Macroeconomic Stability and Wage Inequality: A Model with Credit and Labor Market Frictions

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Abstract

While macroeconomic volatility in the US economy decreased since the early 1980’s, individual earnings volatility and wage inequality increased. This paper argues that increasing financial development can contribute to both changes. I develop a real business cycle model with sectoral productivity shocks and labor as well as credit market frictions. Credit market frictions take the form of collateral-based credit constraints. It is shown that there are interactions between the labor and the credit market that matter for the development of wages and output. When workers are not perfectly mobile between sectors, financial development comes along with an increase in the volatility of individual earnings and in wage inequality, although aggregate output volatility is lower.

JEL classification: E32; E44; J60
Keywords: financial development, labor market frictions, sectoral shocks, volatility, wage inequality

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1 Introduction

Since the early 1980’s, the US experienced a decline in the volatility of real GDP growth (Stock and Watson, 2002; Davis and Kahn, 2008). One possible explanation is the increase in financial development in the meaning of easier access to credit for firms (Dynan, Elmendorf, and Sichel, 2006). Support for the positive link between macroeconomic stability and financial development is given in the cross-country study of Denizer, Iyigun, and Owen (2002). There is however no evidence for higher stability at the household level. Kambourov and Manovskii (2009) find that individual earnings volatility and earnings inequality increased since the 1970’s. The increase occurred especially within narrowly defined age-education subgroups. Figures 1 and 2 show the decline in the volatility of GDP growth as well as the increase in the ratio of gross earnings at the top decile to those at the bottom decile, that have been accompanied by an increase in the share of credit in GDP. The present paper argues that an increase in financial development can contribute to both changes: increasing macroeconomic stability and increasing wage inequality within groups of similar workers.

An increasing amount of theoretical work demonstrates the link between financial development, macroeconomic fluctuations and growth. The models often dispense with labor as a production input (Kiyotaki and Moore, 1997; Kiyotaki, 1998; Azariadis and Kaas, 2009) or they assume perfect mobility of labor (Aghion, Banerjee, and Piketty, 1999; Kiyotaki and Moore, 2008; Kocherlakota, 2009; Kaas, 2009). However, labor reallocation has been identified as an important factor in explaining macroeconomic variables (Lilien, 1982; Burgess and Mawson, 2003). Lagos (2006), for example, shows in a model of frictional labor market and sectoral shocks how labor market policies affect total factor productivity (TFP).

There are few papers that study the effect of financial development on within-group wage inequality. Jerzmanowski and Nabar (2011) find empirical evidence for a positive link between financial development and within-group wage inequality in the US. Their theoretical approach focuses on how financial development leads to organizational change, and thereby to higher wage inequality for skilled workers.

The objective of this paper is to explore the link between macroeconomic stability,
Figure 1: Volatility and Credit: 
Domestic credit to private sector (% of GDP). Data source: World Development Indicators (World Bank).
Aggregate Volatility: Rolling standard deviation of GDP growth (annual %). Data source for GDP growth: World Development Indicators.

volatility of individual earnings, and wage inequality when debt constraints slow down capital reallocation and when workers are not perfectly mobile. It is shown that looking at frictions in financial markets and labor markets separately may be misleading. There are important interactions between both markets. As a result, the effectiveness of improvements in the credit market depend on the degree of labor market frictions.

I develop a model with two sectors. Credit market frictions arise in the form of collateral-based credit constraints as for example in Kiyotaki and Moore (1997), Kiyotaki (1998), and Azariadis and Kaas (2009). I introduce a simple form of labor market frictions: In each period only a given fraction of workers employed in the low-wage sector can move into the high-wage sector. One can think of various factors that make it difficult for a worker to switch sectors. Barriers may arise from sector-specific skills or workers may have to move to another town if they want to switch sectors. Wages within one sector are determined competitively. In the model,
volatility is the result of sectoral productivity shocks. I do not consider aggregate productivity shocks that would affect both sectors in the same way. One sector always produces at the technology frontier. If capital and labor were perfectly mobile, all production factors would flow to the productive sector, and there would be no volatility and no wage inequality. In addition, output and wage income would be maximized. When capital market frictions are introduced, aggregate output depends on the distribution of wealth between sectors. When financial development increases, the wealth distribution becomes less important and the volatility of aggregate output decreases. When labor is mobile, each worker earns the same wage, which behaves similar to aggregate output. Wage income increases in financial development as more capital is employed in the high-TFP sector, and wage volatility decreases in financial development. If, in addition, labor market frictions are introduced, wage inequality can arise. Increasing financial development now increases the correlation of wages with sector-specific TFP, and thereby wage inequality and volatility of individual
A related strand of literature introduces capital market frictions into a search and matching model of equilibrium unemployment (Wasmer and Weil, 2004; Petrosky-Nadeau, 2009; Petrosky-Nadeau and Wasmer, 2010; Dromel, Kolakez, and Lehmann, 2010). While these papers explore the impact of credit market frictions on unemployment in the presence of macroeconomic shocks, the present paper examines the impact of credit market frictions on wage inequality in the presence of sector-specific productivity shocks.

The rest of the paper is organized as follows: Section 2 describes the model environment. Equity returns, wages, and the sectoral distribution of wealth in equilibrium are determined in Section 3. Section 4 explores the effect of financial development on volatility and inequality. Section 5 concludes.

2 The Model

Consider a discrete-time economy with two labor markets or sectors indexed by $j = 1, 2$ and infinitely-lived workers and entrepreneurs. There is one representative worker household in the economy consisting of a continuum of workers. The worker household does not save or borrow, but simply consumes its labor income each period. The worker household assigns its members to the sectors in order to maximize labor income. The labor market within one sector is competitive, but wages may differ between sectors when labor is not perfectly mobile. Workers wish to move to the sector with the highest wage. However, each period only up to a fraction $\gamma$ of workers in the low-wage sector can move into the high-wage sector. The parameter $\gamma$ captures stochastic costs of switching sectors. Each sector consists of a continuum of entrepreneurs that can be represented by one firm in each sector. Entrepreneurs may lend or borrow capital, hire workers, and produce. They derive logarithmic utility from consumption. The expected utility of firm $j$ at date $t$ is $E_t \sum_{\tau \geq t} \beta^{\tau-t} \ln(C^j_\tau)$, where $E_t$ denotes expectations formed at date $t$, $\beta \in (0, 1)$ is the discount factor of firms, and $C^j_\tau$ is firm $j$’s consumption at date $\tau$. All firms produce the same homo-

\footnote{Workers may not borrow since they have no collateral. There is one big representative worker household that insures its members against sectoral income shocks and that does not save. The assumption that workers do not save is a common simplification in models with credit market frictions (e.g. Kocherlakota, 2009).}
geneous good with the Cobb-Douglas production technology $Y_t^j = A_t^j(K_t^j)^\alpha(L_t^j)^{1-\alpha}$, where $L_t^j$ and $K_t^j$ denote labor and capital input in sector $j$. The good produced in period $t$ is used for next period’s consumption and investment. There are two states of productivity ($s_t = 1, 2$) given by

$$A_t^j = A^j(s_t) = \begin{cases} A & \text{if } j = s_t \\ zA & \text{if } j \neq s_t \end{cases}$$

with $0 < z < 1$,

and with transition probabilities

$$\pi(s_{t+1}|s_t) = \begin{cases} \pi \in (0.5, 1) & \text{if } s_{t+1} = s_t \\ 1 - \pi & \text{if } s_{t+1} \neq s_t \end{cases}.$$

Firms in one sector produce at the productivity frontier $A$, and firms in the other sector have low TFP $zA$. Productivity states are positively autocorrelated. In the following, the sector with high (low) TFP $A$ ($zA$) is indexed by $i = H$ ($L$).

