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ESTIMATING STABLE FACTOR MODELS BY INDIRECT INFERENCE

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Abstract

Financial returns exhibit common behavior described at best by factor models, but also fat tails, which may be captured by α -stable distributions. This paper concentrates on estimating factor models with multivariate α -stable distributed and independent factors and idiosyncratic noises under the assumption of time constant distribution (static factor models) or time-varying conditional distribution (GARCH factor models). While the simulation from such a distribution is straightforward, the estimation encounters difficulties. These difficulties are overcome in this paper by implementing the indirect inference estimation method with the multivariate Student's t as the auxiliary distribution.

Keywords: Symmetric Multivariate α -stable Distribution, Factor Models, Indirect Inference, Multivariate Student's t Distribution, Discrete Spectral Measures, GARCH Models

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1 Introduction

Many economic time series, such as financial returns or macroeconomic variables exhibit common behavior, which is captured by means of factor models. These models have the advantage of providing a parsimonious framework of describing large panels of data with only a few parameters: it is assumed that a vector of series of dimension $n \times 1$ is driven by only a few ($k \ll n$) common factors and the vector of idiosyncratic terms specific to the vector components. These models are widely used in applied macroeconomics to provide forecasts of economic variables based on a large volume of data stemming from surveys of households and businesses, as well as from various macroeconomic series that describe the economic activity. The factor models have found so far also a wide application in finance, such as in the asset pricing theory or in the portfolio theory and risk management when applied to parsimoniously estimate and forecast the (conditional) variance-covariance matrices of a large number of financial assets.

It is a very well known fact that many economic variables, especially financial returns, are far from being normally distributed and exhibit, in general, heavy-tailedness. A common approach of dealing with this issue is to assume a fat-tailed distribution for the underlying series, such as the Student's t distribution. Although very applied in practice, the Student's t distribution lacks in stability under aggregation, which is particularly relevant in portfolio applications and risk management. A valuable alternative to the Student's t distribution is the α -stable distribution, which is a generalization of the normal and accounts for fat-tailedness. The main drawback of this distribution is its difficulty to be implemented in practice: it has, in general, no closed-form specification of the density (with a few exceptions for certain values of the parameters) and the theoretical moments larger than one (with a few exceptions on the choice of the parameters) do not exist.

In this paper we provide a feasible way of estimating factor models with jointly α -stable distributed factors and idiosyncratic noises by means of the Indirect Inference (Ind-Inf) method proposed by Gouriéroux et al. (1993), Smith (1993) and Gallant and Tauchen (1996). This estimation approach is particularly attractive, as one can easily simulate pseudo-random numbers from a multivariate stable distribution with independent components. We consider as auxiliary distribution the multivariate Student's t , which describes fat-tailedness by means of degrees of freedom that are counterparts of the parameter of stability or tail index in the stable distribution.

In particular, we focus on two types of factor models where the dependency between the observed variables, the factors and the noises is contemporaneous: a static factor model with an underlying distribution, which is constant in time and a GARCH factor model with conditionally varying factors and noises. The choice of the GARCH models is motivated by their wide popularity among practitioners and academics: they are able to straightforwardly capture empirical features of financial data, such as clustering and fat-thickness.

Thus, the contribution of the paper is twofold: first, we develop on factor models, which are able to capture dynamics of large panels of data with heavy-tailed distributions by means of multivariate stable distributions; and second, we alleviate their estimation problems by adopting the IndInf procedure, which is straightforward to implement when

choosing an adequate auxiliary model.

The application of the stable distribution in economics dates back to the early work of Benoit Mandelbrot (e.g., Mandelbrot (1963*b*), Mandelbrot (1963*a*)) and Eugene Fama (Fama (1965)), who rejected the normal distributional assumption of common financial assets and proposed the stable distribution as an alternative to capture the excess kurtosis. This distribution seems to be particularly attractive for finance applications as (1) it has domains of attraction, i.e., it is robust to misspecifications in the model distribution, (2) it is closed under linear transformations, i.e., linear combinations of stable distributed variables remain stable and (3) it is the limiting distribution in the generalized version of the central limit theorem (Gnedenko and Kolmogorov (1954)), where the condition of finite variance is replaced by a much less restricting one regarding a regular behavior of the tails (see Rachev and Mittnik (2000), Doganoglu et al. (2007), Calzolari et al. (2014) among others).

Although theoretically very attractive, the stable distribution has found so far little application in finance modeling, not just because of its estimation difficulties, but also because it usually lacks in moments larger than one, and, thus, can not be easily incorporated in the classical finance theory, which assumes that financial return distributions are described by the mean and the variance: e.g., the Markowitz model for portfolio optimization, option valuation, the Value at Risk (VaR) measure based on the location-scale model or asset pricing models. However, as described at length in Rachev and Mittnik (2000), most of the finance applications are already adapted to account for the fat-thickness in the underlying data by means of stable distributions.

The factor models treated in this paper are particularly relevant in finance applications, as they provide a parsimonious framework of dealing with large dimensional problems typical to investments in many financial assets. More precisely, the stable static factor model can be applied to the capital asset pricing model for stable returns as introduced by Fama (1970) and extended by Ross (1978) or to the multi-factor pricing model, called Arbitrage Pricing Theory developed by Ross (1976). The GARCH stable factor model is particularly relevant in risk management and portfolio management, as it accounts for the conditional distribution of asset returns, which typically exhibit temporal dependence: Bawa et al. (1979) and Rachev and Han (2000), among others apply the stable distribution for portfolio selection and Rachev and Mittnik (2000) and Gamrowski and Rachev (1996) use the stable distribution for VaR modeling.

The underlying distribution of our models is the multivariate symmetric stable distribution, characterized by a tail index α , a location vector and a spectral measure. For our purposes, we assume that the factors and the idiosyncratic noises are independent of each other and, therefore, the multivariate stable distribution is characterized by a discrete spectral measure, which is easy to handle. The parameters of such a distribution are so far estimated, among others, by Rachev and Xin (1993) and Cheng and Rachev (1995) based on a random sample from the domain of attraction of the original multivariate stable random variable and by Nolan and Panorska (1997), Nolan et al. (2001) and Ogata (2013) based on the empirical characteristic function. In the context of factor models, Tsionas (2013) applies the MCMC procedure to estimate the parameters of a stable static factor model and Doganoglu et al. (2007) apply a polynomial approximation of

the log-likelihood in order to estimate the parameters of a stable GARCH factor model by means of maximum likelihood. The conditioning information set in the approach of Doganoglu et al. (2007) is, however, not completely available to the econometrician, as it includes only past observations of the variable of interest, and no (past) observations on the factors. A similar problem face Sentana et al. (2008) and Harvey et al. (1992), when estimating GARCH factor models with normally distributed factors and idiosyncratic noises. To circumvent this problem, Harvey et al. (1992) use to approximate the unobserved factors by corrections of their conditional expectations given the observables and Sentana et al. (2008) applies the IndInf procedure with the approach of Harvey et al. (1992) as the auxiliary specification.

The application of the IndInf to estimate stable distributions has regarded so far only univariate cases: Lombardi and Calzolari (2008), Lombardi and Veredas (2009) and Garcia et al. (2011) apply it to estimate the parameters of the α -stable distribution, while Lombardi and Calzolari (2009) use it to estimate a stochastic volatility model and Calzolari et al. (2014) to estimate a GARCH model with α -stable innovations. However, the remarkably good estimation results provided by the IndInf in these cases motivate its choice in the multivariate setting described in this paper.

Because the aim of the paper is to show how the IndIn method can feasibly and accurately estimate the parameters of factor models with stable distributed underlying series, we mainly focus on discussing the estimation results from applying the method to simulated and real data. Thus, a direct application of these factor models to finance modeling, such as, for example, the static factor model to the asset pricing theory or the GARCH factor model to the portfolio allocation, does not make the scope of this paper and is left for further research.

Within a thorough Monte Carlo experiment we provide empirical evidence that the IndInf estimation procedure produces very good results in terms of biasedness and efficiency for a wide range of parameter choices. Moreover, within an empirical application to thirty time series of financial returns of components of Dow Jones Industrial Average index, we provide further empirical evidence on the performance of the IndInf method on estimating stable distributions and on the appropriateness of the models to capture empirical features of real data, such as clustering and fat-tailedness.

The rest of the paper is organized as follows: Section 2 provides a short theoretical introduction to multivariate stable distributions, Section 3 introduces the model of interest, namely the static stable factor model in Section 3.1 and the GARCH stable factor model in Section 3.2. Section 4 describes shortly the IndInf estimation method and its practical implementation for estimating the models of interest. Section 5 presents the results of the Monte Carlo experiments, while Section 6 shows empirical results from real data. Section 7 concludes.

2 Multivariate Stable Distribution

The multivariate stable distribution, as its univariate counterpart, is the generalization of the Gaussian distribution that allows for asymmetry and fat-tails. The random vector

$\mathbf{X} = (X_1, X_2, \dots, X_n)$ is said to be a stable vector in $\mathbb{R}^n$ if, for any positive number A and B , there is a positive number C and a vector $\mathbf{D} \in \mathbb{R}^n$ such that

$$A\mathbf{X}^{(1)} + B\mathbf{X}^{(2)} \stackrel{d}{=} C\mathbf{X} + \mathbf{D}, \quad (1)$$

where $\mathbf{X}^{(1)}$ and $\mathbf{X}^{(2)}$ are independent and have the same distribution as $\mathbf{X}$ and $\stackrel{d}{=}$ stands for equality in distribution. One can also say that " $X_1, X_2, \dots, X_n$ " are jointly stable or that " $\mathbf{X}$ has a multivariate stable distribution".

Similar to their univariate counterparts, the multivariate stable distributions are most commonly described by their characteristic functions, as they lack general closed-form expressions for both the density and distribution functions. Thus, the joint characteristic function of the vector $\mathbf{X}$ is given by:

$$\Phi(\theta) = E[\exp\{i\langle\theta, \mathbf{X}\rangle\}] = \exp\left\{-\int_{S_n} \Psi(\langle\theta, \mathbf{s}\rangle)\Gamma(ds) + i\langle\theta, \boldsymbol{\mu}\rangle\right\}, \quad (2)$$

where $S_n = \{\mathbf{s} : \|\mathbf{s}\| = 1\}$, with $\|\cdot\|$ being the Euclidean norm, is the unit sphere in $\mathbb{R}^n$, which is a $(n-1)$ -dimensional surface¹, the symbol $\langle\cdot, \cdot\rangle$ denotes the inner product, Γ is a finite measure on the unit sphere S_n called spectral measure, $\boldsymbol{\mu} = (\mu_1, \dots, \mu_n) \in \mathbb{R}^n$ is the location parameter vector and

$$\Psi(u) = \begin{cases} |u|^\alpha \left(1 - i \operatorname{sign}(u) \tan \frac{\pi\alpha}{2}\right) & \text{if } \alpha \neq 1 \\ |u|^\alpha \left(1 + i \frac{2}{\pi} \operatorname{sign}(u) \ln |u|\right) & \text{if } \alpha = 1, \end{cases} \quad (3)$$

where $\alpha \in (0, 2]$ denotes the characteristic component or the shape parameter of the distribution.

$\mathbf{X}$ is said to be strictly stable, if in Equation (1) $\mathbf{D} = 0$ for any $A > 0$ and $B > 0$ or if in Equation (3) $\boldsymbol{\mu}$ is equal to a vector of zeros for $\alpha \neq 1$ and $\int_{S_n} \mathbf{s}_i \Gamma(ds) = 0$ for $\alpha = 1$ and $i = 1, \dots, n$.

The random vector $\mathbf{X}$ is said to be symmetric stable if it is stable and if the probabilities $P(\mathbf{X} \in F) = P(-\mathbf{X} \in F)$ for any Borel set $F \in \mathbb{R}^n$. The symmetric multivariate stable distribution is characterized by a symmetric spectral measure Γ on S_n : i.e., $\Gamma(F) = \Gamma(-F)$ (Samorodnitsky and Taqqu (1994)). The characteristic function of a multivariate symmetric stable random vector is given by:

$$\Phi(\theta) = \exp\left\{-\int_{S_n} |\langle\theta, \mathbf{s}\rangle|^\alpha \Gamma(ds)\right\}$$

If the random vector $\mathbf{X} = (X_1, X_2, \dots, X_n)$ is multivariate stable distributed, then any linear combination of the components of $\mathbf{X}$ of the type $Y = \sum_{i=1}^n a_i X_i$ has an α -stable distribution $S_\alpha(\sigma_{\mathbf{a}}, \beta_{\mathbf{a}}, \mu_{\mathbf{a}})$, where $\sigma_{\mathbf{a}}$, $\beta_{\mathbf{a}}$ and $\mu_{\mathbf{a}}$ represent scale, skewness and location parameters of the univariate stable distribution and are given in equations (2.3.3), (2.3.4) and (2.3.5) in Samorodnitsky and Taqqu (1994) for $\mathbf{a} = (a_1, \dots, a_n)$. If $\mathbf{X}$ is strictly

¹For example, S_1 is the point set $\{-1, 1\}$ and S_2 is the unit circle.

stable, then Y is also strictly stable ($\mu_a = 0$) and, if $\mathbf{X}$ is symmetric stable, then Y is also symmetric stable ($\beta_a = 0$).

