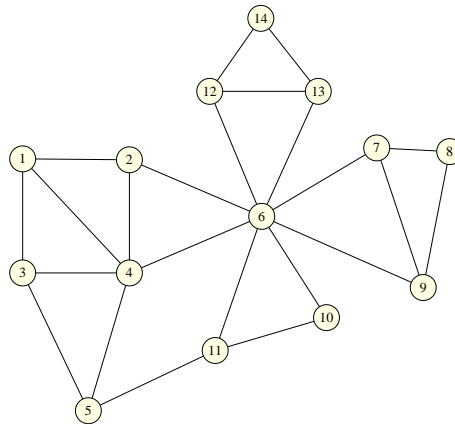


Exercise Sheet 1

Issue date: 18 October 2002 **Hand in by** 29 October 2002

Exercise class: 31 October 2002

Exercise 1.1: Consider the following graph G :



Find

- a) $|E(G)|$
- b) $\Delta(G)$
- c) $d(4)$
- d) whether G is simple or not
- e) a walk that is not a trail, a trail that is not a path, a path that is not a cycle, and a cycle, each of length 5
- f) the distance $d_G(5, 12)$
- g) the adjacency matrix of the graph induced by the vertices $\{1, 4, 5, 6, 8\}$. Is that graph connected?
- h) the largest n such that G contains K_n as a subgraph
- i) a stable set containing 5 vertices

Exercise 1.2:

- a) Draw the graph $G_1 = (V_1, E_1)$ defined by the adjacency matrix

$$\begin{pmatrix} 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 2 & 1 & 0 \\ 1 & 2 & 0 & 2 & 0 \\ 3 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

- b) Determine for each pair of vertices (i, j) ($1 \leq i, j \leq 5$) the number of (i, j) -walks in G_1 of length 2.

Exercise 1.3: Show that every graph has an even number of vertices of odd degree.

Exercise 1.4: Consider the following statement:

At any party with six people there are three mutual acquaintances or three mutual strangers.

- a) Formulate the statement as a graph problem.
- b) Prove that the claim is true.