Firms may borrow and lend at gross interest rate $R_t$. Let $D_t^i$ be the debt position of firm $i$ in period $t$. If firm $i$ is a borrower, $D_t^i > 0$. If firm $i$ is a lender, $D_t^i < 0$. Only a fraction $\lambda \in [0, \alpha z^{\frac{1}{\alpha}})$ of output is pledgeable collateral and firm $i$ may only borrow up to the value of its collateral $D_t^i \leq \frac{\lambda Y_t^i}{R_t}$. Firms observe the state of productivity and decide based on that information whether to borrow or lend, and whether they want to produce and hire workers. Firms collect profits out of production, redeem debt or collect returns from saving.

3 Equilibrium

Firms choose consumption $C_t^i$, capital input $K_t^i$, labor input $L_t^i$, and debt position $D_t^i$ in order to maximize the entrepreneurs’ expected utility subject to budget and debt constraints:

$$\max E_t \sum_{\tau \geq t} \beta^{\tau-t} ln(C_t^i)$$  \hspace{1cm} (1)
s.t. $C^i_t + K^i_t - D^i_t = A^i_{t-1}(K^i_{t-1})^\alpha(L^i_{t-1})^{1-\alpha} - w^i_{t-1}L^i_{t-1} - R_{t-1}D^i_{t-1}, \tau \geq t,$

$R_{t}D^i_{t} \leq \lambda A^i_{t}(K^i_{t})^\alpha(L^i_{t})^{1-\alpha}, \tau \geq t,$

where $w^i_{t}$ denotes the (real) wage paid in period $\tau$ in sector $i$. Equity of firm $i$ is given by $E^i_t = K^i_t - D^i_t$. It is shown below that the budget constraint in period $t$ simplifies to

$C^i_{t+1} + E^i_{t+1} = R^i_t E^i_t,$

where the equity return $R^i_t$ equals the interest rate $R_t$ if firm $i$ is a lender. The equity return is given by $\tilde{R}_t \geq R_t$ if firm $i$ is a borrower. Solving the maximization problem of firm $i$ yields the Euler equation

$$\frac{1}{C^i_t} = \beta R^i_t E^i_t \frac{1}{C^i_{t+1}}$$

with the solutions

$C^i_{t+1} = (1 - \beta)R^i_t E^i_t$ and $E^i_{t+1} = \beta R^i_t E^i_t$.

A *market equilibrium* is defined as a sequence of consumption plans, allocations of capital and labor, as well as debt positions $\{C^i_t, K^i_t, L^i_t, D^i_t\}$ for each sector, consumption for the workers $\{C^w_t\}$, and wages and interest rate $\{w^H_t, w^L_t, R_t\}$ such that for a given initial capital stock, and initial wealth and labor distribution

- the entrepreneurs’ maximization problem (1) is solved
- as many workers as possible are allocated in the high-wage sector and workers consume all their labor income $C^w_t = w^H_t L^H_t + w^L_t L^L_t$
- the markets for output, labor, and capital clear.

Let $D^i_t = D_t > 0$ if firm $i$ is a borrower. Equilibrium in the credit market ensures that $D^i_t = -D_t$ if firm $i$ is a lender. Next period’s aggregate capital stock equals then

$$K_{t+1} = \alpha \beta Y_t,$$

where $Y_t$ denotes aggregate output. In the following, the total amount of labor
is normalized to one and \( k^H_t \equiv \frac{K^H_t}{L^H_t} \). The market equilibrium depends on financial development (\( \lambda \)) as well as on the degree of labor mobility (\( \gamma \)). Equilibrium wages, equity returns, and the development of wealth are separately determined for the case of perfect labor mobility, the case when labor is mobile to some degree, and the case when labor is immobile between sectors.

### 3.1 Perfect Mobility of Labor: \( \gamma = 1 \)

Competitive wages are equal to the marginal productivity of labor. As workers can move freely between sectors, wages are the same in both sectors:

\[
w_t = (1 - \alpha)A \left( k^H_t \right)^\alpha = (1 - \alpha)zA \left( k^L_t \right)^\alpha .
\]

(3)

It follows that \( \frac{k^H_t}{k^L_t} = z^{\frac{1}{\alpha}} \) must hold. Suppose high-TFP firms are debt constrained. Credit is then given by \( D_t = \frac{\lambda Y^H_t}{R_t} \). Wealth of the productive (and credit constrained) firm at the end of period \( t \) is derived as

\[
Y^H_t - w_t L^H_t - R_tD^H_t = \frac{R_t(\alpha - \lambda)}{R_tA^{-1/\alpha} \left[ w_t/(1 - \alpha) \right]^{(1-\alpha)/\alpha} - \lambda} E^H_t .
\]

(4)

Wealth of the low-TFP firm at the end of period \( t \) is given by

\[ R_tE^L_t . \]

Let \( x_t \equiv \frac{E^H_t}{K_t} \) denote the wealth share of the high-TFP firm. The interest rate is determined as a function of \( x_t \) and of the total capital stock \( K_t \). One of three cases can occur:

- **Only high-TFP firms produce and debt constraints are binding**

  When low-TFP firms do not produce, and debt constraints are binding, the
debt/equity ratio of high-TFP firms is
\[ \theta^H_t = \frac{D_t}{K^H_t - D_t} = \frac{\lambda A(k^H_t)^{\alpha-1}}{R_t - \lambda A(k^H_t)^{\alpha-1}}. \]

As low-TFP firms lend all their equity to high-TFP firms in this case, the debt/equity ratio also equals
\[ \theta^H_t = \frac{(1 - x_t)K_t}{x_t K_t}. \]

The interest rate is then
\[ R(x_t, K_t) = \frac{\lambda A K_t^{\alpha-1}}{1 - x_t} \in (MPK^L_t, MPK^H_t), \]
where \( MPK^L_t = \alpha A_t^L (k^L_t)^{\alpha-1} \) denotes the marginal product of capital in sector \( i \).

- **All firms produce and debt constraints are binding**
  The interest rate will not fall below the marginal product of capital of low-TFP firms. If the wealth share of productive firms is small, the interest rate is
  \[ R(x_t, K_t) = MPK^L_t. \]
  The critical value of \( x_t \) below which this happens, is derived as follows:
  \[ \alpha z A(k^L_t)^{\alpha-1} > \lambda A(k^H_t)^{\alpha-1} \frac{1}{1 - x_t} \]
  \[ \Leftrightarrow x_t < 1 - \frac{\lambda}{\alpha z^{1/\alpha}}. \]
  In this case, both types of firms produce, and high-TFP firms are debt constrained. Using \( D_t = \frac{\lambda Y^H_t}{K^H_t} \), \( K^H_t = x_t K_t + D_t \) and \( \frac{K^H_t}{K_t} = z^{\frac{1}{\alpha}} \) yields
  \[ D_t = \frac{\lambda x_t}{\alpha z^{\frac{1}{\alpha}} - \lambda} K_t. \]
• Only high-TFP firms produce and borrowers are not debt constrained
  The interest rate will not exceed the marginal product of capital of high-TFP firms. If the wealth share of high-TFP firms is large, borrowers are not debt constrained and the interest rate is

\[ R(x_t, K_t) = MPK_t^H. \]

The equity return of the productive sector \( \tilde{R}_t \) is then equal to the interest rate. The critical value of \( x_t \) is derived as follows:

\[ \alpha A \left( \frac{1}{\alpha z} - \lambda + \alpha \frac{1}{\alpha x_t} - \alpha \frac{1}{\alpha z} x_t \right)^{\alpha - 1} K_t^{\alpha - 1} \leq \frac{\lambda A \left( k_t^H \right)^{\alpha - 1}}{1 - x_t} \]

\[ \Leftrightarrow x_t \geq 1 - \frac{\lambda}{\alpha}. \]