Of particular interest for our purposes is the multivariate stable distribution with independent components: $X_1, \dots, X_n$. In this case the spectral measure Γ is discrete and concentrated on a finite number of mass points on the unit sphere S_n : i.e., on the intersection points of the Cartesian coordinates with the sphere S_n (Samorodnitsky and Taqqu (1994), Property 2.3.7). The spectral measure Γ takes the following form:

$$\Gamma = \sum_{i=1}^{2n} \lambda_i f(\mathbf{s}_i) \quad (4)$$

where the $\mathbf{s}_i$'s are the $2n$ points of intersection of the unit sphere with the Cartesian coordinates, $f(\mathbf{s}_i)$ is the Dirac-function that assigns unit mass to the point $\mathbf{s}_i$ and $\lambda_i > 0$ with $i = 1, \dots, 2n$ are weights.

Due to the simple form of the spectral measure, this type of multivariate stable distribution is easy to handle in empirical applications, particularly when it comes to simulations. Pseudo-random stable vectors with the discrete spectral measure Γ as given in Equation (4) and stability parameter α can be obtained as follows (Modarres and Nolan (1994), Samorodnitsky and Taqqu (1994), Kozubowski et al. (2003) and Nolan (2003)):

$$Z^* = \begin{cases} \sum_{i=1}^{2n} \lambda_i^{1/\alpha} V_i \mathbf{s}_i & \text{if } \alpha \neq 1 \\ \sum_{i=1}^{2n} \lambda_i \left(V_i + \frac{2}{\pi} \log \lambda_i \right) \mathbf{s}_i & \text{if } \alpha = 1, \end{cases} \quad (5)$$

where V_i 's are i.i.d totally skewed, one-dimensional standard stable variables $S_\alpha(1, 1, 0)$. Univariate pseudo-random stable numbers are simulated based on the algorithm developed by Chambers et al. (1976). Random vectors with a location parameter μ different from zero can be obtained as $Z = Z^* + \mu$.

Example 2.1 *One can simulate bivariate stable distributed random numbers with independent components, location parameter equal to zero and $\alpha \neq 1$ as follows:*

$$Z^* = \lambda_1^{1/\alpha} V_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \lambda_2^{1/\alpha} V_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \lambda_3^{1/\alpha} V_3 \begin{pmatrix} -1 \\ 0 \end{pmatrix} + \lambda_4^{1/\alpha} V_4 \begin{pmatrix} 0 \\ -1 \end{pmatrix} \quad (6)$$

The marginal distributions are also α -stable with location parameters equal to zeros: $S_\alpha(\sigma_1, \beta_1, 0)$ and $S_\alpha(\sigma_2, \beta_2, 0)$, respectively. Samorodnitsky and Taqqu (1994) derive the relationship between the weights $\lambda_1, \lambda_2, \lambda_3$ and λ_4 and the scale and skewness parameters σ_1, σ_2 and, respectively, β_1, β_2 as follows: $\lambda_1 = \sigma_1^\alpha \frac{1+\beta_1}{2}$, $\lambda_2 = \sigma_2^\alpha \frac{1+\beta_2}{2}$, $\lambda_3 = \sigma_1^\alpha \frac{1-\beta_1}{2}$ and $\lambda_4 = \sigma_2^\alpha \frac{1-\beta_2}{2}$. To simulate symmetric bivariate random numbers, one should set in Equation (6) $\lambda_1 = \lambda_3$ and $\lambda_2 = \lambda_4$, which result in $\beta_1 = \beta_2 = 0$.

Similar to the univariate case, the estimation of the parameters of the multivariate stable distribution becomes difficult within standard estimation procedures, due to the lack of closed-form expression for the density and due to the non-existence of moments

higher than α .² The estimation of the parameters of the multivariate stable distribution with discrete spectral measure is so far implemented by Rachev and Xin (1993) and Cheng and Rachev (1995) based on a random sample from the domain of attraction of the original multivariate stable random variable and by Nolan and Panorska (1997), Nolan et al. (2001) and Ogata (2013) based on the empirical characteristic function. However, its easiness in simulating pseudo-random stable vectors with discrete spectral measures makes the IndInf estimation method of Gouriéroux et al. (1993), Smith (1993) and Gallant and Tauchen (1996) a valuable alternative as shown by the empirical results in sections 5 and 6.

3 Stable Factor Models

In this section we introduce two factor models with factors and idiosyncratic error terms that are independent and jointly symmetric multivariate stable distributed. In Section 3.1 there is no time dependency in the factors and in the noises, and therefore it deals with a static factor model and in Section 3.2 the joint distribution of the factors and the idiosyncratic noises is characterized by a time-varying spectral measure whose components are modeled by means of GARCH processes.

3.1 Static Stable Factor Models

Let

$$\mathbf{Y}_t = \mathbf{B}\mathbf{f}_t + \mathbf{u}_t \quad (7)$$

where $\mathbf{Y}_t$ is a $n \times 1$ vector of observable random variables, $\mathbf{f}_t$ is a $k \times 1$ vector of unobserved common factors, with $k \leq n$, $\mathbf{B}$ is the $n \times k$ matrix of factor loadings of $\text{rank}(\mathbf{B}) = k$ and $\mathbf{u}_t$ is a $n \times 1$ vector of idiosyncratic noises. $\mathbf{u}_t$ and $\mathbf{f}_t$ are independent of each other, as well as f_{jt} and f_{lt} for all $j \neq l = 1, \dots, k$ and u_{it} and u_{st} for all $i \neq s = 1, \dots, n$. Moreover, we assume that $\mathbf{u}_t$ and $\mathbf{f}_t$ are jointly symmetric α -stable distributed as follows:

$$\begin{pmatrix} \mathbf{f}_t \\ \mathbf{u}_t \end{pmatrix} \sim S_\alpha(\boldsymbol{\mu}, \boldsymbol{\Gamma}) \quad (8)$$

where α is the stability parameter, with $0 < \alpha \leq 2$ and $\boldsymbol{\Gamma}$ is a discrete and symmetric spectral measure on the unit sphere. To reduce the dimensionality of the problem, we let the location parameter vector in Equation (8) to be equal to the vector of zeros, i.e., $\boldsymbol{\mu} = \mathbf{0}_{(n+k) \times 1}$.³ We denote the model given in equations (7) and (8) to be a Static Stable Factor (StatStabF) model.

According to Theorem 2.1.2. in Samorodnitsky and Taqqu (1994), for each $j = 1, \dots, k$, f_{jt} is symmetric stable distributed: $S_\alpha(\delta_j, 0, 0)$ and for each $i = 1, \dots, n$, u_{it} is

²Samorodnitsky and Taqqu (1994) show that the moments of the marginal stable distributions and the cross moments are finite only up to order $p < \alpha$, for $0 < \alpha < 2$.

³In practical applications, this assumption is not restrictive, as one can consider for $\mathbf{Y}_t$ series of demeaned vectors or simply estimate the vector $\boldsymbol{\mu}$ along with the other parameters.

also symmetric stable distributed: $S_\alpha(\sigma_i, 0, 0)$, where δ_j and σ_i are scale parameters that are related to the spectral measure Γ as follows:

$$\delta_j = \left(\int_{S_{(k+n)}} |(\mathbf{a}_i, \mathbf{s})|^\alpha \Gamma(ds) \right)^{1/\alpha} \quad (9)$$

$$\sigma_i = \left(\int_{S_{(k+n)}} |(\mathbf{a}_j, \mathbf{s})|^\alpha \Gamma(ds) \right)^{1/\alpha} \quad (10)$$

where $\mathbf{a}_i$ ($\mathbf{a}_j$) is a vector of dimension $(n+k) \times 1$ with the i -th (j -th) element equal to 1 and the rest equal to zero.

Denote by $\mathbf{X}_t = \begin{pmatrix} \mathbf{f}_t \\ \mathbf{u}_t \end{pmatrix}$ and define the matrix $\mathbf{A} = (\mathbf{B}, \mathbf{I}_n)$ with $\mathbf{I}_n$ be the identity matrix of dimension $n \times n$. The matrix $\mathbf{A}$ is of dimension $n \times (n+k)$. Then, one can write Equation (7) as follows:

$$\mathbf{Y}_t = \mathbf{A}\mathbf{X}_t. \quad (11)$$

Proposition 3.1 *The variable $\mathbf{Y}_t$ given in Equation (7) is multivariate symmetric α -stable distributed with the location parameter vector equal to the vector of zeros and the spectral measure given by:*

$$\Gamma^Y = \frac{1}{2} \sum_{l=1}^{n+k} \omega_l^\alpha \|a_{\cdot l}\| [f(\iota_l) + f(-\iota_l)], \quad (12)$$

where ω_l with $l = 1, \dots, n+k$ are the diagonal elements of the matrix:

$$\Omega = \begin{pmatrix} \Delta & \mathbf{0}_{k \times n} \\ \mathbf{0}_{n \times k} & \Sigma \end{pmatrix}$$

of dimension $(n+k) \times (n+k)$ with Δ and Σ being diagonal matrices of dimension $k \times k$ and, respectively, $n \times n$, whose elements are the scale parameters of the factors and the idiosyncratic noises, i.e.,

$$\Delta = \begin{pmatrix} \delta_1 & 0 & \dots & 0 \\ 0 & \delta_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \delta_k \end{pmatrix} \quad \Sigma = \begin{pmatrix} \sigma_1 & 0 & \dots & 0 \\ 0 & \sigma_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_n \end{pmatrix}$$

and $\|a_{\cdot l}\| = (\sum_{i=1}^n a_{il}^2)^{\alpha/2}$ and a_{il} be the il -th element of the matrix $\mathbf{A}$, $\iota_l = \frac{a_{\cdot l}}{\|a_{\cdot l}\|}$, where $a_{\cdot l}$ is the l -th column of the matrix $\mathbf{A}$ and $f(\iota_l)$ is the Dirac-function, which assigns an unit mass to the point with coordinates given by ι_l .

The proof of Proposition 3.1 can be derived from the fact that, since $\mathbf{X}_t$ is an $(n+k) \times 1$ vector of independent standard stable random variables, then its spectral measure is discrete and concentrated on $2(n+k)$ points on the unit sphere S_{n+k} . Moreover, according to the Example 2.3.6 in Samorodnitsky and Taqqu (1994) and to the fact the $\mathbf{X}_t$ is symmetric

stable distributed, the vector $\mathbf{Y}_t$, which is a linear combination of the elements of $\mathbf{X}_t$ is also symmetric stable distributed with the spectral measure given in Equation (12).

To eliminate the scale indeterminacy of the common factors, some parameter constraints have to be imposed. One can impose that $\delta_j = 1$ or, alternatively, that the jj -th elements of $\mathbf{B}$, namely b_{jj} , are equal to 1 for $j = 1, \dots, k$. For our purposes, we impose that the scale parameters of the factors are equal to 1: i.e., $\delta_j = 1$. As a result, the vector of parameters to estimate is given by: $\boldsymbol{\vartheta} = (\alpha, \mathbf{b}', \boldsymbol{\sigma}')'$ with $\mathbf{b} = \text{vec}(\mathbf{B}') = (\mathbf{b}_1', \dots, \mathbf{b}_n')'$, $\mathbf{b}_i = (b_{i1}, \dots, b_{ik})'$ with $i = 1, \dots, n$, and $\boldsymbol{\sigma} = (\sigma_1, \dots, \sigma_n)'$. Furthermore, for identification reasons, we also impose the commonly adopted zero upper-triangular parametrization of $\mathbf{B}$ to define identifiable models when the number of factors is larger or equal to two: $b_{12} = \dots = b_{1k} = b_{23} = \dots = b_{k-1k} = 0$, for all $k \geq 2$. Thus, the total number of parameters to estimate is equal to $nk - \frac{k(k-1)}{2} + n + 1$.

To the best of our knowledge, the research on static factor models with stable distributed factors and idiosyncratic error terms is still in its infancy. Tsionas (2013) introduces different procedures to undergo numerical statistical inference in the family of univariate and multivariate stable distributions with direct application to static and dynamic factor models, among others. Particularly, to estimate the parameters of the static factor model, Tsionas (2013) applies the MCMC procedure. In our paper, we take advantage of the fact that one can easily simulate from the StatStabF model regardless of the dimension of the process and implement the Indirect Inference method with the multivariate Student's t as the auxiliary distribution, as described in Section 4.

3.2 GARCH Stable Factor Models

Let

$$\mathbf{Y}_t = \mathbf{B}\mathbf{f}_t + \mathbf{u}_t \quad (13)$$

where $\mathbf{Y}_t$ is a $n \times 1$ vector of observable random variables, $\mathbf{f}_t$ is a $k \times 1$ vector of unobserved common factors, with $k \leq n$, $\mathbf{B}$ is the $n \times k$ matrix of factor loadings of $\text{rank}(\mathbf{B}) = k$ and $\mathbf{u}_t$ is a $n \times 1$ vector of idiosyncratic noises. $\mathbf{u}_t$ and $\mathbf{f}_t$ are independent of each other, as well as f_{jt} and f_{lt} for all $j \neq l = 1, \dots, k$ and u_{it} and u_{st} for all $i \neq s = 1, \dots, n$.

