Progressive Compression of Point-Sampled Models
Short introduction to data compression
The compression method for point-based models proposed in [Waschbüsch et al, Eurographics, 2004]
- point-based models
- the architecture of the algorithm
Experimental results and future development
Developed by Claude Shannon in late 1940s
- Shows how to quantify the information
- The information content of a message resides in the amount of surprise it contains
- Formally: considering a source $S$ with an alphabet of $n$ symbols having the probabilities $P_i$, the entropy of the source is defined as:

$$H(S) = -\sum_{i=1}^{n} P_i \log_2 P_i$$

$H$ (entropy) is the amount of information per symbol generated by the source
- it is measured in $\text{bits}$
Data Compression

- **Lossless compression**
  - compression by removing the redundancy
  - example: text compression

- **Lossy compression**
  - applied in multimedia
  - compression by removing the irrelevancy and redundancy

- **Progressive compression**

- **Examples of redundancy and irrelevancy**
  - redundancy: text (e.g. in English language) in ASCII format
  - irrelevancy: continuous-tone image
Compression for Point Based 3D Models

- **Input:** an unstructured set of 3D points
  - Each point is associated to a surfel (surface element) that represents the surface in the immediate neighborhood of the point
  - The surfel contains information about:
    - position
    - color
    - normal
    - radius
    - ....

- Task: compression of positions, normals and colors
- Compression pipeline:
  - Multiresolution Decomposition
  - Differential Coding
  - Zerotree Coding
  - Arithmetic Coding
Compression Pipeline for Point-Based 3D Models

- **Multiresolution decomposition**
  - hierarchically decomposes the original set to a series of lower resolution subsets

- **Differential coding**
  - predict a higher resolution set from a lower resolution one and store the prediction errors as detail coefficients
  - the original model can be fully reconstructed from the lowest resolution set and the hierarchy of detail coefficient sets
  - quantize the set to reduce irrelevancy

- **Zerotree coding**
  - remove further coherencies among detail coefficients

- **Arithmetic coding**
  - eliminate the redundancy
Multiresolution Decomposition

- Iteratively reduce the size of the set to half
  - define a distance between points
  - compute a set of pairs (minimum weight perfect matching problem) of the higher resolution set ($\lambda^{-i}$)
  - form the lower resolution set ($\lambda^{-i-1}$) by representing each pair by its mean

- After $k$ steps, results a hierarchy of $k+1$ sets with the sizes respectively $N$, $N/2$, ..., $N/(2^{**k})$

- The decomposition algorithm can be seen as building a forest of binary trees, starting from the bottom layer

- Define the $n$-neighborhood of one point (in a certain resolution set) by considering $n$ points of the set that contributed to the same point from a lower resolution set (equivalently, have a common ancestor in tree).
the distance between the points is computed from the distances between the positions, the normals and the cosines of the angles of contraction between current and previous decompositions
Predictive Differential Coding

- Try to predict, based on the information of the lower resolution set $\lambda^{-k-1}$, the next higher resolution set $\lambda^{-k}$
- Save the differences from the predicted values $\lambda^{-k}$ to the real values of $\lambda^{-k}$ as a set of detail coefficients ($\gamma^{-k}$)
- For a better prediction, transform the coordinate system to a local coordinate system, based on the neighborhood of each point
Positions Prediction

- Try to predict from $\lambda_i^{-k}$ the points $\lambda_{2i}^{-k+1}$ and $\lambda_{2i+1}^{-k+1}$

- Coordinate transform:
  - find the 4-neighborhood of the point $\lambda_i^{-k}$
  - form the least squares plane of the neighborhood, represent the points in cylindrical coordinates, align the system to the first principal component and move the origin in $\lambda_i^{-k}$

- Prediction:
  - in the local coordinate system
  - predict elevation to 0
  - predict radius to the average distance in the neighbor set divided by $2\times\sqrt{2}$
  - predict $\theta$ to 0
Coordinate transform based on the 4-neighborhood (red points, marked with x), predicted position and difference to the real point coordinate.
Normals and Colors Prediction

**Normals**
- store the angle between the average normal and one of the normals of the points
- use spherical coordinates
  - align the polar reference to the average normal
  - align the azimuthal reference to the projection of the eigenvector (see the coordinate transform) on the plane perpendicular to the normal
- $\phi$ – azimuthal angle, between 0 and $2\pi$
- $\theta$ – polar angle, between 0 and $\pi$
- predict the changes of $\phi$ and $\theta$ to 0

**Colors**
- use YUV color space (useful in quantization)
- predict the color changes to 0
The data set is decorrelated by multiresolution decomposition and predictive differential coding.

The original data ($\lambda^0$) can be completely reconstructed, before quantization takes place.

The total size is the same.
After multiresolution decomposition:
- the size of the data is still the same
- the data is less correlated
- the detail coefficients have values close to 0

Quantization maps the symbols of a set to symbols of a set with smaller size
- increases the redundancy, keeping the same set size

To prevent recursive error accumulation, after a layer of coefficients is quantized the error is propagated to the next higher resolution layer
Zerotree Coding

- Embedded coding
  - all the encodings of the data set at lower bit rates are encoded at the beginning of the bit stream at a higher rate

- Use the significant maps for coding wavelet coefficients
  - apply successively a set of thresholds (with decreasing values) to code the values that are significant for the given threshold

- Zerotree coding for significance maps
  - do not send the whole significance map, each step
  - hypothesis: if a coefficient at a coarse scale is insignificant (relative to a threshold), then all the coefficients of the same orientation at the same location at finer scale are likely to be insignificant (relative to the same threshold)
  - zerotree coding: introduce a new symbol to code that the current coefficient and all the related finer scale coefficients are insignificant relative to the given threshold
Zerotree Coding

Zerotree algorithm iterates two steps:

- dominant step: creates the significance map for current threshold and passes the significant values to subordinate list
- subordinate step: refines the magnitudes of the values in the list (reduces the uncertainty interval by 1/2)
Arithmetic Coding

- The encoded data after zerotree coding contains 7 symbols
  - POS, NEG, IZ, ZT, Z (from the significance map)
  - 0,1 (interval refining, in the subordinate pass)
- Arithmetic coding:
  - a message is represented by an interval between 0 and 1
  - depending on the current symbol, the interval is refined
  - as the message is longer, the interval is smaller (the number of bits to specify the interval grows)
- Comparison to Huffman codes
  - Huffman compression assigns lower number of bits for more probable symbols; it is optimal if probabilities are negative powers of two
  - arithmetic compression relates symbols to intervals proportional to symbols probabilities; it is always optimal
Experiments

distance = position_distance (1 + 0.6(|cos_1| + |cos_2|) + (1-cos_normal)/2)
Future Development

- Find a distance function that will give good results
- Improve the prediction scheme
- Study other possibilities for the decomposition