The following function sums up the above results and gives the interest rate for all combinations of \( x_t \) and \( K_t \):

\[ R(x_t, K_t) = \begin{cases} \alpha z A \left( \frac{1}{\alpha z} - \lambda + \alpha \frac{1}{\alpha x_t} - \alpha \frac{1}{\alpha z} x_t \right)^{\alpha - 1} K_t^{\alpha - 1} & \text{if } x_t \leq 1 - \frac{\lambda}{\alpha z^{1/\alpha}} \\ \frac{\lambda A}{1 - x_t} K_t^{\alpha - 1} & \text{if } x_t \in \left[ 1 - \frac{\lambda}{\alpha z^{1/\alpha}}, 1 - \frac{\lambda}{\alpha} \right] \\ \alpha A K_t^{\alpha - 1} & \text{if } x_t \geq 1 - \frac{\lambda}{\alpha} \end{cases}. \] (6)

Next period’s wealth share of the high-TFP firm is

\[ x_{t+1} = \begin{cases} X_0(x_t) & \text{if the productivity state does not change} \\ X_1(x_t) = 1 - X_0(x_t) & \text{if the productivity state changes} \end{cases} \]

with
\[ X_0(x_t) = \frac{\tilde{R}(x_t,K_t)x_t}{R(x_t,K_t)x_t + \tilde{R}(x_t,K_t)(1-x_t)} = \begin{cases} \frac{(\alpha-\lambda)x_t}{(\alpha-\lambda)x_t + (\alpha z^\alpha - \lambda)(1-x_t)} & \text{if } x_t \leq 1 - \frac{\lambda}{\alpha z^\alpha} \\ 1 - \frac{\lambda}{\alpha} & \text{if } x_t \in [1 - \frac{\lambda}{\alpha z^\alpha}, 1 - \frac{\lambda}{\alpha}] \\ x_t & \text{if } x_t \geq 1 - \frac{\lambda}{\alpha} \end{cases} \] (7)

The stochastic dynamics of borrower wealth depends on the collateral share \( \lambda \), and is similar to Proposition 4 in Azariadis and Kaas (2009) in which capital is the only production input. Economies with high collateral \( \lambda \geq \frac{\alpha}{2} \equiv \lambda_{2}^{\text{comp}} \) converge to an equilibrium with efficient production, non-binding credit constraints and no volatility. Economies with medium collateral \( \frac{\alpha z^\alpha}{1 + z^\alpha} \leq \lambda < \frac{\alpha}{2} \) converge to a cycle with efficient production. However credit constraints bind in a fraction \( 1 - \pi \) of periods. Economies with small collateral \( \lambda < \frac{\alpha z^\alpha}{1 + z^\alpha} \equiv \lambda_{1}^{\text{comp}} \) converge to a cycle with a finite number of states. Production is efficient only in three states. For a more detailed description of the dynamics and for a proof, see Azariadis and Kaas (2009).

### 3.2 Labor is mobile to some degree: \( 0 < \gamma < 1 \)

Let \( b_t \) be the fraction of workers in the high-TFP sector at the beginning of period \( t \) before the new productivity state is drawn and before labor reallocation takes place. After labor is reallocated, labor input in period \( t \) is \( L_t^H = b_{t+1} \) in the high-TFP sector and \( L_t^L = 1 - b_{t+1} \) in the low-TFP sector. In each period, only a fraction \( \gamma \) of workers in the low-wage sector can move to the high-wage sector.

If the labor market constraint does not bind, all workers earn the same wage after labor reallocation. The distribution of workers between sectors for this case is derived in Appendix A. Labor input in the high-TFP sector in period \( t \) is then

\[ b_{t+1} = B^{mc}(x_t) = \begin{cases} \frac{\alpha x_t}{\alpha z^\alpha (1-x_t) - \lambda + \alpha x_t} & \text{if } x_t < 1 - \frac{\lambda}{\alpha z^\alpha} \\ 1 & \text{if } x_t \geq 1 - \frac{\lambda}{\alpha z^\alpha} \end{cases} \] (8)

When labor is not perfectly mobile, not all workers can leave the low-wage sector
and both sectors produce. Wages do not only depend on sectoral TFP, but also on the amount of capital and labor employed. Workers in the high-TFP sector usually earn the highest wage. However, when labor is sufficiently scarce in the low-TFP sector, workers in the low-TFP sector are paid the highest wage. Given \( x_t \) and \( b_t \), labor input in the high-TFP sector in period \( t \) is given by

\[
b_{t+1} = B_0(x_t, b_t) = \begin{cases} 
  \min (b_t + \gamma(1 - b_t), B^{nc}(x_t)) & \text{if } B^{nc}(x_t) \geq b_t \\
  \max ((1 - \gamma)b_t, B^{nc}(x_t)) & \text{if } B^{nc}(x_t) < b_t
\end{cases}
\]  

(9)

if the productivity state remains the same, and by

\[
b_{t+1} = B^1(x_t, b_t) = B_0(x_t, 1 - b_t) = \begin{cases} 
  \min (1 - b_t + \gamma b_t, B^{nc}(x_t)) & \text{if } B^{nc}(x_t) \geq 1 - b_t \\
  \max ((1 - \gamma)(1 - b_t), B^{nc}(x_t)) & \text{if } B^{nc}(x_t) < 1 - b_t
\end{cases}
\]

if the productivity state changes.

When workers are not perfectly mobile, the high-TFP firm is not necessarily the borrower. The productivity of capital within one sector increases in the amount of labor employed. When labor is sufficiently immobile, it may happen that there is so few labor in the high-TFP sector that the productivity of capital is higher in the low-TFP sector, and the low-TFP firm borrows. A firm borrows capital as long as its marginal productivity of capital exceeds the interest rate. It is shown in Appendix A that high-TFP firms borrow as long as \( x_t \leq \frac{b_{t+1}}{b_{t+1} + (1 - b_{t+1})z^{1-\alpha}} \). Low-TFP firms borrow if \( x_t > \frac{b_{t+1}}{b_{t+1} + (1 - b_{t+1})z^{1-\alpha}} \). The threshold increases in the fraction of labor employed in the high-TFP sector, \( b_{t+1} \).

- **Case 1: High-TFP firms borrow, i.e.** \( x_t \leq \frac{b_{t+1}}{b_{t+1} + (1 - b_{t+1})z^{1-\alpha}} \)

When the high-TFP firm is borrowing constrained, debt is given by \( D_t = \frac{\lambda A(K^H_t)^\alpha (H^H_t)^{1-\alpha}}{R_t} \). Wealth of the productive firm at the end of the period is then

\[
\frac{R_t(\alpha - \lambda)}{R_t A^{-1} (k^H_t)^{1-\alpha} - \lambda} E_t^H.
\]

(10)
Since some workers stay in the low-TFP sector, both sectors produce and the interest rate equals the marginal product of capital of low-TFP firms. When borrower wealth is large enough, credit constraints are not binding, and the interest rate equals also the marginal product of capital of high-TFP firms.\footnote{The corresponding value of }\( x_t \)\footnote{The corresponding value of } is derived in Appendix A. The interest rate is therefore given by
\[
R_t = MPK_t^L = \begin{cases} 
\alpha z A (f(x_t, b_{t+1}) b_{t+1} + 1 - b_{t+1})^{1-\alpha} K_t^{\alpha-1} & \text{if } x_t < \frac{(\alpha-\lambda) b_{t+1}}{\alpha (b_{t+1} + (1-b_{t+1}) z)^{1/\alpha}} \\
MPK_t^H = \alpha A \left( b_{t+1} + z^{1/\alpha} (1 - b_{t+1}) \right)^{1-\alpha} K_t^{\alpha-1} & \text{if } x_t \geq \frac{(\alpha-\lambda) b_{t+1}}{\alpha (b_{t+1} + (1-b_{t+1}) z)^{1/\alpha}} 
\end{cases}
\]
(11)

where \( f(x_t, b_{t+1}) \) gives the equilibrium value of \( k^H_t \) and is derived in Appendix A. The transitional dynamics of the wealth share of high-TFP firms is described by
\[
X_0(x_t, b_{t+1}) = \begin{cases} 
\frac{(\alpha-\lambda) x_t}{(\alpha-\lambda) x_t + (\alpha z [f(x_t, b_{t+1})]^{1-\alpha} - \lambda)(1-x_t)} & \text{if } x_t < \frac{(\alpha-\lambda) b_{t+1}}{\alpha (b_{t+1} + (1-b_{t+1}) z)^{1/\alpha}} \\
x_t & \text{if } x_t \geq \frac{(\alpha-\lambda) b_{t+1}}{\alpha (b_{t+1} + (1-b_{t+1}) z)^{1/\alpha}} 
\end{cases}
\]
(12)

if the productivity state does not change. If the productivity state changes, \( x_{t+1} = 1 - X_0(x_t, b_{t+1}) \).