Moreover, we assume that $\mathbf{f}_t$ and $\mathbf{u}_t$ are conditionally on the past information set jointly symmetric α -stable distributed: let $\mathbf{X}_t = (\mathbf{f}_t', \mathbf{u}_t')'$; then $\mathbf{X}_t$ is distributed as follows

$$\mathbf{X}_t | \mathcal{F}_{t-1} \sim S_\alpha(\mathbf{0}_{(n+k) \times 1}, \boldsymbol{\Gamma}_t), \quad (14)$$

where the discrete symmetric spectral measure $\boldsymbol{\Gamma}$ defined in Equation (8) becomes here time-varying and $\mathcal{F}_t$ is an information set that contains the values of $\mathbf{Y}_t$ and $\mathbf{f}_t$ up to time t .⁴ As a consequence, each factor and each idiosyncratic noise is conditionally on $\mathcal{F}_{t-1}$

⁴Without loss of generality and similar to the static case, we also assume here a location vector equal to zero. Such an assumption is also imposed by Sentana et al. (2008) in the context of conditionally normally distributed factors and idiosyncratic noises, who apply the GARCH factor model on a vector of demeaned series of observed variables. In our empirical exercise, we follow the same idea, and apply our models to demeaned series of daily log-returns of financial stocks. Alternatively, one could estimate the location parameter vector additionally to the other parameters, however, at higher computational costs.

univariate stable distributed with the tail index α and a time varying scale parameter. In order to account for the time variation in the scale parameters, we define the j -th factor in the vector $\mathbf{f}_t$ to have the following representation:

$$f_{jt} = z_{jt} \sqrt{\delta_{jt}} \quad (15)$$

$$\delta_{jt} = (1 - \phi_j - \rho_j) \delta_j + \phi_j f_{jt-1}^2 + \rho_j \delta_{jt-1}, \quad (16)$$

where $\delta_j > 0$, $\phi_j \geq 0$, $\rho_j \geq 0$, δ_j is the unconditional scale of f_{jt} and z_{jt} is i.i.d standard symmetric stable distributed with α tail index for $j = 1, \dots, k$ and the i -th noise in the vector $\mathbf{u}_t$ to follow:

$$u_{it} = \epsilon_{it} \sqrt{\sigma_{it}} \quad (17)$$

$$\sigma_{it} = (1 - \varphi_i - \varrho_i) \sigma_i + \varphi_i u_{it-1}^2 + \varrho_i \sigma_{it-1}, \quad (18)$$

where $\sigma_i > 0$, $\varphi_i \geq 0$, $\varrho_i \geq 0$, σ_i is the unconditional scale of u_{it} and ϵ_{it} is i.i.d standard symmetric stable distributed with α tail index for $i = 1, \dots, n$. We denote the model given in equations (13), (15), (16), (17) and (18) to be a GARCH Stable Factor (GARCHStabF) model.

The GARCH(1,1) processes in equations (16) and (18) admit strictly stationary solutions if and only if the top-Lyapunov conditions $E \log(\phi_j z_{jt}^2 + \rho_j) < 0$ and $E \log(\varphi_i \epsilon_{it}^2 + \varrho_i) < 0$ are satisfied for all $j = 1, \dots, k$ and $i = 1, \dots, n$ (Nelson (1990), Mittnik et al. (2002)).

The discrete spectral measure Γ_t from Equation (14) has $2(k + n)$ mass points at the intersection of the unit sphere S_{n+k} with the axes in each t and can be written as a function of the time varying scale parameters of the factors given in Equation (16) and of the idiosyncratic errors given in Equation (18) as follows (Samorodnitsky and Taqqu (1994)):

$$\Gamma_t = \frac{1}{2} \sum_{l=1}^{n+k} \omega_{lt}^\alpha [f_t(\mathbf{s}_l) + f_t(-\mathbf{s}_l)], \quad (19)$$

where $f_t(\mathbf{s}_l)$ assigns unit mass to the point $\mathbf{s}_l$, which is the intersection of the unit sphere S_{n+k} with the axes at time t and ω_{lt} are the diagonal elements of the matrix:

$$\Omega_t = \begin{pmatrix} \Delta_t & \mathbf{0}_{k \times n} \\ \mathbf{0}_{n \times k} & \Sigma_t \end{pmatrix}$$

of dimension $(n + k) \times (n + k)$ with Δ_t and Σ_t being diagonal matrices of dimension $k \times k$ and, respectively, $n \times n$: i.e.,

$$\Delta_t = \begin{pmatrix} \delta_{1t} & 0 & \dots & 0 \\ 0 & \delta_{2t} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \delta_{kt} \end{pmatrix} \quad \Sigma_t = \begin{pmatrix} \sigma_{1t} & 0 & \dots & 0 \\ 0 & \sigma_{2t} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_{nt} \end{pmatrix}.$$

We derive below the distribution of $\mathbf{Y}_t$ conditionally on $\mathcal{F}_{t-1}$.

Proposition 3.2 *The variable $\mathbf{Y}_t$ given in Equation (13) is conditionally on $\mathcal{F}_{t-1}$ multivariate symmetric α -stable distributed with the location parameter vector equal to the vector of zeros and the spectral measure given by:*

$$\Gamma_t^Y = \frac{1}{2} \sum_{l=1}^{n+k} \omega_{lt}^\alpha \|a_{\cdot l}\| [f_t(\iota_l) + f_t(-\iota_l)], \quad (20)$$

where ω_{lt} are defined above and $\|a_{\cdot l}\| = (\sum_{i=1}^n a_{il}^2)^{\alpha/2}$, with $l = 1, \dots, n+k$ and a_{il} be the il -th element of the matrix $\mathbf{A}$ defined in Section 3.1, $\iota_l = \frac{a_{\cdot l}}{\|a_{\cdot l}\|}$, where $a_{\cdot l}$ is the l -th column of the matrix $\mathbf{A}$ and $f_t(\iota_l)$ is the Dirac-function which assigns an unit mass to the point with coordinates given by ι_l at each time t .

The proof of Proposition 3.2 can be derived similarly to the proof of Proposition 3.1.

As in the StatStabF model, to eliminate the scale indeterminacy of the common factors, one can impose that $\delta_j = 1$ or, alternatively, that jj -th elements of $\mathbf{B}$, namely b_{jj} , are equal to 1 for $j = 1, \dots, k$. For our purposes, we impose that the scale parameters of the factors are equal to 1: i.e., $\delta_j = 1$ for all $j = 1, \dots, k$. Moreover, without loss of generality, but for computational reasons, we assume that the conditional scale of each idiosyncratic noise has the same GARCH parameters: $\varphi_i = \varphi$ and $\rho_i = \rho \forall i$. As a result, the vector of parameters to estimate is given by: $\boldsymbol{\vartheta} = (\alpha, \mathbf{b}', \boldsymbol{\sigma}', \boldsymbol{\theta}', \boldsymbol{\eta}')'$ with $\mathbf{b} = \text{vec}(\mathbf{B}') = (\mathbf{b}_1', \dots, \mathbf{b}_n')'$, $\mathbf{b}_i = (b_{i1}, \dots, b_{ik})'$ with $i = 1, \dots, n$, $\boldsymbol{\sigma} = (\sigma_1, \dots, \sigma_n)'$, $\boldsymbol{\theta} = (\boldsymbol{\theta}_1', \dots, \boldsymbol{\theta}_k')'$ with $\boldsymbol{\theta}_j = (\phi_j, \rho_j)'$ for $j = 1, \dots, k$ and $\boldsymbol{\eta} = (\varphi, \varrho)'$. Furthermore, we impose the zero upper-triangular parametrization of $\mathbf{B}$, similar to the static case. Thus, the total number of parameters to estimate is equal to $nk - \frac{k(k-1)}{2} + n + 2k + 3$.

Conditionally heteroskedastic factor models are first introduced by Diebold and Nerlove (1989) and further developed by Harvey et al. (1992). These approaches assume normally distributed factors and idiosyncratic error terms. Doganoglu et al. (2007) are the first who relax the normality assumption in favor to the stable distribution. Differently from our approach, they restrict the number of factors to one and assume time varying scale processes only for the factor, while the idiosyncratic error terms exhibit scales constant in time. All resulting scales of the elements of the vector $\mathbf{Y}_t$ are time varying, as the spectral measure of $\mathbf{Y}_t$ changes at each time instant t following the changes in the scale of the factor. However in this framework, one can construct combinations of the elements of $\mathbf{Y}_t$ with constant scale parameters as explained in Sentana et al. (2008), which may be in many empirical applications not plausible: e.g., $\mathbf{Y}_t$ are vectors of asset returns with time varying scale parameters and some portfolios of these assets may exhibit constant scale parameter. Although the scale parameter does not have a direct interpretation as the volatility, in the portfolio theory adapted to non-Gaussian stable portfolios (Bawa et al. (1979) and Rachev and Han (2000), among others), it replaces the volatility as a measure of risk. Consequently, it is not possible to construct portfolios with risk constant in time while all underlying assets display a time varying risk.

Doganoglu et al. (2007) derive the log-likelihood for their model as a function of univariate stable densities due to the fact that the idiosyncratic errors and the factors are independent. However, the lack of closed form expressions for the univariate stable densities makes the direct implementation of the maximum likelihood difficult. Therefore,

they use a polynomial approximation as developed in Doganoglu and Mittnik (1998), which is based on fast Fourier transformations as described in Mittnik et al. (1999). The problem with this estimation is that the information set $\mathcal{F}_{t-1}$ is not completely available to the econometrician, as the values of $\mathbf{f}_t$ up to $t - 1$ are not known (for a more detailed discussion, see Sentana et al. (2008)). Therefore, in the paper, we implement the indirect inference method to estimate the parameters of interest, which circumvents both the lack of closed form expression of the stable density and the incompletely available information set $\mathcal{F}_{t-1}$.

4 Indirect Estimation

The lack of closed-form expression of the stable density and, consequently, of the log-likelihood function, as well as the non-existence of most of the moments, makes the implementation of standard methods to estimate the models given in Section 3 a very difficult task. However, the easiness in simulating pseudo-random numbers from the multivariate stable distribution with independent components as described in Section 2 makes the indirect inference estimation methods developed by Gouriéroux et al. (1993), Smith (1993) and Gallant and Tauchen (1996) a valuable alternative. The IndInf methods have found already some application in dealing with the estimation difficulties of the parameters of the stable distributions, however only in the univariate context: Lombardi and Calzolari (2008) and Garcia et al. (2011) for estimating the parameters of an univariate stable distribution and Calzolari et al. (2014) for the estimation of GARCH models with stable distributed error terms. Sentana et al. (2008) apply IndInf to circumvent the problem of unknown information set $\mathcal{F}_t$ within a GARCH factor model with normal distributed factors and idiosyncratic error terms.

The idea behind the IndInf estimation method is to replace the model of interest (in the following called "true model") with an approximated model, which is easier to handle and estimate (in the following called "auxiliary model"). For identification purposes, the dimension of the parameter vector of the auxiliary model should be equal or larger than the dimension of the parameter vector of the true model.

Below, we give a very short introduction to the IndInf method, while in the following subsections we concentrate on describing its implementation for estimating the models introduced in Section 3.

Let $\mathbf{y}_t$, $t = 1, \dots, T$ be a series of observed values of the random vector $\mathbf{Y}_t$ and characterized by the density function $f_0(\mathbf{y}_t; \boldsymbol{\vartheta})$, where $\boldsymbol{\vartheta}$ is the vector of unknown parameters of dimension $r \times 1$. Denote $\boldsymbol{\vartheta}_0$ to be the true value of the parameter vector $\boldsymbol{\vartheta}$. The density function $f_0(\mathbf{y}_t; \boldsymbol{\vartheta})$ has no closed-form or it is very difficult to implement.

Let $f^a(\mathbf{y}_t; \boldsymbol{\psi})$ be an auxiliary density function, which is easier to handle than $f_0(\mathbf{y}_t; \boldsymbol{\vartheta})$ and which is characterized by the parameter vector $\boldsymbol{\psi}$ of dimension $q \times 1$ with $q \geq r$. The corresponding log-likelihood function is given by $\mathcal{L}^a(\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_T; \boldsymbol{\psi})$, which is available analytically.

The IndInf method implies the following steps:

Step 1: Compute the pseudo-ML (PML) estimator of the pseudo-true ψ_0 :

$$\hat{\psi} = \arg \max_{\psi} \mathcal{L}^a(\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_T; \psi). \quad (21)$$

Step 2: For a given value of ϑ , simulate H paths of length T from the model of interest: $\mathbf{y}_{h,1}(\vartheta), \dots, \mathbf{y}_{h,T}(\vartheta)$, with $h = 1, \dots, H$.

Step 3: Find the IndInf estimator $\hat{\vartheta}$ such that:

- (a) $\hat{\psi}$ and $\widehat{\psi}_{HT}(\vartheta)$ are as close as possible (Gouriéroux et al. (1993), see also Smith (1993)):

$$\hat{\vartheta}(\Xi) = \arg \min_{\vartheta} [\hat{\psi} - \widehat{\psi}_{HT}(\vartheta)]' \Xi [\hat{\psi} - \widehat{\psi}_{HT}(\vartheta)], \quad (22)$$

where Ξ is a weighting matrix, which is symmetric nonnegative definite and

$$\widehat{\psi}_{HT}(\vartheta) = \arg \max_{\psi} \frac{1}{H} \sum_{h=1}^H \mathcal{L}^a(\mathbf{y}_{h,1}(\vartheta), \mathbf{y}_{h,2}(\vartheta), \dots, \mathbf{y}_{h,T}(\vartheta); \psi). \quad (23)$$

or

- (b) the score, computed on the HT simulated observations and at value $\hat{\psi}$, is as close as possible to the vector of zeros (Gallant and Tauchen (1996)):

$$\tilde{\vartheta}(\Upsilon) = \arg \min_{\vartheta} \frac{\partial \mathcal{L}_{H,T}^a}{\partial \psi'}(\vartheta, \hat{\psi}) \Upsilon \frac{\partial \mathcal{L}_{H,T}^a}{\partial \psi}(\vartheta, \hat{\psi}), \quad (24)$$

where $\mathcal{L}_{H,T}^a(\vartheta, \psi) \equiv \frac{1}{H} \sum_{h=1}^H \frac{1}{T} \mathcal{L}^a(\mathbf{y}_{h,1}(\vartheta), \mathbf{y}_{h,2}(\vartheta), \dots, \mathbf{y}_{h,T}(\vartheta); \psi)$ and Υ is a weighting matrix, which is symmetric nonnegative definite.

