Sketchy Graph Drawing

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Interests:

• Graph Drawing
• Graph Algorithms and Network Analysis
• Algorithmic Geometry
• Efficient Algorithms
Fun Part: Small Worlds I

Small World networks: Model for empirical networks, characterized by

- Limited node *degree* (number of acquaintances)
- Locally *dense subgraphs* (a friend of a friend is a friend)
- Low characteristic *path length* (“six degrees of separation”)

Modeled by a *torus* and random *short-cuts*:
Fun Part: Small Worlds II

Six Degrees of Separation (with U. Brandes and M. Broghammer)

Installation at the Ars Electronica Festival, “Language of Networks”, Sept 1-7, Linz, Austria

http://www.aec.at/

Also see http://www.cs.virginia.edu/oracle/
Graph Drawing? Sketchy?

Sketchy Graph Drawing:

depicting a graph $G=(V,E)$

Objective: *aesthetic criteria*
- few crossings
- display symmetries
- equal edge lengths
- reveal graph-theoretic properties
- ...

Problems:
- criteria are contradictory
- optimization problems are typically *NP*-hard
- ...

Sketchy Graph Drawing I

Problem 1:

Some graphs are too large to be completely *processed* (analyzed, drawn)

- information overflow
- Runtime problems

Problem 2:

Some graphs are even too large to be *read* completely

- very very very large
- only given implicitly
Sketchy Graph Drawing II

Idea:

- Find a way to draw a graph *without knowing it in advance*
- Sketch the graph with what we know as yet
- Provide means for navigation (zooming into regions of interest, thereby providing smooth transitions)
Issues:

• How shall we draw a sketch? (D. Knuth, 1970: “How shall we draw a tree?”)

• What is the best way to explore an unknown graph? DFS? BFS? A hybrid search? Something else? Use randomization?

• How can we ensure that the transitions between navigation steps be smooth?
Sketchy Graph Drawing IV

Graph Mining

Abstraction & Navigation

Network Analysis

Sublinear Algorithms

Graph Exploration

Sketchy Graph Drawing
Abstraction & Navigation I

Zooming
- geometric
- semantic

Focus&Context
- Fisheye Distortion
- Topological Fisheye Views

Incremental Exploration
- display only central parts of the graph
- enhance when demanded


Abstraction & Navigation II

Display Abstraction in a graph:

- ghosting
- hiding
- grouping

Decompose a graph into hierarchical layers (graph slices, graph sketches), clustering


Graph Exploration I

Original Problem: Complete traversal of an unknown graph/environment, motion planning

- Create a map of the partial traversal from a given starting node
- Use low number of edge traversals
- Variation: tethered to a rope
- Variation: limited fuel capacity

It's an on-line problem: We know nothing about the graph beforehand, but we gain knowledge

Deng, Papadimitriou: Exploring an Unknown Graph. Proc. 31\textsuperscript{th} IEEE FOCS, 1990.
Graph Exploration II

Search Strategies
• Breadth First
• Depth First
• Best First
• A*
• ...

Relaxation
• Only partial traversal, but
• sufficient amount of information to work with

Parallel Graph Algorithms
• pivot set of starting nodes
• combine the partial results

Sublinear Time Algorithms I

Often even $O(n)$ is too slow

- very large graphs ("large information spaces")
- can/want to view only at a small fraction of the input

Goal: Sublinear Time Algorithms

- $O(\log n)$, $O(\sqrt{n})$, independent of $n$
- typically: use sampling, randomization
- relaxation of an exact problem to an approximation, estimation

(sublinear memory: streaming algorithms)

Sublinear Time Algorithms II

Application: Property Testing

- Exact problem: yes/no
- Relaxated problem: is the object \( \varepsilon\)-far from having a property, with constant probability?

Example: Is the list \( x = \{x_1, \ldots, x_n\} \) \( \varepsilon \)-far from being monotone with prob. 2/3, i.e. does it have an increasing subsequence of length \( (1 - \varepsilon) n \)?

- Select \( i \) at random and perform ordinary binary search as if to find \( x_i \)
- Repeat \( O(1/\varepsilon) \) times, report FAIL if the search does not find \( x_i \). Eventually report PASS.
- Running time in \( O((1/\varepsilon) \log n) \).

Graph Mining

Find frequent relevant subgraphs in a large graph

Uses methods from Classical Data Mining (finding regularities, basket analysis)

Network Analysis I

Friendships in a US school

[Picture by James Moody]

Find out:

• Who is the most important actor in a network? Who is central? Who isn't? Why?
• Are there clusters?
• How did the network evolve over time?
• What will it look like in the future?
Network Analysis II

Graph models, network statistics

Newman, 2003: “How can I tell what this network looks like when I can't actually look at it?”

Summary

Sketchy Graph Drawing should employ methods from manifold areas, including

- Visual Abstraction and Navigation, Multilevel Visualization
- Graph Exploration and Network Searching
- Sublinear Time Algorithms
- Graph Mining
- Network Analysis
Thanks for your attention!