- **Case 2: Low-TFP firms borrow**, i.e. \( x_t > \frac{b_{t+1}}{b_{t+1} + (1-b_{t+1}) z^{1/\alpha}} \)
Consider the case when the low-TFP firm is borrowing constrained. Using \( D_t = \frac{\lambda Y_t^L}{R_t} \) one obtains wealth of the low-TFP firm at the end of the period as
\[
\frac{R_t (\alpha - \lambda)}{R_t (zA)^{-1} (k_t^L)^{1-\alpha} - \lambda} E_t^L.
\]
(13)

The interest rate equals the marginal product of capital of high-TFP firms. When wealth of the low-TFP firm is sufficiently high, credit constraints are not
binding and the interest rate equals also the marginal product of capital of low-
TFP firms.\footnote{The corresponding value of $x_t$ is derived in Appendix A.} It follows that the interest rate is given by
\[ R_t = MPK^H_t = \]
\[
\begin{cases}
\alpha A \left( \frac{g(x_t, b_{t+1})}{g(x_t, b_{t+1}) - b_{t+1} - b_{t+1}} \right)^\alpha - 1 K_t^{a-1} & \text{if } x_t > \frac{\alpha b_{t+1} + \lambda (1 - b_{t+1}) x_t}{\alpha [b_{t+1} + (1 - b_{t+1}) x_t]} \\
MPK^L_t = \alpha A \left( b_{t+1} + \frac{1}{\alpha} (1 - b_{t+1}) \right)^{1-\alpha} K_t^{a-1} & \text{if } x_t \leq \frac{\alpha b_{t+1} + \lambda (1 - b_{t+1}) x_t}{\alpha [b_{t+1} + (1 - b_{t+1}) x_t]} 
\end{cases}
\]

where $g(x_t, b_{t+1})$ gives the equilibrium value of $\frac{k^H_t}{k^L_t}$.\footnote{The derivation of $g(x_t, b_{t+1})$ is similar to the derivation of $f(x_t, b_{t+1})$ in Appendix A. The value of $g(x_t, b_{t+1})$, with $x_t > \frac{\alpha b_{t+1} + \lambda (1 - b_{t+1}) x_t}{\alpha [b_{t+1} + (1 - b_{t+1}) x_t]}$ is uniquely determined as the value of $\frac{k^H_t}{k^L_t}$ that solves}

\[ \alpha x_t - \alpha (1 - x_t) \frac{b_{t+1}}{1 - b_{t+1}} \frac{k^H_t}{k^L_t} = \lambda_{x_t} \left( \frac{k^H_t}{k^L_t} \right)^{1-\alpha}. \]

The values of $b_t$ and $x_t$ determine whether the high-TPF firm is a lender or a borrower in period $t$, and which sector can attract workers. Figure 3 illustrates the thresholds of $x_t$ as functions of $b_t$ indicating which case occurs when the productivity state does not change. The threshold functions are derived in Appendix B. In the beginning of period $t$, before labor is reallocated between sectors, a fraction $b_t$ of workers is in the high-TPF sector, and the wealth share of high-TPF firms is given by $x_t = X_0(x_{t-1}, b_t)$. When $(b_t, x_t)$ is located above the $T_B$ curve, the high-TPF firm has a high wealth share and employs relatively few workers. It will therefore lend capital to the low-TPF firm. Below the $T_B$ threshold, in contrast, the high-TPF firm borrows capital. It is profitable for a firm to borrow capital as long as its marginal productivity
exceeds the interest rate. When \((b_t, x_t)\) is located in the area between the \(T_{RH}\) and the \(T_{RL}\) curve, the borrower’s wealth share is sufficiently large that credit constraints are not binding. Marginal productivities of capital are equalized across sectors. Equity returns are equalized as well. The wealth distribution does not change. When \((b_t, x_t)\) is located above the \(T_{RH}\) curve, the interest rate equals \(MPK_t^H\) and the wealth share of the high-TFP sector decreases. Below the \(T_{RL}\) curve, the interest rate equals \(MPK_t^L\) and the wealth share of the high-TFP sector increases.

Workers earn a wage equal to the marginal productivity of labor within their sector. They wish to move to the sector with the highest wage. That is usually the sector with higher TFP. However, when the wealth share in the high-TFP sector is very low, workers in the low-TFP sector earn the highest wage. This occurs in the area below the \(T_{WL}\) curve. Workers move to the high-TFP (low-TFP) sector when \((b_t, x_t)\) is located above (below) the \(T_L\) curve. Wages in both sectors are equalized by labor reallocation, when \((b_t, x_t)\) is located in the area between the \(T_{WH}\) and the \(T_{WL}\) curve. Above the \(T_{WH}\) curve wages in the high-TFP sector exceed wages in the low-TFP sector.

The arrows in Figure 3 indicate whether \(x\) and/or \(b\) decrease or increase. When the productivity state changes, the fraction of workers in the high-TFP sector before labor reallocation is given by \(1 - b_t\), and the wealth share in the new high-TFP sector is \(x_t = 1 - X_0(x_{t-1}, b_t)\).

Assume the productivity state does not change for several periods. When a lot of workers and only few capital is allocated in the high-TFP sector, workers will leave and capital flows to the high-TFP sector. The wealth share increases and eventually the sector can again attract workers. When there are only few workers and a lot of capital in the high-TFP sector, workers will move to the high-TFP sector while capital leaves. When the productivity state changes, the wealth share and labor input in the new high-TFP sector is again located below the \(T_{RL}\) threshold. When financial development is sufficiently high, \((b_t, x_t)\) eventually stays in the area between the \(T_{RH}\) and the \(T_{RL}\) curve where equity returns are equalized across sectors.

**Proposition 1.** If \(\lambda \geq \frac{\alpha}{2}\), equity returns are equalized across sectors in the long run for a given \(\gamma \in (0, 1]\).
Figure 3: Dynamics when the productivity state remains the same \((s_t = s_{t-1})\). When \(s_t \neq s_{t-1}\), \(b_t\) and \(X_0(x_{t-1}, b_t)\) are replaced by \(1 - b_t\) and \(1 - X_0(x_{t-1}, b_t)\).

Appendix C contains the proof of Proposition 1. The critical value of \(\lambda\) equals the threshold \(\lambda_{comp}^2\) that holds in the case of perfect labor mobility.