Gouriéroux et al. (1993) show that the two family of estimators, $\hat{\vartheta}(\Xi)$ and $\tilde{\vartheta}(\Upsilon)$ are asymptotically equivalent.

Gouriéroux et al. (1993) show that the indirect inference estimator is consistent and asymptotically normal distributed for H fixed and $T \rightarrow \infty$ with the variance-covariance matrix as given in Appendix A.1. When $r = q$, which is our case in the Monte Carlo exercise and in the empirical application presented below, the results are independent of the choice of Ξ , respectively of Υ , for which we consider identity matrices. Moreover, if the closed form for the gradient of the auxiliary model is available, then the approach of Gallant and Tauchen (1996) has computational advantages over the approach of Gouriéroux et al. (1993) and Smith (1993) as it avoids the repetition of the numerical optimization given in Equation (22).

4.1 Estimation of StatStabF model

For the estimation of the StatStabF model introduced in Section 3.1 by means of IndInf, we consider the following auxiliary model:

$$\mathbf{Y}_t = \mathbf{B}^* \mathbf{f}_t^* + \mathbf{u}_t^* \quad (25)$$

where $\mathbf{f}_t^*$ is a $k \times 1$ vector of unobserved common factors, with $k \leq n$, $\mathbf{B}^*$ is the $n \times k$ matrix of factor loadings of $\text{rank}(\mathbf{B}^*) = k$ and $\mathbf{u}_t^*$ is a $n \times 1$ vector of idiosyncratic noises. We further assume that $\mathbf{f}_t^*$ and $\mathbf{u}_t^*$ are independent of each other, as well as f_{jt}^* and f_{lt}^* for all $j \neq l = 1, \dots, k$ and u_{it}^* and u_{st}^* for all $i \neq s = 1, \dots, n$ and that they have finite variances. Let Δ^* and Σ^* be diagonal matrices of dimension $k \times k$ and, respectively, $n \times n$ whose elements are the variances of the factors and of the idiosyncratic noises respectively, i.e.,

$$\Delta^* = \begin{pmatrix} \delta_1^* & 0 & \dots & 0 \\ 0 & \delta_2^* & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \delta_k^* \end{pmatrix} \quad \Sigma^* = \begin{pmatrix} \sigma_1^* & 0 & \dots & 0 \\ 0 & \sigma_2^* & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_n^* \end{pmatrix}. \quad (26)$$

Similar to the model of interest, for identification reasons we impose that the variances of the factors are equal to 1: i.e., $\delta_j^* = 1$ for $j = 1, \dots, k$.

The variable $\mathbf{Y}_t$ is assumed to be n -variate Student's t distributed with ν degrees of freedom and covariance matrix $\mathbf{\Pi}$.⁵ The choice of the Student's t distribution is motivated by the fact that the degrees of freedom parameter ν has a clear and interpretable matching to the tail parameter α in the stable distribution, as both describe the thickness of the tail. Moreover, $\mathbf{\Pi} = \mathbf{B}^* \mathbf{B}^{*'} + \Sigma^*$. The probability density function of $\mathbf{y}_t$, which is the observed value of $\mathbf{Y}_t$ is given by (Kotz and Nadarajah (2004))

$$f^a(\mathbf{y}_t, \boldsymbol{\psi}) = \frac{\Gamma[(\nu + n)/2]}{\Gamma(\nu/2) \nu^{n/2} \pi^{n/2} |\mathbf{\Pi}|^{1/2} \left[1 + \frac{1}{\nu} \mathbf{y}_t' \mathbf{\Pi}^{-1} \mathbf{y}_t\right]^{(\nu+n)/2}}, \quad (27)$$

where $\boldsymbol{\psi} = (\nu, \mathbf{b}^{*'}, \boldsymbol{\sigma}^{*'})'$ with $\mathbf{b}^* = \text{vec}(\mathbf{B}^{*'}) = (\mathbf{b}_1^{*'}, \dots, \mathbf{b}_n^{*'})'$, $\mathbf{b}_i^* = (b_{i1}^*, \dots, b_{ik}^*)'$ with $i = 1, \dots, n$, and $\boldsymbol{\sigma}^* = (\sigma_1^*, \dots, \sigma_n^*)'$. Similar to the model of interest, for identification reasons we also impose the zero upper-triangular parametrization of $\mathbf{B}^*$. Thus, the log-likelihood of the auxiliary model is given by $\mathcal{L}^a(\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_T; \boldsymbol{\psi}) = \sum_{t=1}^T \ln f^a(\mathbf{y}_t, \boldsymbol{\psi})$ with $\mathbf{y}_1, \dots, \mathbf{y}_T$ independent of each other due to the time independency assumption among the factors and among the idiosyncratic error terms.

The only constraints that must be imposed during the maximization of the pseudo-likelihood of the auxiliary model are on σ_i and σ_i^* , where $i = 1, \dots, n$ that must be non-negative during each iteration of the process and upon convergence. We must also be prepared to deal with possible values of σ_i and σ_i^* that reach the lower boundary of zero (the so called "Heywood case"): they would imply that factors are no more latent, but to some extent, they become "observable", and, thus, the pseudo-likelihood and the score must be changed accordingly. Such cases, however, never occur in any of our empirical experiments.

The total number of parameters to estimate is equal to number of parameters of the model of interest given in Section 3.1. As we are able to provide closed forms for the gradients of the auxiliary model (see Appendix A.2), we implement the approach of Gallant

⁵ As in the model of interest, we assume here that the location vector is equal to zero. However, this assumption can be easily relaxed in practical applications by considering demeaned series for $\mathbf{Y}_t$ or simply estimate the location vector along with the other parameters.

and Tauchen (1996) to obtain the IndInf estimator. In order to avoid that the degrees of freedom reach the boundary of zero, for which the Student's t distribution is not defined, we could implement a constrained indirect inference procedure as introduced by Calzolari et al. (2004). However, given that this is never the case in any of our empirical exercises, we stay with the unconstrained routine as described above.

4.2 Estimation of GARCHStabF model

In order to estimate the GARCHStabF model introduced in Section 3.2 by means of IndInf we consider the following auxiliary model

$$\mathbf{Y}_t = \mathbf{B}^* \mathbf{f}_t^* + \mathbf{u}_t^* \quad (28)$$

where $\mathbf{f}_t^*$ is a $k \times 1$ vector of unobserved common factors, with $k \leq n$, $\mathbf{B}^*$ is the $n \times k$ matrix of factor loadings of rank($\mathbf{B}^*$) = k and $\mathbf{u}_t^*$ is a $n \times 1$ vector of idiosyncratic noises. We further assume that $\mathbf{f}_t^*$ and $\mathbf{u}_t^*$ are independent of each other, as well as f_{jt}^* and f_{lt}^* for all $j \neq l = 1, \dots, k$ and u_{it}^* and u_{st}^* for all $i \neq s = 1, \dots, n$ and that they have finite unconditional variances. Let Δ^* and Σ^* be diagonal matrices of dimension $k \times k$ and, respectively, $n \times n$ whose elements are the unconditional variances of the factors and of the idiosyncratic noises respectively, as given in Equation (26).

Similar to Section 4.1, for identification reasons we impose that the unconditional variances of the factors are equal to 1: i.e., $\delta_j^* = 1$ for $j = 1, \dots, k$ and that $\mathbf{B}^*$ has a zero upper-triangular parametrization. Let $\boldsymbol{\psi}_1 = (\mathbf{b}^{*'}, \boldsymbol{\sigma}^{*'})'$ with $\mathbf{b}^* = \text{vec}(\mathbf{B}^{*'}) = (\mathbf{b}_1^{*'}, \dots, \mathbf{b}_n^{*'})'$, $\mathbf{b}_i^* = (b_{i1}^*, \dots, b_{ik}^*)'$ with $i = 1, \dots, n$, and $\boldsymbol{\sigma}^* = (\sigma_1^*, \dots, \sigma_n^*)'$.

For estimation purposes, we have available only the observations $\mathbf{y}_1, \dots, \mathbf{y}_T$ and not the factors $\mathbf{f}_1^*, \dots, \mathbf{f}_T^*$. For this reason, in order to estimate the parameters of the GARCHStabF model, we adopt a sequential procedure based on three steps. Each step helps at estimating a subset of the parameter vector of the auxiliary model by possibly involving different estimation procedure. Although the framework of the auxiliary model is misspecified, we are able to identify the parameters of the model of interest through the ones of the auxiliary approach as follows:

Step 1: Similar to Section 4.1, assume that $\mathbf{Y}_t$ given in Equation (28) follows a multivariate Student's t distribution with ν degrees of freedom, an unconditional variance-covariance matrix $\boldsymbol{\Pi} = \mathbf{B}^* \mathbf{B}^{*'} + \Sigma^*$ and a location vector equal to zero. Following the estimation procedure described in Section 4.1, compute the IndInf estimates of ν and $\boldsymbol{\psi}_1$.

Step 2: Extract k "approximated" common factors $g_{1t}^*, \dots, g_{kt}^*$ and corresponding residuals $w_{1t}^*, \dots, w_{nt}^*$ based on the classical approach (Lawley and Maxwell (1962)):

$$g_{jt}^* = \mathbf{B}^{*'} (\mathbf{B}^* \mathbf{B}^{*'} + \Sigma^*)^{-1} \mathbf{y}_t \quad (29)$$

$$w_{it}^* = [\mathbf{I}_n - \mathbf{B}^* \mathbf{B}^{*'} (\mathbf{B}^* \mathbf{B}^{*'} + \Sigma^*)^{-1}] \mathbf{y}_t. \quad (30)$$

Step 3: Treat the extracted factors and noises from Step 2 as being conditionally heteroskedastic with conditional variances given by the following GARCH(1,1) processes:

$$\delta_{jt}^* = (1 - \phi_j^* - \rho_j^*)\delta_j^* + \phi_j^* g_{jt-1}^{*2} + \rho_j^* \delta_{jt-1}^*, \quad (31)$$

$$\sigma_{it}^* = (1 - \varphi_i^* - \varrho_i^*)\sigma_i^* + \varphi_i^* w_{it-1}^{*2} + \varrho_i^* \sigma_{it-1}^*, \quad (32)$$

where $\delta_j^* = 1$, $\sigma_i^* > 0$ is derived in Step 1 and $\phi_j^* \geq 0$, $\rho_j^* \geq 0$, $\varphi_i^* \geq 0$, $\varrho_i^* \geq 0$. Moreover, we impose that the GARCH parameters are equal for all idiosyncratic noises: $\varphi_i^* = \varphi^*$ and $\varrho_i^* = \varrho^* \forall i$.

To assure stationarity of the GARCH processes in equations (31) and (32), we impose that $\phi_j^* + \rho_j^* < 1$ for $j = 1, \dots, k$ and $\varphi^* + \varrho^* < 1$.

In order to estimate the parameters of the GARCH processes given in equations (31) and (32), which are stacked in $\psi_2 = (\theta^{*'}, \eta^{*'})'$ with $\theta^* = (\theta_1^{*'}, \dots, \theta_k^{*'})'$ where $\theta_j^* = (\phi_j^*, \rho_j^*)'$ with $j = 1, \dots, k$ and $\eta^* = (\varphi^*, \varrho^*)'$, we treat the extracted factors and the noises as if they are conditionally multivariate Student's t distributed with ν degrees of freedom, where ν is fixed to the value obtained in Step 1 and the conditional variance covariance matrix is given by

$$\Omega_t^* = \begin{pmatrix} \Delta_t^* & \mathbf{0}_{k \times n} \\ \mathbf{0}_{n \times k} & \Sigma_t^* \end{pmatrix}$$

of dimension $(n+k) \times (n+k)$ with Δ_t^* and Σ_t^* being diagonal matrices of dimension $k \times k$ and, respectively, $n \times n$ whose elements are the conditional variances of the extracted factors and of the noises respectively, i.e.,

$$\Delta_t^* = \begin{pmatrix} \delta_{1t}^* & 0 & \dots & 0 \\ 0 & \delta_{2t}^* & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \delta_{kt}^* \end{pmatrix} \quad \Sigma_t^* = \begin{pmatrix} \sigma_{1t}^* & 0 & \dots & 0 \\ 0 & \sigma_{2t}^* & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_{nt}^* \end{pmatrix}.$$

where δ_{jt}^* with $j = 1, \dots, k$ and σ_{it}^* with $i = 1, \dots, n$ are given in equations (31) and (32), respectively.

