

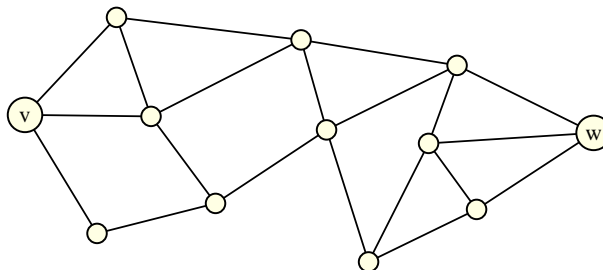
Exercise Sheet 7

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Exercise 7.1: The 'harem problem': Let B be a set of boys, G a set of girls, and suppose that each boy b in B wishes to marry more than one, say k_b , of his girl friends $G(b) \subset G$. Find a necessary and sufficient condition for the harem problem to have a solution. [Hint: replace each boy by several identical copies of himself, and then use Hall's theorem.]

Exercise 7.2 A connected graph G is called k -edge-connected if at least k edges need to be deleted to disconnect G . Prove that a graph G is k -edge-connected if and only if any two distinct vertices of G are connected by at least k edge-disjoint paths.

Exercise 7.3: Determine a maximum number of edge-disjoint v - w -paths in the graph below. Is the graph 3-edge-connected? If yes, remove a minimum number of edges such that the graph is no longer 3-edge-connected; if not, add a minimum number of edges such that the graph becomes 3-edge-connected.



Exercise 7.4: We want to find a maximal matching in the following bipartite graph. Modify the graph by adding two vertices s and t such that s is connected to all vertices on the left side and t is connected to all vertices of the right side. Try to direct the edges and define capacity values for the edges such that from a maximum s - t -flow for which only integral values are allowed a maximum matching can be determined by taking all edges that are assigned flow 1.