3.3 Labor is immobile between sectors: \(\gamma = 0\)

Consider now the case \(\gamma = 0\). Workers are stuck in their present sector. Each period \(t\), the fraction of workers employed in the high-TFP sector is either \(\mu \geq 0.5\) or \(1 - \mu\). The dynamics of the model become clear by inspection of Figures 4 and 5. The horizontal axis displays now the labor input of high-TFP firms in period \(t\), \(b_{t+1}\). The threshold functions are derived in Appendix B. Assume that financial development is high, \(b_{t+1}\) equals \(\mu\), and \(x_t\) falls between the \(T_{RH}\) and the \(T_{RL}\) threshold. When productivity stays constant, next period’s wealth share of the high-TFP sector as well as its labor input remain the same. Otherwise, they equal \(1 - \mu\) and \(1 - x_t\).
When the new wealth share of high-TFP firms is also located in the area between the $T_{RH}$ and the $T_{RL}$ threshold, this will also be the case in all following periods and firms are not credit constrained. When it is located below the $T_{RL}$ line, the wealth share of the high-TFP sector will approach the $T_{RL}$ line from below. The long run dynamics are illustrated in Figure 4 by the double arrows. Note that the $T_{RH}$ curve has the same curvature as the $T_{RL}$ curve. In the long run, $x$ will never lie above the $T_{RH}$ curve. This follows from concavity of both curves. Figure 5 displays the dynamics for low financial development.

**Proposition 2.**

(a) Economies with high collateral $\lambda \geq \frac{\alpha \mu (1-\mu)(1-z)z}{2\mu(1-z)^{1+z\alpha} - 2\mu^2(1-z)^{1+z\alpha} + z^{1+z\alpha}} \equiv \lambda_C^{im}$ converge to a cycle with two states $x_2 = 1 - x_1 \in [T_{RL}(1-\mu), 1 - T_{RL}(1-\mu)]$. 

Figure 4: Labor is immobile and $\lambda \geq \frac{\alpha \mu (1-\mu)(1-z)z}{2\mu(1-z)^{1+z\alpha} - 2\mu^2(1-z)^{1+z\alpha} + z^{1+z\alpha}}$
Equity returns are equalized across sectors in the long run and credit constraints do not bind.

(b) Economies with small collateral $\lambda < \lambda^m_C$ converge to a cycle with $x \in [1 - T_{RL}(\mu), T_{RL}(1 - \mu)] \cup [1 - T_{RL}(1 - \mu), T_{RL}(\mu)]$. Capital flows to the high-TPF sector and debt constraints are binding.

Appendix C contains the proof of Proposition 2.

Figure 5: Labor is immobile and $\lambda < \frac{\alpha\mu(1-\mu)(1-z\frac{2}{\alpha})}{2\mu(1-z\frac{1}{\alpha})-2\mu^2(1-z\frac{1}{\alpha})+z\frac{1}{\alpha}}$
4 Simulation

This section examines the dynamics of the model for \( 0 < \gamma \leq 1 \) by varying the value of \( \lambda \).\(^5\) The simulation is not meant to replicate real data, but to highlight the effects of different degrees of capital and labor market imperfections on the development of output and wages. The model period is one year.\(^6\) The discount factor is set to \( \beta = 0.95 \). Let \( Y \) include output as well as undepreciated capital. Using this interpretation, it is reasonable to choose a capital share \( \alpha = 0.8 \) (Kaas, 2009). The remaining parameters are set to \( A = 1, z = 0.9, \) and \( \pi = 0.6 \). The initial values of \( x \) and \( b \) are 0.5 and 0.7. One obtains the thresholds \( \lambda_{comp}^1 = 0.37 \) and \( \lambda_{comp}^2 = \frac{\alpha}{2} = 0.40 \). The variables of interest are determined by the sample means of

- aggregate output: \( Y_t = Y_t^H + Y_t^L \)
- the share of credit in aggregate output: \( D_t/Y_t \)
- the average wage: \( b_{t+1}w_t^H + (1 - b_{t+1})w_t^L \)
- wage inequality: \( w_t^H/w_t^L \)

The volatility of a variable is measured as its standard deviation over all periods. The volatility of individual wages is calculated as the standard deviation of wages within one sector.

The simulation results are illustrated in Figures 6 to 11. Since labor and capital are complementary input factors, the effect of a policy improving the mobility of capital depends on the mobility of labor. I simulated each series for three different degrees of labor market frictions: low labor mobility \( (\gamma = 0.1) \), high labor mobility \( (\gamma = 0.9) \), and perfect mobility of labor \( (\gamma = 1) \). The simulation results show that increasing financial development has, in general, a higher effect when workers are more mobile. Higher financial development increases the share of credit in aggregate output (Figure 6). Recall that the credit share in GDP has been taken as a measure of financial development in the introductory section of this paper. Financial development has a similar effect on aggregate output as it has on the average wage. Aggregate output

\(^5\)The case when labor cannot move between sectors differs mainly in the threshold of \( \lambda \) above which equity returns are equalized.

\(^6\)I simulated time series of 50,000 periods.
increases in financial development (Figure 7). The volatility of aggregate output decreases in financial development (Figure 8). Financial development has a higher potential effect on volatility when the labor market is more flexible. Aggregate volatility is zero only if neither the capital nor the labor market constraint binds. Note, however, that for economies with poor financial development, volatility is higher when workers are more mobile. When capital mobility is low, in some periods, a lot of capital is allocated in the low-TFP sector. Hence, the low-TFP sector withdraws workers from the high-TFP sector and aggregate output is low. In other periods, a lot of capital is allocated in the high-TFP sector and workers want to work in the high-TFP sector. As a result, aggregate output is high. These fluctuations are amplified when worker mobility is increased. Wage inequality as well as the volatility of individual earnings increase in financial development when labor is not perfectly mobile between sectors (Figures 9 and 10). When labor is assumed to be perfectly mobile, each worker earns the same wage and the simulation shows that volatility of wages decreases in financial development. It is by the introduction of labor market
frictions that a positive relationship between wage inequality, volatility of individual earnings, and financial development emerges.

Figure 7: The effect of financial development on aggregate output for $\gamma = 0.1$ (dotted line), $\gamma = 0.9$ (dashed line), $\gamma = 1$ (solid line)
Figure 8: The effect of financial development on the volatility of aggregate output for $\gamma = 0.1$ (dotted line), $\gamma = 0.9$ (dashed line), $\gamma = 1$ (solid line)

Figure 9: The effect of financial development on wage inequality for $\gamma = 0.1$ (dotted line), $\gamma = 0.9$ (dashed line), $\gamma = 1$ (solid line)
What is the intuition behind the results? Volatility in the model framework is the result of sectoral productivity shocks. When there are no credit and no labor market frictions, capital and labor always flow to the sector with high TFP. There is no volatility, and no inequality. Labor income and output are maximized. When capital market frictions are introduced, the sector with lower TFP also produces and the distribution of wealth between the high and the low TFP sector matters. The wealth distribution becomes less important when financial development increases. Increasing financial development decreases the volatility of aggregate output and of the single wage. When labor market frictions are introduced as well, it may occur that not enough workers manage to move to the high-wage sector to equalize marginal productivities of labor across sectors. Workers in one sector earn then lower wages than workers in the other sector. If financial development increases now, more capital flows to the high-TFP sector. As a result, wages in the high-TFP sector increase even more while wages in the low-TFP sector decrease even more. Wages
are more correlated with sectoral TFP (Figure 11). Wage inequality and volatility of individual earnings increase.