If $\mathbf{B}^*$ and Σ^* were known, the extracted factors and noises in equations (29) and (30) would be conditionally multivariate stable distributed with the same tail index as $\mathbf{Y}_t$ defined in Section 3.2. However, $\mathbf{B}^*$ and Σ^* are not known and therefore are estimated in Step 1; so, the joint distribution of the extracted factors and noises becomes unknown. However, empirically we detect some fat-tailness for the extracted factors and noises after being standardized by their conditional variances that is inherited from the fat-tailness of the observed original data. Therefore, adopting a multivariate Student's t distribution, which is easy to implement and accounts for fat-tails might be a valuable choice. Alternatively, one can adopt for the estimation purposes of ψ_2 a multivariate normal distribution with mean zero and conditional variance-covariance matrix Ω_t^* . Both procedures have been implemented and the empirical comparison of results suggests that the Student's t should be preferred: although the estimated values of the parameters are comparable, their efficacy is in general larger if we consider the Student's t rather than the normal distribution.

In addition to the constraints imposed in Step 1, we have to impose here that the GARCH parameters of each factor and of the noises must be nonnegative and they assure stationarity of the processes. In a GARCH model, when the ARCH parameter is zero or very small, the corresponding GARCH parameter is not identified or poorly identified. To avoid such cases, we impose that $\phi_j, \phi_j^*, \rho_j, \rho_j^*$ with $j = 1, \dots, k$ and $\varphi, \varphi^*, \varrho, \varrho^*$ must not be less than 0.01. Moreover, to assure stationarity we constrain that $\phi_j^* + \rho_j^*$, and $\varphi^* + \varrho^*$ must not exceed 0.999.⁶ Upon the convergence, we verify if the estimated values of ϕ_j, ρ_j, φ and ϱ satisfy the top-Lyapunov conditions imposed in Section 3.2, which is always the case in all our Monte Carlo experiments and in the empirical application described below.

Although arbitrary, these bounds are wider than those obtained in all the empirical applications we perform on real data. However, in the Monte Carlo experiments these bounds are binding in less than 1% of the cases. The inequality constraints are imposed during the IndInf calibration process as in the constrained indirect estimation of Calzolari et al. (2004), where it is shown that the arbitrarily chosen bounds do not affect the consistency of the indirect estimator.

The total number of parameters to estimate is equal to the number of parameters of the model of interest given in Section 3.2. Therefore, in order to derive the IndInf estimates of the parameter vector ϑ , we implement the approach of Gallant and Tauchen (1996), where the vector of scores is obtained by stacking up together the scores derived in Step 1 and Step 3 for which we provide in Appendix A.2 and A.3, respectively the closed form expressions.

The (asymptotic) variance covariance matrix of the indirect inference estimator of the parameter vector ϑ specific to GARCHStabF model cannot be computed from Equation (A.37) given in the appendix as the indirect estimation procedure is done in several steps. More precisely, the Fisher information matrix of the auxiliary model, namely $I(\psi_0)$ cannot be fully estimated. Therefore, we replace it by a consistent estimator, namely the sample variance-covariance matrix of 1000 independently simulated score vectors. The score vectors are computed after the last iteration, upon convergence. The values of $\frac{\partial^2 \mathcal{L}_{H,\infty}^a}{\partial \psi \partial \vartheta'}(\vartheta_0, \psi_0)$ in Equation (A.37) are also computed in the last iteration, upon convergence. The good performance of such procedure is preliminarily verified with several Monte Carlo experiments, where upon convergence of each replication, not only the parameter estimates are retrieved, but also the estimate of their (asymptotic) variance covariance matrix computed as described above. After a large number of replications (such as 1000), the Monte Carlo variance covariance matrices of the parameter estimates and the averages of the estimated (asymptotic) variances covariance matrix always appear to be very similar.

⁶ Dealing with stable distributions, we cannot take advantage of the result proved in Proposition 1 of Sentana et al. (2008) and derived under normality assumption concerning the case when ARCH parameters are "equal to zero". The main advantage of the result of Sentana et al. (2008) is that, when the estimated values of the ARCH parameters of the factors and of the noises are very small, their values (as well as the corresponding values of the GARCH parameter) could be taken as final results, avoiding the indirect calibration procedure.

5 Monte Carlo Study

A detailed set of Monte Carlo experiments is performed to assess the finite sample properties of the IndInf estimates for the two models introduced in Section 3. All estimations in this section and in the following one are done with Fortran 77 and most of them are ran on the parallel cluster bwUniCluster, which is a distributed memory parallel computer where each node has sixteen Intel Xeon processors⁷.

For each of the models, we perform a total of $R = 1000$ Monte Carlo replications with a length of the series of $T = 3000$. This sample size corresponds to about 13 years of daily data, as in our empirical application. We consider here two cases: the first case regards a total of three series ($n = 3$) for which we estimate only one factor ($k = 1$) and the second case regards a larger panel of series of about ten ($n = 10$) for which we estimate two factors ($k = 2$). For implementing the IndInf procedure, we choose $H = 10$ and $H = 100$ for the StatStabF Model and GARCHStabF model for three series and one factor, which implies a total of 30000 and 300000, respectively simulated observations to compute the mean scores in the auxiliary models. Increasing H reduces the (theoretical) variance covariance matrix of the estimates according to Equation (A.37), however at high computational costs. We choose to report here results only for $H = 100$, as they do not significantly differ from the ones for $H = 10$, except for the variances which are slightly smaller. To estimate GARCHStabF model for ten series and two factors we also choose $H = 10$ and $H = 100$. However, the implementation of IndInf for $H = 100$ becomes computational burdensome, and, given that in the other cases, there are no significant differences between the results, we choose to proceed in this case with $H = 10$.

We chose the values of the parameters to mimic real-case values, as reported in Section 6. Thus, to generate StatStabF processes, we choose five cases with α ranging from a very "fat-tailed" value of 1.5 to a moderate-up-to-Gaussian "fat-value" of 1.9. The values of the elements of $\mathbf{B}$ and Σ are chosen to be all equal to 1 for the case of three series and one factor and some of the elements of $\mathbf{B}$ to be equal to -1 for the case of ten series and two factors⁸. Table B.1 in the appendix reports the average and standard deviations over the 1000 Monte Carlo replications. Overall, one may say that the Monte Carlo results are quite remarkable: all estimates "seem unbiased" (differences between the average estimates and the parameters used to generate the data are observable only after the third digit). The results improve by increasing the value of α , and, thus, by reducing the extremes in the simulated data.

Regarding the efficiency of the estimates, the tail index is estimated with the highest precision regardless of the choice of the number of series and factors. Moreover, as similar to the bias, the precision improves with increasing the value of α . The same effect is observed for the elements of the matrix $\mathbf{B}$ and Σ , which are in general estimated with a lower precision than the tail index.

In order to generate GARCHStabF processes, we choose again five cases with α ranging from 1.75 to 1.95. The choice of larger α in the GARCHStabF model compared to the

⁷For further details about the cluster, please visit: http://www.bwhpc-c5.de/wiki/index.php/Main_Page.

⁸If we choose all elements of $\mathbf{B}$ to be equal to 1, we would face identification problems.

StatStabF model is due to the fact that in the real application presented in the following section, when GARCH effects are included, the fat-tailedness of the standardized residuals reduces. This might be due to the fact that, although not directly applicable here due to lack of moments larger or equal to α , the GARCH specification is able to capture some of the "overkurtosis" present in the original series. Although all empirical experiments done on real data indicate that, in the presence of GARCH effects, the smallest realistic α is estimated around 1.81, we consider here also the case where the standardized residuals may have a even fatter tail, described by a value of 1.75 or 1.8. Moreover, as the empirical results show, the largest realistic α is estimated to be about 1.96, which is much closer to the normality assumption than in the StatStabF case. For this reason, we choose in this Monte Carlo experiment as upper limit of 1.95. The values of the elements of $\mathbf{B}$ and $\mathbf{\Sigma}$ are chosen to be all equal to 1 for the case of three series and one factor and some of the elements of $\mathbf{B}$ to be equal to -1 for the case of ten series and two factors. Moreover, the values of the GARCH parameters are chosen to mimic the values obtained from the real data. Tables B.2 and B.3 in the appendix report the average and standard deviations over the 1000 Monte Carlo replications for the case of three series and one factor and for the case of ten series and two factors, respectively.

Overall, one may say that compared to the StatStabF model, the introduction of the GARCH effects induces some bias in the estimates. From the results reported in Table B.2, three conclusions can be drawn: First, the bias reduces when α increases, and this might be due to less extremes in the generated data. Second, the bias in the estimated α and in the GARCH parameters is much smaller than in the other parameters and it is observable only after three digits. Third, the variance of the estimates also reduces when α increases, which might be due again to a reduction in the extremes in the generated data. Of special interest is the case of $\alpha = 1.75$, for which two of the three diagonal elements of the matrix $\mathbf{\Sigma}$ exhibit very large standard deviations. For this reason, we complement the information on the distribution of the estimates and report in Table B.4 the median and interquartile ranges over the 1000 replications.⁹ One may observe that, for this special case, the median becomes much closer to the simulated value and the interquartile range becomes much smaller compared to the standard errors reported in Table B.2. This strengthens the presumption that the relatively poor average and standard deviation results might be due to a high rate of extremes within the sample generated for small values of α . However, when one increases the number of observations to an extreme case of two millions and the number of replications to 2000, the estimates become very precise and accurate: Table 1 from below reports simulation results for the GARCHStabF model for the case of $\alpha = 1.75$ and $\alpha = 1.8$ and $H = 1$. One should keep in mind that, if the experiment is repeated for $H = 100$, it is expected a reduction in the variance approximately by $1 + 1/H$ (see Equation (A.37)).

Thus, one can conclude that, for the extreme cases of $\alpha = 1.75$ and $\alpha = 1.8$, it takes more than 3000 observations to obtain reliable finite sample properties. However, although these cases might be of interest for some applications, there are not very realistic. When estimated from real data, for the three series and one factor case, the coefficient α varies between 1.92 and 1.96 for which Table B.2 reports very good results: the bias in

⁹A further approach of dealing with biased IndInf estimates is by implementing the procedures of Gouriéroux et al. (2000) and Arvanitis and Demos (2006).

Table 1: **Monte Carlo results for estimating the GARCHStabF model by Indirect Inference:** average, median, standard errors (std.dev.) and interquartile ranges (IQ-range) over $R = 2000$ Monte Carlo replications, based on $T = 2000000$ observations, $n = 3$, $k = 1$ and $H = 1$ simulation paths.

	True Values	1.750		1.800	
		average (std. dev.)	median (IQ-range)	average (std. dev.)	median (IQ-range)
α		1.749 (0.003)	1.749 (0.005)	1.799 (0.003)	1.799 (0.004)
B	1.00	0.994 (0.017)	0.994 (0.023)	0.995 (0.012)	0.996 (0.016)
	1.00	0.994 (0.017)	0.995 (0.022)	0.995 (0.012)	0.996 (0.016)
	1.00	0.994 (0.017)	0.994 (0.023)	0.995 (0.012)	0.996 (0.016)
diag Σ	1.00	0.991 (0.042)	0.992 (0.057)	0.992 (0.028)	0.993 (0.040)
	1.00	0.989 (0.042)	0.988 (0.056)	0.990 (0.028)	0.989 (0.039)
	1.00	0.989 (0.044)	0.989 (0.059)	0.990 (0.030)	0.991 (0.038)
ρ	0.93	0.930 (0.001)	0.930 (0.002)	0.930 (0.001)	0.930 (0.002)
ϕ	0.04	0.040 (0.001)	0.040 (0.001)	0.040 (0.001)	0.040 (0.001)
ϱ	0.93	0.930 (0.001)	0.930 (0.001)	0.930 (0.001)	0.930 (0.001)
φ	0.04	0.040 (0.000)	0.040 (0.001)	0.040 (0.000)	0.040 (0.001)

the estimates is observable only after three digits.

By analyzing the results for the case of ten series and two factors given in Table B.3, one can derive the same conclusions as in the previous case. However, the biases and standard deviations increase compared to the case of three series and one factor. This is due to the fact that now we estimate 25 parameters more on the same number of observations. Furthermore, also for $\alpha = 1.8$ one gets standard deviations which seem to be too large. But the median and interquartile ranges reported in Table B.5 indicate an improvement in the estimates for the extreme cases of $\alpha = 1.75$ and $\alpha = 1.8$, which, however, are of little relevance in our empirical application: the estimated α parameter varies between 1.83 and 1.88. Moreover, similar to the previous case, the average and standard deviations can be improved by increasing the number of observations.

6 Empirical Application

In this section we provide further evidence on the performance of the IndInf method in estimating the parameters of the models described in Section 3 when applied to real data. The practical implementaton of these models in finance applications, such as asset pricing or risk management does not represent the scope of this paper and is left for further research.

We estimate the StatStabF and GARCHStabF models on daily log-returns of thirty individual components¹⁰ of the Dow Jones Industrial Average stock index computed for the open-to-close period (from 9:30 to 16:00) on the sampling window 01.11.2001 – 30.09.2014 ($T = 3220$ trading days). The data stems from the Trade and Quotations (TAQ) database¹¹ and the daily log-returns are scaled up by 100.

Figure B.1 in Appendix plots the line graphs of the log-return series and Table B.6 provides their descriptive statistics. From Figure B.1 one may observe that there is strong commonality in the dynamics of daily returns, especially during the previous financial crisis characterized by clustered extreme returns. Moreover, the descriptive statistics reported in Table B.6 provide further insight that the distributions of the underlying series are far from being normal, due to the large heavy-tailedness, in particular for the assets belonging to the financial sector. This leads to the conclusion that factor models with fat-tailed factors or, additionally, with time-varying dispersion might be appropriate to capture such dynamics. Although the means of the series are in general close to zero, in what follows, we present results from using demeaned log-returns.

