a section is given by the point labeled '1', and a suboptimal one by the point '2'. The plots on the right illustrate some of the points in the corresponding Poincaré sections together with circles with radii equal to the estimated noise.

An analysis of the quality of Poincaré sections computed using the modified DBA and registration algorithm shows that for large phase intervals there are cycles that do not intersect properly with the section as required, see Figure 6. For the remaining intervals the figure shows the SNR. For many phases the SNR is negative, indicating that the measurement noise dominates the signal, rendering such Poincaré sections useless for motion analysis. The figure also shows the preferred Poincaré section with maximal SNR where the circles indicate the noise of the samples.

3   Discussion and conclusion

We proposed a new method to calculate the average of a set of cycles based on dynamic time warping and a modification of DTW barycentric averaging. Our method allows the study of features of kinematic variables in cyclic motion depending on the phase using an average cycle as reference. In addition, a new definition of the quality of intersections of cycles with Poincaré sections is given based on signal-to-noise ratios. In our future work we will augment the cost function in the alignment method by the motion velocity in state space and apply the method to time series obtained from a reconstruction of the embedding space using the Taken's theorem. Further, we will extend the method to continuous time.

Literature

Harris G. F. & Smith P. A. (1996). *Human Motion Analysis: Current Applications and Future Directions*, IEEE.

Moon, F. C. (1992). *Chaotic and Fractal Dynamics*. New York: John Wiley & Sons.

Müller, M. (2007). *Information Retrieval for Music and Motion* (*Vol 2*). *69–84*. Springer.

Petitjean, F., Ketterlin, A., & Gancarski, P. (2011). A global averaging method for dynamic time warping. with applications to clustering. *Pattern Recognition*, 44(3), 678–693.

Schablowski-Trautmann, M., & Gerner, H. (2006). State-space analysis of joint angle kinematics in normal treadmill walking. *Biomedizinische Technik*, 51(5/6), 294–298.

Vieten, M. M., Sehle, A., & Jensen, R. L. (2013). A novel approach to quantify time series differences of gait data using attractor attributes. *PloS one*, 8(8), e71824.