Figure 11: The effect of financial development on the correlation of wages with sector-specific TFP for $\gamma = 0.1$ (dotted line), $\gamma = 0.9$ (dashed line), $\gamma = 1$ (solid line)

5 Conclusions

A real business cycle model with sectoral productivity shocks and labor as well as credit market frictions can explain a simultaneous increase in macroeconomic stability and in wage inequality. In line with other theoretical work on financial frictions, it was shown that financial development has a positive effect on output and macroeconomic stability. The main contribution of the present paper is to make visible the interaction between the labor and the credit market. In the presence of labor market frictions, an increase in financial development increases the correlation of wages with sector-specific TFP and thereby wage inequality, and volatility of individual earnings.
References


APPENDIX

A Derivations

Derivation of equation (8)

The following derivation makes use of results obtained in Section 3.1. In a competitive labor market, only high-TFP firms produce in period $t$ if $x_t \geq 1 - \frac{\lambda}{\alpha z^\frac{1}{\alpha}}$. All workers are then employed in the high-TFP sector, i.e. $b_{t+1} = 1$. When $x_t < 1 - \frac{\lambda}{\alpha z^\frac{1}{\alpha}}$ both firms produce, the high-TFP firm borrows, and debt constraints are binding. As the marginal products of labor are equalized between both sectors, $k_t^H = k_t^L z^\frac{1}{\alpha}$. This is equivalent to

$$\frac{x_t K_t + D_t}{b_{t+1}} = \frac{(1 - x_t) K_t - D_t}{1 - b_{t+1}} z^\frac{1}{\alpha}.$$

Using equation (5), one obtains

$$\Leftrightarrow b_{t+1} = \frac{\alpha x_t}{\alpha z^\frac{1}{\alpha}(1 - x_t) - \lambda + \alpha x_t}.$$

Threshold of $x_t$ below which high-TFP firms borrow

The high-TFP firm borrows as long as $MPK_t^H \geq MPK_t^L$. This is equivalent to

$$\frac{K_t^H}{K_t^L} \leq \frac{b_{t+1}}{1 - b_{t+1}} z^\frac{1}{\alpha - 1} \Leftrightarrow \frac{x_t}{1 - x_t} \leq \frac{b_{t+1}}{1 - b_{t+1}} z^\frac{1}{\alpha - 1} \Leftrightarrow x_t \leq \frac{b_{t+1}}{b_{t+1} + (1 - b_{t+1}) z^\frac{1}{\alpha - 1}}.$$

Derivation of $f(x_t, b_{t+1})$

Using $\frac{K_t^H}{K_t^L} = \frac{K_t x_t + D_t}{K_t (1 - x_t) - D_t}$ and $\frac{K_t^H}{K_t^L} = \frac{b_{t+1} k_t^H}{1 - b_{t+1} k_t^L}$ yields

$$\frac{K_t x_t + D_t}{K_t (1 - x_t) - D_t} = \frac{b_{t+1} k_t^H}{1 - b_{t+1} k_t^L}.$$
When \( x_t < \frac{(\alpha - \lambda) b_{t+1}}{\alpha [b_{t+1} + (1 - b_{t+1}) z^{1-\alpha}]} \), the debt constraint is binding and \( D_t = \frac{\lambda x_t}{\alpha z \left( \frac{k_H}{k_L} \right)^{1-\alpha} - \lambda} K_t \).

Substituting this into equation (A.2) yields

\[
\alpha z (1 - x_t) \frac{k_H}{k_L} - \alpha z x_t \frac{1 - b_{t+1}}{b_{t+1}} = \lambda \left( \frac{k_H}{k_L} \right)^\alpha.
\] (A.3)

The value of \( f(x_t, b_{t+1}) \) is determined as the value of \( \frac{k_H}{k_L} \) that solves equation (A.3). The left-hand side of this equation is linear and increasing in \( \frac{k_H}{k_L} \). The right-hand side is increasing at a decreasing rate. Hence, \( f(x_t, b_{t+1}) \) is determined as the unique solution of equation (A.2). The solution is illustrated in Figure 12.

![Figure 12: Determination of \( f(x_t, b_{t+1}) \)](image)

**Critical value of borrower wealth above which credit constraints do not bind for \( 0 < \gamma < 1 \)**

There are two cases to distinguish:
Case 1: The high-TFP firm borrows

When the high-TFP firm is not borrowing constrained, the following conditions are fulfilled:

\[ D_t \leq \frac{\lambda Y_t^H}{R_t} \]  
\[ R_t = MPK_t^L = MPK_t^H \]  
\[ \frac{k_t^H}{k_t^L} = z^{-\frac{1}{1-\alpha}} \]  
\[ K_t^H = x_t K_t + D_t \]

Equation (A.6) follows from (A.5). Substituting (A.5) and (A.7) into (A.4) yields

\[ D_t \leq \frac{\lambda x_t}{\alpha - \lambda} K_t. \]  

Since \( \frac{x_t K_t + D_t}{(1-x_t)K_t - D_t} \) is increasing in \( D_t \),

\[ \frac{x_t K_t + D_t}{(1-x_t)K_t - D_t} \frac{1 - b_{t+1}}{b_{t+1}} \leq \frac{x_t K_t + \frac{\lambda x_t}{\alpha - \lambda} K_t}{(1-x_t)K_t - \frac{\lambda x_t}{\alpha - \lambda} K_t} \frac{1 - b_{t+1}}{b_{t+1}}. \]

Using \( \frac{x_t K_t + D_t}{(1-x_t)K_t - D_t} \frac{1 - b_{t+1}}{b_{t+1}} = \frac{k_t^H}{k_t^L} \) and (A.6), one obtains

\[ x_t \geq \frac{(\alpha - \lambda) b_{t+1}}{\alpha \left[ b_{t+1} + (1 - b_{t+1}) z^{\frac{1}{1-\alpha}} \right]} \]  

(A.9)
Case 2: The low-TFP firm borrows

When the low-TFP firm is not borrowing constrained, the following conditions are fulfilled:

\[ D_t \leq \frac{\lambda Y_t^L}{R_t} \quad (A.10) \]

\[ R_t = MPK_t^H = MPK_t^L \quad (A.11) \]

\[ \frac{k_t^H}{k_t^L} = z^{-\frac{1}{1-\alpha}} \quad (A.12) \]

\[ K_t^L = (1 - x_t)K_t + D_t \quad (A.13) \]

Substituting (A.11) and (A.13) into (A.10) yields

\[ D_t \leq \frac{\lambda(1 - x_t)}{\alpha - \lambda}K_t. \quad (A.14) \]

Since \( \frac{x_tK_t - D_t}{(1 - x_t)K_t + D_t} \) is decreasing in \( D_t \),

\[ \frac{x_tK_t - D_t}{(1 - x_t)K_t + D_t} \cdot \frac{1 - b_{t+1}}{b_{t+1}} \geq \frac{x_tK_t - \frac{\lambda(1 - x_t)}{\alpha - \lambda}K_t}{(1 - x_t)K_t + \frac{\lambda(1 - x_t)}{\alpha - \lambda}K_t} \cdot \frac{1 - b_{t+1}}{b_{t+1}}. \]

Using \( \frac{x_tK_t - D_t}{(1 - x_t)K_t + D_t} \cdot \frac{1 - b_{t+1}}{b_{t+1}} = \frac{k_t^H}{k_t^L} \) and (A.12), one obtains

\[ x_t \leq \frac{\alpha b_{t+1} + \lambda(1 - b_{t+1})z^{\frac{1}{1-\alpha}}}{\alpha \left[ b_{t+1} + (1 - b_{t+1})z^{\frac{1}{1-\alpha}} \right]} \quad (A.15) \]

**B  Threshold functions**

**Thresholds when 0 < \gamma < 1**

The following calculations hold when the productivity state remains the same (\( s_t = s_{t-1} \)).
Worker reallocation and wages

- Workers want to move to the high-TFP sector in period t when $b_t \leq B^{nc}(x_t)$ which is equivalent to

$$x_t \geq \frac{\left(\alpha z^{\frac{1}{\alpha}} - \lambda\right) b_t}{\alpha \left[1 - \left(1 - z^{\frac{1}{\alpha}}\right) b_t\right]} \equiv T_L(b_t).$$

- Wages in the high-TFP sector exceed wages in the low-TFP sector if $b_t + \gamma(1 - b_t) < B^{nc}(x_t)$ which is equivalent to

$$x_t > \frac{\left(\alpha z^{\frac{1}{\alpha}} - \lambda\right) \left(\gamma + (1 - \gamma)b_t\right)}{\alpha \left[1 - \left(1 - z^{\frac{1}{\alpha}}\right) \left(\gamma + (1 - \gamma)b_t\right)\right]} \equiv T_{WH}(b_t).$$

- Wages in the low-TFP sector exceed wages in the high-TFP sector if $(1 - \gamma)b_t > B^{nc}(x_t)$, i.e.

$$x_t < \frac{\left(\alpha z^{\frac{1}{\alpha}} - \lambda\right) (1 - \gamma)b_t}{\alpha \left[1 - \left(1 - z^{\frac{1}{\alpha}}\right) (1 - \gamma)b_t\right]} \equiv T_{WL}(b_t).$$

- All workers earn the same wage if

$$T_{WL}(b_t) \leq x_t \leq T_{WH}(b_t).$$

Borrowing

Using equations (9) and (A.1), one obtains the condition under which high-TFP firms borrow. The conditions under which credit constraints are binding is derived by substitution of (9) into (A.9) and (A.15).