Even if we report and discuss at length here results only for $n = 10$ series, namely for AXP, BAC, C, GS and JPM from the financial industry and DD, JNJ, MRK, PFE and PG from the chemical industry, we also refer briefly to results obtained for different combinations of stocks with n taking further values, such as 3, 5, 7, 15 and 30.¹² Moreover, the results we discuss are from estimating both the StatStabF and GARCHStabF models with $k = 1$ and $k = 2$ factors and we shortly refer to results from estimating $k = 3$ factors. Table B.7 in appendix reports estimates and standard errors computed with $H = 100$. The standard errors of the estimates stemming from the StatStabF model are obtained by the empirical counterparts of the diagonal elements of the covariance matrix given in Equation (A.37) and the standard errors of the estimates of the GARCHStatF model are computed as described in Section 4.2.

The estimation results show that the factors and idiosyncratic noises exhibit large fat-tails with α varying between 1.6 and 1.86 in StatStabF model. Moreover, increasing the number of factors increases the value of estimated α in both models, with a more pronounced effect in the StatStabF model than in the GARCHStabF (the effect is further observable when k becomes 3). The common behavior of daily returns, which seems to be also very extreme, especially during turbulent times, can be captured by a factor with fat-tails or by two factors, both with less fatter tails than the one factor. The fact that the StatStabF model provides in general smaller values of the estimated α is due to the fact that through the GARCH effects one may capture some proportion of the heavy-tailedness

¹⁰The stocks are: Alcoa Inc. (AA), American Express Company (AXP), Boeing Corporation (BA), Bank of America Corporation (BAC), Citigroup Inc. (C), Caterpillar Inc. (CAT), Chevron Corporation (CVX), Dupont (DD), Walt Disney Company (DIS), General Electric Company (GE), The Goldman Sachs Group, Inc. (GS), Home Depot Inc. (HD), Honeywell International Inc. (HON), Hewlett-Packard Company (HPQ), International Business Machines (IBM), International Paper Company (IP), Johnson & Johnson (JNJ), J.P. Morgan Chase & Company (JPM), Coca-Cola Company Kraft Foods Inc. (KO), McDonald's Corporation (MCD), 3M Company (MMM), Altria Group Inc. (MO), Merck & Company Inc. (MRK), Nike, Inc. (NKE), Pfizer Inc. (PFE), Procter & Gamble Company (PG), United Technologies Corporation (UTX), Verizon Communications Inc. (VZ), Wal-Mart Stores Inc. (WMT), Exxon Mobil Corporation (XOM).

¹¹We are grateful to Sebastian Bayer for preparing the data.

¹²The detailed results for these cases can be obtained from the authors upon request.

in the series, similar to the GARCH effects in standard frameworks (i.e., under normal or Student's t distributional assumptions).

The estimates of the $\mathbf{B}$ and Σ are all significant at 5% level and comparable in values among the models. The nonexistence of moments larger or equal to α within the stable distribution makes it difficult to provide a classical interpretation of the GARCH parameters in terms of degree of clustering and overkurtosis. However, the GARCH effects for both the factors and the noises are significant and the values are similar to the ones obtained in a classical GARCH framework: the estimate of the GARCH parameter is very large (varying around 0.9) relative to the estimate of the ARCH parameter (varying around 0.05), while the sum of the two estimates is close to the value of 1. Similar results are obtained by Calzolari et al. (2014) when estimating GARCH models with conditionally α -distributed standardized errors and by Sentana et al. (2008) when estimating GARCH parameters within a conditionally heteroskedastic factor structure with normally distributed factors and idiosyncratic noises.

When extending the analysis to different number of assets, one observes that the estimates of the parameter α in the GARCHStabF model decreases by increasing the number of assets. Applying the model to several combinations of stocks, for $n = 3$ the estimates of the α coefficient vary between 1.92 and 1.96, for $n = 5$ between 1.88 and 1.91, for $n = 7$ between 1.85 and 1.9, for $n = 15$ between 1.86 and 1.87 and for $n = 30$, the estimate of α is equal to 1.81. This might be explained by the fact that by increasing the number of series one also increases the degree of heterogeneity in terms of heavy-tailedness among the assets. Thus, for example, in the set of three series with the estimated α of about 1.92 we include the assets BAC, C and CAT, while in the set of five series with the estimated α of about 1.88, we include the assets belonging to the financial sector, namely AXP, BAC, C, GS and JPM. Compared to the set of three, in the set of five assets we drop CAT, which according to Table B.6 has a small empirical over kurtosis relative to the assets from the financial sector and add the rest of the financial assets, which increases the overall fat-tailedness of the system. This has a direct effect on the value of the tail index estimate, common to the factors and idiosyncratic noises, which decreases accordingly. Therefore, in the set of 30 stocks, where we include all assets available, the estimated α is the smallest among all possible combinations of stocks in our sample.

7 Conclusions

In this paper we apply the indirect inference method to estimate factor models with α -stable distributed factors and idiosyncratic noises. The factor models have found so far wide applications in the economic and finance empirical literature, as they parsimoniously capture common dynamics of large panels of data. Most of the existing factor models assume a normal distribution for the underlying variables. However, many economic variables, especially in the finance area, such as returns from stock prices or exchange rates, exhibit heavy-tails that are usually captured by means of the Student's t distribution assumptions. Although very popular, the Student's t distribution lacks of stability under aggregation.

A valuable alternative is the α -stable distribution that possesses closure under aggregation, domain of attraction and applicability of the (generalized) central limit theory. These features make it particularly attractive for finance applications, such as portfolio optimization or asset pricing. The main difficulty, however, is the estimation of its parameters: classical estimation methods, such as maximum likelihood or methods of moments can not be applied, as the α -stable distribution lacks in closed-form solutions of the density or in moments larger than one. However, its easiness in simulating pseudo-random numbers makes feasible the application of the indirect inference method to estimate the parameters of interest.

In this paper we focus on two types of factor models, both assuming that the factors and the idiosyncratic noises are independently jointly multivariate α -stable distributed: a static factor model with the underlying distribution constant in time and a GARCH factor model with a time varying conditional distribution. The simulation exercise reveals very good estimation results for a wide range of parameter choices: the estimates seem unbiased and their efficiency increases with the tail index of the distribution. Moreover, the empirical results on a set of thirty daily stock log-returns provides further evidence on the properties of the estimates and on the performance of the indirect inference estimation method when applied to real data.

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Appendix A

A.1 The asymptotic properties of the Indirect Inference estimators

Gouriéroux et al. (1993) show that, under certain regularity conditions, the indirect inference estimator $\hat{\vartheta}(\Xi)$ is consistent and asymptotically normal for H fixed and $T \rightarrow \infty$.

$$\sqrt{T}(\hat{\vartheta}(\Xi) - \vartheta_0) \xrightarrow{d} N(\mathbf{0}, \mathbf{W}(H, \Xi)), \quad (\text{A.33})$$

where

$$\begin{aligned} \mathbf{W}(H, \Xi) = & \left(1 + \frac{1}{H}\right) \left[\frac{\partial p'}{\partial \vartheta}(\vartheta_0) \Xi \frac{\partial p}{\partial \vartheta'}(\vartheta_0) \right]^{-1} \frac{\partial p'}{\partial \vartheta}(\vartheta_0) \times \\ & \times \Xi J(\psi_0)^{-1} I(\psi_0) J(\psi_0)^{-1} \Xi \frac{\partial p}{\partial \vartheta'}(\vartheta_0) \left[\frac{\partial p'}{\partial \vartheta}(\vartheta_0) \Xi \frac{\partial p}{\partial \vartheta'}(\vartheta_0) \right]^{-1}, \end{aligned} \quad (\text{A.34})$$

where $p(\vartheta) = \lim_{T \rightarrow \infty} \widehat{\psi}_{HT}(\vartheta)$ to be the link between ϑ and ψ as a binding function, such that $p(\vartheta_0) = \psi_0$. $J(\psi_0)$ is minus the expectation of the Hessian of the log-likelihood of the auxiliary model and $I(\psi_0)$ is the Fisher information matrix of the auxiliary model.

When the problem is just identified, i.e. the dimension of the two parameter vectors is equal, $r = q$, the results are independent of the choice of the matrices that define the metric, Ξ . On the contrary, when $q > r$, it would be necessary to choose a metric Ξ to measure the distance between $\hat{\psi}$ and $\widehat{\psi}_{HT}(\vartheta)$. The optimal choice of Ξ is

$$\Xi^* = J(\psi_0) I(\psi_0)^{-1} J(\psi_0). \quad (\text{A.35})$$

The corresponding IndInf estimator is denoted by $\hat{\vartheta}^*$ and its variance-covariance matrix is given by

$$\mathbf{W}(H, \Xi^*) = \left(1 + \frac{1}{H}\right) \left[\frac{\partial p'}{\partial \vartheta}(\vartheta_0) J(\psi_0) I(\psi_0)^{-1} J(\psi_0) \frac{\partial p}{\partial \vartheta'}(\vartheta_0) \right]^{-1}. \quad (\text{A.36})$$

As shown by Gouriéroux et al. (1993), $\hat{\vartheta}(\Xi)$ and the $\tilde{\vartheta}(\Upsilon)$ estimator of Gallant and Tauchen (1996) are asymptotically equivalent and the optimal value of Υ is $\Upsilon^* = I(\psi_0)^{-1}$. In this case, the variance-covariance matrix of $\tilde{\vartheta}^*$ is given by

$$\mathbf{W}(H, \Upsilon^*) = \left(1 + \frac{1}{H}\right) \left[\frac{\partial^2 \mathcal{L}_{H,\infty}^a}{\partial \vartheta \partial \vartheta'}(\vartheta_0, \psi_0) I(\psi_0)^{-1} \frac{\partial^2 \mathcal{L}_{H,\infty}^a}{\partial \psi \partial \psi'}(\vartheta_0, \psi_0) \right]^{-1}, \quad (\text{A.37})$$

where $\mathcal{L}_{H,\infty}^a(\vartheta, \psi) = \lim_{T \rightarrow \infty} \mathcal{L}_{H,T}^a(\vartheta, \psi)$.

A.2 The score of the auxiliary model for the StatStabF Model

The log-likelihood of the auxiliary model is given by $\mathcal{L}^a(\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_T; \psi) = \sum_{t=1}^T \ln f^a(\mathbf{y}_t, \psi)$, where

$$f^a(\mathbf{y}_t, \psi) = \frac{\Gamma[(\nu + n)/2]}{\Gamma(\nu/2) \nu^{n/2} \pi^{n/2} |\mathbf{\Pi}|^{1/2} \left[1 + \frac{1}{\nu} \mathbf{y}_t' \mathbf{\Pi}^{-1} \mathbf{y}_t\right]^{(\nu+n)/2}}, \quad (\text{A.38})$$

where $\psi = (\nu, \mathbf{b}^{*'}, \boldsymbol{\sigma}^{*'})'$ with $\mathbf{b}^* = \text{vec}(\mathbf{B}^{*'}) = (\mathbf{b}_1^{*'}, \dots, \mathbf{b}_n^{*'})'$, $\mathbf{b}_i^* = (b_{i1}^*, \dots, b_{ik}^*)'$ with $i = 1, \dots, n$ and where some of the b^* 's coefficients are set to zero as explained in the main text

of the paper and $\boldsymbol{\sigma}^* = (\sigma_1^*, \dots, \sigma_n^*)'$. We compute the score of the logarithm transformation of the likelihood given in Equation (A.38) with respect to $\boldsymbol{\psi}$ element by element. Thus the derivative of $\ln f^a(\mathbf{y}_t, \boldsymbol{\psi})$ w.r.t. ν is given by:

$$\frac{\partial \ln f^a(\mathbf{y}_t, \boldsymbol{\psi})}{\partial \nu} = \frac{1}{2} \left[\frac{\Gamma'[(\nu+n)/2]}{\Gamma[(\nu+n)/2]} - \frac{\Gamma'(\nu/2)}{\Gamma(\nu/2)} - \ln \left(1 + \frac{1}{\nu} \mathbf{y}_t' \boldsymbol{\Pi}^{-1} \mathbf{y}_t \right) - \frac{\nu+n}{\nu + \mathbf{y}_t' \boldsymbol{\Pi}^{-1} \mathbf{y}_t} + 1 \right], \quad (\text{A.39})$$

where $\Gamma'(\cdot)$ is the derivative of the Γ function computed with the built-in Fortran function.

The derivatives of $\ln f^a(\mathbf{y}_t, \boldsymbol{\psi})$ w.r.t. the unique elements of the symmetric matrix $\boldsymbol{\Pi}$ are given by:

$$\begin{aligned} \frac{\partial \ln f^a(\mathbf{y}_t, \boldsymbol{\psi})}{\partial \pi_{ij}} &= -\frac{\pi^{ij}}{2} - \sum_{r=1}^n \sum_{s=1}^n \frac{\partial \ln \left([1 + \frac{1}{\nu} \mathbf{y}_t' \boldsymbol{\Pi}^{-1} \mathbf{y}_t]^{(\nu+n)/2} \right)}{\partial \pi^{rs}} \cdot \frac{\partial \pi^{rs}}{\partial \pi_{ij}} \\ &= -\frac{\pi^{ij}}{2} + \frac{\nu+n}{2\nu [1 + \frac{1}{\nu} \mathbf{y}_t' \boldsymbol{\Pi}^{-1} \mathbf{y}_t]} \sum_{r=1}^n \sum_{s=1}^n (\mathbf{y}_t \mathbf{y}_t')_{r,s} \pi^{ri} \pi^{js} \end{aligned} \quad (\text{A.40})$$

where $i, j = 1, \dots, n$ and $j \geq i$, $(\mathbf{y}_t \mathbf{y}_t')_{r,s}$ is the (r, s) -th element of the matrix $\mathbf{y}_t \mathbf{y}_t'$ and π^{ij} is the ij -th element of $\boldsymbol{\Pi}^{-1}$.

In order to compute the elements of the derivative of $\ln f^a(\mathbf{y}_t, \boldsymbol{\psi})$ w.r.t. the elements of $\mathbf{B}^*$ and $\boldsymbol{\Sigma}^*$, we make use of the fact that $\boldsymbol{\Pi} = \mathbf{B}^* \mathbf{B}^{*'} + \boldsymbol{\Sigma}^*$, i.e.,

$$\pi_{ij} = \begin{cases} b_{i1}^* b_{j1}^* + b_{i2}^* b_{j2}^* + b_{i3}^* b_{j3}^* + \dots + b_{ik}^* b_{jk}^* + \sigma_i^* & \text{if } i = j \\ b_{i1}^* b_{j1}^* + b_{i2}^* b_{j2}^* + b_{i3}^* b_{j3}^* + \dots + b_{ik}^* b_{jk}^* & \text{if } i \neq j \end{cases} \quad (\text{A.41})$$

Thus

$$\frac{\partial \pi_{ij}}{\partial b_{rq}^*} = \begin{cases} 0 & \text{if } i \neq r \text{ and } j \neq r \\ 2b_{rq}^* = 2b_{iq}^* = 2b_{jq}^* & \text{if } i = j = r \\ b_{jq}^* & \text{if } i = r \text{ and } j \neq r \\ b_{iq}^* & \text{if } i \neq r \text{ and } j = r \end{cases} \quad (\text{A.42})$$

$$\frac{\partial \pi_{ij}}{\partial \sigma_r^*} = \begin{cases} 1 & \text{if } i = j = r \\ 0 & \text{otherwise} \end{cases} \quad (\text{A.43})$$

$\forall q$ with $q = 1, \dots, k$ and $i, j, r = 1, \dots, n, j \geq i$.

Consequently

$$\begin{aligned} \frac{\partial \ln f^a(\mathbf{y}_t, \boldsymbol{\psi})}{\partial b_{pq}^*} &= \sum_{i=1}^n \sum_{j=1}^n \frac{\partial \ln f^a(\mathbf{y}_t, \boldsymbol{\psi})}{\partial \pi_{ij}} \cdot \frac{\partial \pi_{ij}}{\partial b_{pq}^*} \\ &= \sum_i^n \left[-\frac{\pi^{ip}}{2} + \frac{\nu+n}{2\nu [1 + \frac{1}{\nu} \mathbf{y}_t' \boldsymbol{\Pi}^{-1} \mathbf{y}_t]} \sum_{r=1}^n \sum_{s=1}^n (\mathbf{y}_t \mathbf{y}_t')_{r,s} \pi^{ri} \pi^{ps} \right] b_{iq}^* \\ &\quad + \sum_j^n \left[-\frac{\pi^{pj}}{2} + \frac{\nu+n}{2\nu [1 + \frac{1}{\nu} \mathbf{y}_t' \boldsymbol{\Pi}^{-1} \mathbf{y}_t]} \sum_{r=1}^n \sum_{s=1}^n (\mathbf{y}_t \mathbf{y}_t')_{r,s} \pi^{rp} \pi^{js} \right] b_{jq}^* \end{aligned} \quad (\text{A.44})$$

Due to symmetry of $\mathbf{\Pi}$ and $\mathbf{y}_t \mathbf{y}_t'$, we get:

$$\frac{\partial \ln f^a(\mathbf{y}_t, \boldsymbol{\psi})}{\partial b_{pq}^*} = - \sum_{i=1}^n \pi^{ip} b_{iq}^* + \frac{\nu + n}{\nu [1 + \frac{1}{\nu} \mathbf{y}_t' \mathbf{\Pi}^{-1} \mathbf{y}_t]} \sum_{i=1}^n \sum_{r=1}^n \sum_{s=1}^n (\mathbf{y}_t \mathbf{y}_t')_{r,s} \pi^{ri} \pi^{ps} b_{iq}^* \quad (\text{A.45})$$

Moreover

$$\begin{aligned} \frac{\partial \ln f^a(\mathbf{y}_t, \boldsymbol{\psi})}{\partial \sigma_i^*} &= \frac{\partial \ln f^a(\mathbf{y}_t, \boldsymbol{\psi})}{\partial \pi_{ii}} \cdot \frac{\partial \pi_{ii}}{\partial \sigma_i^*} \\ &= -\frac{\pi^{ii}}{2} + \frac{\nu + n}{2\nu [1 + \frac{1}{\nu} \mathbf{y}_t' \mathbf{\Pi}^{-1} \mathbf{y}_t]} \sum_{r=1}^n \sum_{s=1}^n (\mathbf{y}_t \mathbf{y}_t')_{r,s} \pi^{ri} \pi^{is}. \end{aligned} \quad (\text{A.46})$$

The correctness of the above closed form expressions of the derivatives is carefully checked empirically by comparing their numerical values to finite differences (as Δ/Δ from the log-likelihood given in Equation (A.38)).

A.3 The score of the auxiliary model for the GARCHstabF Model

The estimation of the parameters of the GARCHstabF model implies three steps and the scores of Step 1 and 3 are stacked up together in one vector. As the scores corresponding to Step 1 are already provided in Appendix A.2, we give here the scores corresponding to Step 3 when we assume a conditionally multivariate Student's t distribution with ν degrees of freedom for the extracted factors $\mathbf{g}_t^* = (g_{1t}^*, \dots, g_{kt}^*)'$ and noises $\mathbf{w}_t^* = (w_{1t}^*, \dots, w_{nt}^*)'$. Given the diagonal structure of the covariance matrix $\mathbf{\Omega}_t^*$, each of the extracted factors is treated as conditionally univariately Student's t distributed with ν degrees of freedom with the conditional variance as given in Equation (31). The log-likelihood for each extracted factor is given by:

$$\ln f^a(g_{jt}^*, \boldsymbol{\theta}_j^*) = \ln \left[\Gamma\left(\frac{\nu+1}{2}\right) - \Gamma\left(\frac{\nu}{2}\right) \right] - \frac{1}{2} \ln(\delta_{jt}^*) - \frac{\nu+1}{2} \ln \left(1 + \frac{g_{jt}^{*2}}{\nu \cdot \delta_{jt}^*} \right) - \frac{1}{2} \ln(\pi) - \frac{1}{2} \ln(\nu),$$

where $\boldsymbol{\theta}_j^* = (\phi_j^*, \rho_j^*)'$ and δ_{jt}^* are given in Equation (31) with $j = 1, \dots, k$. Thus, the scores of the log-likelihood $\ln f^a(g_{jt}^*, \boldsymbol{\theta}_j^*)$ w.r.t. the elements of $\boldsymbol{\theta}_j^* = (\phi_j^*, \rho_j^*)'$ are given by:

$$\frac{\partial \ln f^a(g_{jt}^*, \boldsymbol{\theta}_j^*)}{\partial \phi_j^*} = \frac{\nu(g_{jt}^{*2} - \delta_{jt}^*)}{2\delta_{jt}^*(\nu\delta_{jt}^* + g_{jt}^{*2})} \cdot (-\delta_j^* + g_{jt-1}^{*2}) \quad (\text{A.47})$$

$$\frac{\partial \ln f^a(g_{jt}^*, \boldsymbol{\theta}_j^*)}{\partial \rho_j^*} = \frac{\nu(g_{jt}^{*2} - \delta_{jt}^*)}{2\delta_{jt}^*(\nu\delta_{jt}^* + g_{jt}^{*2})} \cdot \frac{\partial \delta_{jt}^*}{\partial \rho_j^*}, \quad (\text{A.48})$$

where δ_j^* is constrained to be equal to 1 and $\frac{\partial \delta_{jt}^*}{\partial \rho_j^*}$ is obtained recursively from $\frac{\partial \delta_{jt}^*}{\partial \rho_j^*} = -\delta_j^* + \rho_j^* \frac{\partial \delta_{jt-1}^*}{\partial \rho_j^*}$ and the starting value is the derivative of the unconditional variance of g_{jt}^* with respect to ρ_j^* .

From the multivariate Student's t distribution assumption, we get that the vector of extracted noises $\mathbf{w}_t^*$ is also jointly Student's t distributed with ν degrees of freedom and diagonal matrix

Σ_t^* , where the diagonal elements are as given in Equation (32). Given that the noises $w_{1t}^*, \dots, w_{nt}^*$ are independent of each other, the log-likelihood of their vector becomes:

$$\ln f^a(\mathbf{w}_t^*, \boldsymbol{\eta}^*) = \sum_{i=1}^n \left\{ \ln \left[\Gamma\left(\frac{\nu+1}{2}\right) - \Gamma\left(\frac{\nu}{2}\right) \right] - \frac{1}{2} \ln(\sigma_{it}^*) - \frac{\nu+1}{2} \ln \left(1 + \frac{w_{it}^{*2}}{\nu \cdot \sigma_{it}^*} \right) - \frac{1}{2} \ln(\pi) - \frac{1}{2} \ln(\nu) \right\},$$

where σ_{it}^* is given in Equation (32) and $\boldsymbol{\eta}^* = (\varphi^*, \varrho^*)'$.

Thus, the scores of $\ln f^a(\mathbf{w}_t^*, \boldsymbol{\eta}^*)$ with respect to φ^* and ϱ^* are given by:

$$\frac{\partial \ln f^a(\mathbf{w}_t^*, \boldsymbol{\eta}^*)}{\partial \varphi^*} = \sum_{i=1}^n \left\{ \frac{\nu(w_{it}^{*2} - \sigma_{it}^*)}{2\sigma_{it}^*(\nu\sigma_{it}^* + w_{it}^{*2})} \cdot (-\sigma_{it}^* + w_{it}^{*2}) \right\} \quad (\text{A.49})$$

$$\frac{\partial \ln f^a(\mathbf{w}_t^*, \boldsymbol{\eta}^*)}{\partial \varrho^*} = \sum_{i=1}^n \left\{ \frac{\nu(w_{it}^{*2} - \sigma_{it}^*)}{2\sigma_{it}^*(\nu\sigma_{it}^* + w_{it}^{*2})} \cdot \frac{\partial \sigma_{it}^*}{\partial \varrho^*} \right\}, \quad (\text{A.50})$$

where $\frac{\partial \sigma_{it}^*}{\partial \varrho^*} = -\sigma_{it}^* + \varrho^* \frac{\partial \sigma_{it-1}^*}{\partial \varrho^*}$ is computed recursively and the starting value is the derivative of the unconditional variance of w_{it}^* with respect to ϱ^* .

Appendix B: Figures and Tables

Figure B.1: **Line Graph of daily log-returns of the DJIA 30 stocks over the period 01.11.2001 – 30.09.2014** ($T = 3220$ trading days.) The log-returns are scaled up by 100. On X -axis we plot the years.

Table B.1: Monte Carlo results for estimating the StatStabF model by Indirect Inference: average estimates and standard errors (in parentheses) over $R = 1000$ Monte Carlo replications, based on $T = 3000$ observations and $H = 100$ simulation paths.