- When $w_t^H > w_t^L$, the high-TFP sector borrows if

$$x_t \leq \frac{\gamma + (1 - \gamma)b_t}{\gamma + (1 - \gamma) \left(b_t + (1 - b_t)z^{\frac{1}{1-\alpha}}\right)} \equiv T_B(b_t).$$
The high-TFP firm is credit constrained if
\[ x_t < \frac{\alpha - \lambda}{\alpha} \frac{\gamma + (1 - \gamma)b_t}{\gamma + (1 - \gamma)(b_t + (1 - b_t)z^{1/\alpha})} \equiv T_{RL}(b_t). \]

When the low-TFP sector borrows, credit constraints bind if
\[ x_t > \frac{\alpha [\gamma + (1 - \gamma)b_t] + \lambda(1 - \gamma)(1 - b_t)z^{1/\alpha}}{\alpha [\gamma + (1 - \gamma)(b_t + (1 - b_t)z^{1/\alpha})]} \equiv T_{RH}(b_t). \]

- When \( w^L_t > w^H_t \), the high-TFP sector borrows if
  \[ x_t \leq \frac{(1 - \gamma)b_t}{z^{1/\alpha} + (1 - \gamma)b_t \left(1 - z^{1/\alpha}\right)} \equiv T_{HB}(b_t). \]

The high-TFP sector is borrowing constrained if
\[ x_t < \frac{\alpha - \lambda}{\alpha} \frac{(1 - \gamma)b_t}{z^{1/\alpha} + (1 - \gamma)b_t \left(1 - z^{1/\alpha}\right)} \equiv T_{HC}(b_t). \]

When the low-TFP firm borrows, the credit constraint binds if
\[ x_t > \frac{\alpha(1 - \gamma)b_t + \lambda(1 - (1 - \gamma)b_t)z^{1/\alpha}}{\alpha \left[z^{1/\alpha} + (1 - \gamma)b_t \left(1 - z^{1/\alpha}\right)\right]} \equiv T_{LC}(b_t). \]

- If \( w^L_t = w^H_t \), always high-TFP firms borrow. To see this, substitute equation (8) into (A.1). This yields
  \[ x_t \leq \frac{\alpha - \lambda}{\alpha} \frac{\alpha z^{1/\alpha} - \lambda}{\alpha - \alpha z^{1/\alpha} z^{1/\alpha}}. \]

Since the right-hand side is larger than 1, the condition that high-TFP firms borrow is always fulfilled if \( w^L_t = w^H_t \). Further, we know from Section 3.1 that credit constraints bind in this case.\(^7\)

\(^7\)It was shown in Section 3.1 that credit constraints bind if both firms produce, \( R_t = MPK^L_t \), and \( w^L_t = w^H_t \).
Location of the threshold curves in \((b_t, x_t)\) space

**Proposition 3.** When \(w_t^L > w_t^H\), high-TFP firms borrow and credit constraints bind.

**Proof.** Simple algebra proves that the \(T_{LC}\) curve and the \(T_{HB}\) curve are located above the \(T_{HC}\) curve. It remains to show that the \(T_{HC}\) curve lies above the \(T_{WL}\) curve for all \(b_t \in (0, 1)\). Since the \(T_{WL}\) curve is convex and the \(T_{HC}\) curve is concave, and since both curves are increasing and start at \(b_t = 0\) and \(x_t = 0\), it suffices to show that the \(T_{HC}\) threshold exceeds the \(T_{WL}\) threshold for \(b_t = 1\):

\[
\frac{\alpha - \lambda}{\alpha} \frac{1 - \gamma}{\gamma + (1 - \gamma) z^{\frac{1}{1-\alpha}}} \geq \frac{\left(\alpha z^{\frac{1}{\alpha}} - \lambda\right) (1 - \gamma)}{\alpha \left[1 - \left(1 - z^{\frac{1}{\alpha}}\right)\right] (1 - \gamma)}
\]

\[
\Leftrightarrow \gamma \left[\alpha - \lambda - z^{\frac{1}{1-\alpha}} \left(\alpha z^{\frac{1}{\alpha}} - \lambda\right)\right] + \lambda (1 - \gamma) \left(1 - z^{\frac{1}{\alpha}}\right) \geq 0
\]

The condition is satisfied for \(0 < \gamma < 1\) and \(\lambda \in [0, \alpha z^{\frac{1}{\alpha}}]\). QED

![Figure 13: Threshold functions for 0 < γ < 1](image)

It remains to show that \(T_{RH}(b_t) > T_B(b_t) > T_{RL}(b_t) > T_{WH}(b_t) > T_L(b_t) > T_{WL}(b_t)\) for all \(b_t \in (0, 1)\). Figure 13 illustrates the threshold functions and includes
their corresponding values at $b_t = 0$ and $b_t = 1$. Since the $T_{RL}(b_t)$ curve is increasing and concave, and the $T_{WH}(b_t)$ curve is increasing and convex, it suffices to compare both threshold functions at $b_t = 0$ and $b_t = 1$. It can be shown that $T_{RL}(0) > T_{WH}(0)$ if

$$\gamma < \frac{\alpha - \lambda - (\alpha z^{\frac{1}{\alpha}} - \lambda)z^{\frac{1}{1-\alpha}}}{\alpha - \lambda - (\alpha z^{\frac{1}{\alpha}} - \lambda)z^{\frac{1}{1-\alpha}} - \lambda(z^{\frac{1}{\alpha}} - 1)}.$$ 

Since the right-hand side is larger than 1, $T_{RL}(0) > T_{WH}(0)$ is always satisfied for $0 < \gamma < 1$. Further, it can be shown that $T_{RL}(1) > T_{WH}(1)$. The relative location of the other threshold functions is obtained by simple algebra.

**Thresholds when $\gamma = 0$**

When $\gamma = 0$, wages are the same in both sectors only if

$$x_t = \frac{(\alpha z^{\frac{1}{\alpha}} - \lambda) b_{t+1}}{\alpha \left[1 - \left(1 - z^{\frac{1}{\alpha}}\right) b_{t+1}\right]} \equiv T_L(b_{t+1}). \quad (B.1)$$

Equation (B.1) is obtained from (8). The high-TFP firm borrows if (A.1) is satisfied:

$$x_t \leq \frac{b_{t+1}}{b_{t+1} + (1 - b_{t+1}) z^{\frac{1}{1-\alpha}}} \equiv T_B(b_{t+1}).$$

Credit constraints do not bind if (A.9) or (A.15) is satisfied:

$$x_t \geq \frac{\alpha - \lambda}{\alpha} \frac{b_{t+1}}{b_{t+1} + (1 - b_{t+1}) z^{\frac{1}{1-\alpha}}} \equiv T_{RL}(b_{t+1}).$$

or

$$x_t \leq \frac{\alpha b_{t+1} + \lambda (1 - b_{t+1}) z^{\frac{1}{1-\alpha}}}{\alpha \left[b_{t+1} + (1 - b_{t+1}) z^{\frac{1}{1-\alpha}}\right]} \equiv T_{RH}(b_{t+1}).$$
C Proofs

Proof of Proposition 1

Step 1. Development of $x$

$s_t = s_{t-1}$:

1. If the high-TFP firm borrows, the transitional dynamics of the wealth share of high-TFP firms is described by

$$x_t = X_0(x_{t-1}, b_t) = \frac{\tilde{R}_{t-1} x_{t-1}}{\tilde{R}_{t-1} x_{t-1} + R_{t-1}(1 - x_{t-1})}.$$  

The wealth share of the high-TFP firm increases if

$$X_0(x_{t-1}, b_t) > x_{t-1},$$

which is equivalent to $\tilde{R}_{t-1} > R_{t-1}$. This condition holds if the high-TFP firm is credit constrained, i.e. if $(b_t, x_t)$ is located below the $T_{RL}$ curve.

2. If the low-TFP firm borrows, the transitional dynamics of the wealth share of high-TFP firms is described by

$$x_t = X_0(x_{t-1}, b_t) = \frac{R_{t-1} x_{t-1}}{R_{t-1} x_{t-1} + R_{t-1}(1 - x_{t-1})}.$$  

The wealth share of the high-TFP firm decreases if

$$X_0(x_{t-1}, b_t) < x_{t-1},$$

which is equivalent to $\tilde{R}_{t-1} > R_{t-1}$. This condition holds if the low-TFP firm is credit constrained, i.e. if $(b_t, x_t)$ is located above the $T_{RH}$ curve.

3. It follows that the wealth distribution does not change if $(b_t, x_t)$ is located in the area between the $T_{RL}$ and the $T_{RH}$ curve.

If $s_t \neq s_{t-1}$, the wealth share of high-TFP firms at the end of period $t-1$ is given by $1 - X_0(x_{t-1}, b_t)$. 

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Step 2. The functions $T_{RL}(b_t)$ and $T_{RH}(b_t)$ are monotonously increasing in $b_t$ and have the same derivative:

$$T'_{RL}(b_t) = T'_{RH}(b_t) = \frac{\alpha - \lambda}{\alpha} \frac{(1 - \gamma)z^{\frac{1}{1+\alpha}}}{(\gamma + (1 - \gamma)(b_t + (1 - b_t)z^{\frac{1}{1+\alpha}}))^2} > 0.$$ 

It follows that $(b_t, x_t)$ stays in the area between the $T_{RL}$ and the $T_{RH}$ curve if the following conditions are true:

Condition 1: $T_{RL}(1) \leq 0.5$

Condition 2: $T_{RH}(0) \geq 0.5$

Using $T_{RL}(1) = \frac{\alpha - \lambda}{\alpha}$, Condition 1 is satisfied if

$$\lambda \geq \frac{\alpha}{2}.$$ 

Using $T_{RH}(0) = \frac{\alpha \gamma + (1 - \gamma)z^{\frac{1}{1+\alpha}}}{\alpha [\gamma + (1 - \gamma)z^{\frac{1}{1+\alpha}}]}$, Condition 2 is satisfied if

$$\alpha \gamma + (2 \lambda - \alpha)(1 - \gamma)z^{\frac{1}{1+\alpha}} \geq 0.$$ 

If Condition 1 is satisfied, $\lambda \geq \frac{\alpha}{2}$. This also ensures that Condition 2 is satisfied.

Hence, $\lambda \geq \frac{\alpha}{2}$ is a sufficient condition that $(b_t, x_t)$ stays in the area between the $T_{RL}$ and the $T_{RH}$ curve in the long run. QED

Proof of Proposition 2

The inspection of Figures 14 and 15 already suggests that two cases have to be considered. For reasons of clarity let $a, e, c$, and $d$ denote the line segments between points $(1 - \mu, T_{RH}(1 - \mu))$ and $(1 - \mu, 1)$, $(\mu, T_{RH}(\mu))$ and $(\mu, 1)$, $(1 - \mu, 0)$ and $(1 - \mu, T_{RL}(1 - \mu))$, as well as $(\mu, 0)$ and $(\mu, T_{RL}(\mu))$, respectively. The length of a line segment is denoted by the symbol $\|\cdot\|$. Since the $T_{RH}$ and $T_{RL}$ threshold functions have the same derivatives, and since they are increasing and concave, $\|a\| < \|d\|$ and $\|e\| < \|c\|$.
Figure 14: Case 1: $T_{RL}(\mu) \leq 1 - T_{RL}(1 - \mu)$

Figure 15: Case 2: $T_{RL}(\mu) > 1 - T_{RL}(1 - \mu)$
Case 1. $T_{RL}(\mu) \leq 1 - T_{RL}(1 - \mu)$

1. Suppose $(x_t, b_{t+1})$ is located on $a$ in Figure 14. As long as the productivity state does not change, the economy converges to $T_{RH}(1 - \mu)$. When the productivity state changes, the new value of $(x_t, b_{t+1})$ is located on $d$. Since $\|a\| < \|d\|$, the economy eventually converges to $T_{RL}(\mu)$ and then fluctuates between two states $x_1 = T_{RL}(\mu)$ and $x_2 = 1 - T_{RL}(\mu)$.

2. Suppose $(x_t, b_{t+1})$ is located on $e$. As long as the productivity state does not change, the economy converges to $T_{RH}(\mu)$. When the productivity state changes, the new value of $(x_t, b_{t+1})$ is located on $c$. Since $\|e\| < \|c\|$, the economy eventually converges to $T_{RL}(1 - \mu)$ and then fluctuates between two states $x_1 = T_{RL}(1 - \mu)$ and $x_2 = 1 - T_{RL}(1 - \mu)$.

Case 2. $T_{RL}(\mu) > 1 - T_{RL}(1 - \mu)$

1. Suppose $(x_t, b_{t+1})$ is located on $a$. As long as the productivity state does not change, the economy converges to $T_{RH}(1 - \mu)$. When the productivity state changes, the new values of $(x_t, b_{t+1})$ are located on $d$. Since $\|a\| < \|d\|$, the economy eventually converges to $T_{RL}(\mu)$. When the productivity state changes then, the new value of $(x_t, b_{t+1})$ is located on $c$ and $x_t = 1 - T_{RL}(\mu)$. The economy converges to $T_{RL}(1 - \mu)$ as long as the productivity state does not change. When the productivity state changes then, the new value of $(x_t, b_{t+1})$ is located on $d$ and $x_t = 1 - T_{RL}(1 - \mu)$. The economy converges to a cycle with $x \in [1 - T_{RL}(\mu), T_{RL}(1 - \mu)] \cup [1 - T_{RL}(1 - \mu), T_{RL}(\mu)]$.

2. Suppose $(x_t, b_{t+1})$ is located on $e$. As long as the productivity state does not change, the economy converges to $T_{RH}(\mu)$. When the productivity state changes, the new value of $(x_t, b_{t+1})$ is located on $c$. Since $\|e\| < \|c\|$, the economy again converges to a cycle with $x \in [1 - T_{RL}(\mu), T_{RL}(1 - \mu)] \cup [1 - T_{RL}(1 - \mu), T_{RL}(\mu)]$.

Hence, if and only if the condition $T_{RL}(\mu) \leq 1 - T_{RL}(1 - \mu)$ is satisfied, the economy converges to a cycle with two states $x_2 = 1 - x_1 \in [T_{RL}(1 - \mu), 1 - T_{RL}(1 - \mu)]$. This is equivalent to $\lambda \geq \frac{\alpha(1 - \mu)(1 - \frac{1}{1-\alpha})}{2\mu(1 - \frac{1}{1-\alpha}) - 2\mu^2(1 - \frac{1}{1-\alpha}) + \frac{1}{1-\alpha}}$. QED

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