n = 3, k = 1											
α	True Values	1.500		1.600		1.700		1.800		1.900	
		1.500 (0.020)		1.600 (0.020)		1.701 (0.020)		1.801 (0.020)		1.901 (0.017)	
B	1.00	1.001 (0.032)		1.001 (0.031)		1.001 (0.030)		1.001 (0.030)		1.001 (0.029)	
	1.00	1.001 (0.032)		1.001 (0.031)		1.001 (0.031)		1.000 (0.030)		1.000 (0.029)	
	1.00	0.999 (0.033)		0.999 (0.031)		0.999 (0.030)		0.998 (0.029)		0.998 (0.028)	
diag Σ	1.00	1.002 (0.060)		1.001 (0.057)		1.000 (0.054)		1.000 (0.051)		0.999 (0.049)	
	1.00	0.999 (0.059)		0.999 (0.055)		0.999 (0.052)		1.000 (0.049)		1.000 (0.046)	
	1.00	1.000 (0.062)		1.000 (0.058)		1.000 (0.054)		1.000 (0.051)		1.000 (0.047)	
n = 10, k = 2											
α	True Values	1.500		1.600		1.700		1.800		1.900	
		1.501 (0.015)		1.601 (0.016)		1.701 (0.015)		1.801 (0.014)		1.901 (0.012)	
B	1.00	1.002 (0.032)		1.002 (0.030)		1.002 (0.028)		1.002 (0.027)		1.002 (0.025)	
	1.00 1.00	1.000 (0.039)	1.000 (0.036)	1.001 (0.038)	1.000 (0.035)	1.001 (0.037)	1.000 (0.034)	1.001 (0.036)	1.000 (0.033)	1.001 (0.034)	1.000 (0.032)
	1.00 -1.00	1.001 (0.039)	-0.999 (0.037)	1.001 (0.038)	-1.000 (0.036)	1.001 (0.037)	-1.000 (0.034)	1.001 (0.037)	-1.000 (0.033)	1.001 (0.037)	-1.000 (0.032)
	1.00 1.00	1.000 (0.039)	1.001 (0.037)	1.001 (0.037)	1.001 (0.035)	1.001 (0.036)	1.001 (0.034)	1.001 (0.036)	1.001 (0.033)	1.001 (0.036)	1.001 (0.032)
	1.00 -1.00	1.000 (0.039)	-0.999 (0.036)	1.000 (0.039)	-0.999 (0.035)	1.000 (0.038)	-0.999 (0.034)	1.000 (0.038)	-1.000 (0.033)	1.000 (0.037)	-1.000 (0.032)
	1.00 1.00	1.000 (0.040)	0.999 (0.037)	1.000 (0.038)	0.999 (0.035)	1.000 (0.037)	0.999 (0.034)	0.999 (0.036)	1.000 (0.033)	0.999 (0.036)	1.000 (0.032)
	1.00 -1.00	1.000 (0.039)	-1.000 (0.036)	0.999 (0.038)	-1.000 (0.034)	0.999 (0.037)	-1.000 (0.033)	0.999 (0.037)	-1.000 (0.032)	1.000 (0.037)	-1.000 (0.031)
	1.00 1.00	1.001 (0.040)	1.000 (0.037)	1.001 (0.038)	1.000 (0.035)	1.001 (0.037)	1.000 (0.034)	1.001 (0.037)	1.000 (0.033)	1.002 (0.036)	1.000 (0.032)
	1.00 -1.00	1.000 (0.039)	-1.000 (0.036)	0.999 (0.038)	-1.000 (0.034)	0.999 (0.037)	-1.000 (0.033)	0.999 (0.037)	-1.000 (0.032)	0.999 (0.037)	-1.000 (0.031)
	1.00 1.00	1.001 (0.040)	1.000 (0.037)	1.001 (0.038)	1.000 (0.035)	1.001 (0.037)	1.000 (0.034)	1.001 (0.037)	1.000 (0.033)	1.002 (0.036)	1.000 (0.032)
	1.00 -1.00	1.000 (0.039)	-1.000 (0.036)	0.999 (0.038)	-1.000 (0.034)	0.999 (0.037)	-1.000 (0.033)	0.999 (0.037)	-1.000 (0.032)	0.999 (0.037)	-1.000 (0.031)
	1.00 1.00	1.001 (0.040)	0.999 (0.037)	1.001 (0.039)	0.999 (0.035)	1.001 (0.038)	0.999 (0.033)	1.001 (0.037)	0.999 (0.032)	1.001 (0.037)	0.999 (0.031)
diag Σ	1.00	1.001 (0.056)		1.000 (0.051)		1.000 (0.046)		1.000 (0.041)		1.001 (0.037)	
	1.00	1.001 (0.064)		1.001 (0.058)		1.000 (0.053)		1.000 (0.047)		1.000 (0.042)	
	1.00	1.003 (0.068)		1.003 (0.061)		1.002 (0.055)		1.001 (0.050)		1.001 (0.044)	
	1.00	1.001 (0.061)		1.001 (0.056)		1.001 (0.050)		1.000 (0.045)		1.000 (0.040)	
	1.00	1.002 (0.063)		1.002 (0.056)		1.001 (0.051)		1.001 (0.046)		1.000 (0.041)	
	1.00	1.002 (0.064)		1.001 (0.058)		1.001 (0.053)		1.001 (0.048)		1.001 (0.043)	
	1.00	1.003 (0.063)		1.002 (0.057)		1.001 (0.051)		1.000 (0.046)		0.999 (0.041)	
	1.00	1.002 (0.061)		1.002 (0.055)		1.001 (0.049)		1.001 (0.044)		1.001 (0.040)	
	1.00	1.003 (0.065)		1.002 (0.059)		1.001 (0.053)		1.001 (0.048)		1.000 (0.042)	
	1.00	1.006 (0.065)		1.006 (0.058)		1.005 (0.052)		1.004 (0.047)		1.004 (0.041)	

Table B.2: **Monte Carlo results for estimating the GARCHStabF model by Indirect Inference:** average estimates and standard errors (in parentheses) over $R = 1000$ Monte Carlo replications, based on $T = 3000$ observations, $n = 3$, $k = 1$ factors and $H = 100$ simulation paths.

n = 3, k=1						
		<i>1.750</i>	<i>1.800</i>	<i>1.850</i>	<i>1.900</i>	<i>1.950</i>
α	<i>True Values</i>	1.795 (0.041)	1.832 (0.035)	1.873 (0.034)	1.915 (0.031)	1.959 (0.024)
B	<i>1.00</i>	1.159 (0.673)	1.110 (0.183)	1.056 (0.114)	1.027 (0.087)	1.008 (0.093)
	<i>1.00</i>	1.156 (0.672)	1.110 (0.183)	1.055 (0.114)	1.027 (0.088)	1.008 (0.093)
	<i>1.00</i>	1.156 (0.672)	1.108 (0.182)	1.053 (0.114)	1.025 (0.088)	1.006 (0.092)
diag Σ	<i>1.00</i>	1.529 (1.733)	1.234 (0.495)	1.132 (0.318)	1.073 (0.217)	1.038 (0.148)
	<i>1.00</i>	1.662 (7.352)	1.255 (0.572)	1.136 (0.342)	1.074 (0.227)	1.035 (0.150)
	<i>1.00</i>	1.462 (1.009)	1.243 (0.516)	1.132 (0.316)	1.070 (0.213)	1.034 (0.142)
ρ	<i>0.93</i>	0.908 (0.123)	0.920 (0.062)	0.926 (0.041)	0.929 (0.028)	0.929 (0.039)
ϕ	<i>0.04</i>	0.044 (0.042)	0.042 (0.028)	0.040 (0.019)	0.039 (0.017)	0.039 (0.018)
ϱ	<i>0.93</i>	0.927 (0.030)	0.925 (0.021)	0.926 (0.015)	0.926 (0.013)	0.927 (0.014)
ς	<i>0.04</i>	0.044 (0.023)	0.044 (0.015)	0.042 (0.011)	0.041 (0.009)	0.041 (0.008)

Table B.3: Monte Carlo results for estimating the GARCHStabF model by Indirect Inference: average estimates and standard errors (in parentheses) over $R = 1000$ Monte Carlo replications, based on $T = 3000$ observations, $n = 3$, $k = 1$ and $H = 10$ simulation paths.

n = 10, k=2												
α	True Values		1.750		1.800		1.850		1.900		1.950	
			1.818 (0.046)		1.842 (0.037)		1.874 (0.033)		1.912 (0.028)		1.955 (0.021)	
B	1.00	.	1.156 (0.770)	.	1.102 (0.277)	.	1.052 (0.168)	.	1.024 (0.138)	.	1.016 (0.076)	.
	1.00	1.00	1.154 (0.770)	1.175 (0.295)	1.101 (0.280)	1.101 (0.144)	1.052 (0.170)	1.050 (0.110)	1.024 (0.139)	1.025 (0.087)	1.016 (0.079)	1.010 (0.071)
	1.00	-1.00	1.155 (0.769)	-1.177 (0.294)	1.102 (0.279)	-1.103 (0.144)	1.053 (0.169)	-1.052 (0.110)	1.025 (0.139)	-1.028 (0.086)	1.017 (0.079)	-1.013 (0.070)
	1.00	1.00	1.155 (0.768)	1.177 (0.296)	1.101 (0.279)	1.103 (0.144)	1.052 (0.169)	1.052 (0.110)	1.024 (0.138)	1.026 (0.086)	1.016 (0.079)	1.011 (0.070)
	1.00	-1.00	1.154 (0.769)	-1.177 (0.294)	1.101 (0.277)	-1.103 (0.144)	1.051 (0.169)	-1.052 (0.111)	1.023 (0.139)	-1.027 (0.087)	1.016 (0.079)	-1.013 (0.071)
	1.00	1.00	1.153 (0.767)	1.176 (0.295)	1.100 (0.278)	1.102 (0.144)	1.050 (0.169)	1.051 (0.110)	1.022 (0.139)	1.026 (0.086)	1.015 (0.079)	1.012 (0.070)
	1.00	-1.00	1.154 (0.771)	-1.177 (0.295)	1.101 (0.278)	-1.103 (0.144)	1.050 (0.171)	-1.052 (0.111)	1.022 (0.141)	-1.027 (0.087)	1.014 (0.082)	-1.013 (0.071)
	1.00	1.00	1.154 (0.770)	1.174 (0.295)	1.101 (0.279)	1.101 (0.146)	1.051 (0.171)	1.050 (0.111)	1.023 (0.141)	1.025 (0.086)	1.015 (0.082)	1.010 (0.070)
	1.00	-1.00	1.154 (0.770)	-1.176 (0.294)	1.101 (0.278)	-1.102 (0.144)	1.051 (0.169)	-1.051 (0.110)	1.022 (0.141)	-1.027 (0.086)	1.014 (0.081)	-1.012 (0.071)
	1.00	1.00	1.153 (0.770)	1.175 (0.296)	1.100 (0.280)	1.101 (0.146)	1.050 (0.171)	1.050 (0.111)	1.022 (0.141)	1.025 (0.087)	1.013 (0.081)	1.011 (0.071)
diag Σ	1.00		1.617 (2.310)		1.265 (0.906)		1.138 (0.680)		1.066 (0.313)		1.029 (0.166)	
	1.00		1.563 (2.022)		1.263 (1.372)		1.131 (0.879)		1.053 (0.334)		1.018 (0.182)	
	1.00		1.592 (1.669)		1.246 (0.659)		1.115 (0.381)		1.056 (0.252)		1.023 (0.163)	
	1.00		1.538 (1.748)		1.246 (0.883)		1.122 (0.627)		1.057 (0.272)		1.026 (0.172)	
	1.00		1.659 (2.123)		1.319 (1.316)		1.162 (0.810)		1.078 (0.290)		1.037 (0.181)	
	1.00		1.500 (1.496)		1.225 (0.623)		1.105 (0.365)		1.051 (0.248)		1.021 (0.164)	
	1.00		1.578 (2.172)		1.293 (1.491)		1.124 (0.581)		1.056 (0.296)		1.024 (0.181)	
	1.00		1.591 (3.193)		1.227 (0.627)		1.107 (0.372)		1.052 (0.247)		1.023 (0.161)	
	1.00		1.535 (1.938)		1.233 (0.813)		1.106 (0.441)		1.051 (0.278)		1.021 (0.176)	
	1.00		1.641 (2.626)		1.280 (1.577)		1.120 (0.448)		1.057 (0.259)		1.025 (0.164)	
ρ_j	0.93	0.83	0.898 (0.083)	0.802 (0.077)	0.914 (0.038)	0.811 (0.047)	0.922 (0.027)	0.820 (0.036)	0.925 (0.023)	0.825 (0.032)	0.926 (0.021)	0.827 (0.033)
ϕ_j	0.04	0.10	0.065 (0.048)	0.130 (0.058)	0.053 (0.029)	0.119 (0.039)	0.045 (0.021)	0.109 (0.030)	0.043 (0.016)	0.104 (0.025)	0.041 (0.013)	0.101 (0.022)
ϱ	0.89		0.864 (0.041)		0.876 (0.021)		0.885 (0.015)		0.888 (0.011)		0.888 (0.011)	
φ	0.06		0.091 (0.035)		0.077 (0.019)		0.067 (0.012)		0.063 (0.008)		0.061 (0.006)	

Table B.4: **Monte Carlo results for estimating the StatStabF model by Indirect Inference**: median estimates and interquartile ranges (in parentheses) over $R = 1000$ Monte Carlo replications, based on $T = 3000$ observations, $n = 3$, $k = 1$ and $H = 100$ simulation paths.

n = 3, k=1						
α	<i>True Values</i>	1.750	1.800	1.850	1.900	1.950
		1.799 (0.058)	1.834 (0.049)	1.877 (0.044)	1.921 (0.037)	1.965 (0.026)
B	1.00	1.198 (0.317)	1.120 (0.186)	1.064 (0.131)	1.035 (0.106)	1.017 (0.086)
	1.00	1.192 (0.309)	1.119 (0.182)	1.064 (0.132)	1.035 (0.107)	1.014 (0.091)
	1.00	1.195 (0.308)	1.114 (0.178)	1.064 (0.131)	1.032 (0.107)	1.012 (0.086)
diag Σ	1.00	1.296 (0.898)	1.190 (0.565)	1.123 (0.358)	1.079 (0.254)	1.037 (0.173)
	1.00	1.283 (0.928)	1.195 (0.583)	1.114 (0.395)	1.070 (0.266)	1.039 (0.179)
	1.00	1.314 (0.948)	1.190 (0.610)	1.126 (0.403)	1.068 (0.277)	1.030 (0.178)
ρ	0.93	0.928 (0.055)	0.928 (0.043)	0.929 (0.035)	0.930 (0.031)	0.931 (0.036)
ϕ	0.04	0.034 (0.033)	0.036 (0.027)	0.037 (0.024)	0.038 (0.023)	0.038 (0.022)
ϱ	0.93	0.929 (0.023)	0.927 (0.022)	0.927 (0.018)	0.928 (0.016)	0.927 (0.017)
φ	0.04	0.039 (0.015)	0.040 (0.015)	0.041 (0.013)	0.041 (0.011)	0.040 (0.008)

Table B.5: Monte Carlo results for estimating the GARCHStabF model by Indirect Inference: median estimates and interquartile ranges (in parentheses) $R = 1000$ Monte Carlo replications, based on $T = 3000$ observations, $n = 10$, $k = 2$ and $H = 10$ simulation paths.

n = 10, k=2											
α	True Values	1.750		1.800		1.850		1.900		1.950	
		1.821 (0.062)		1.846 (0.046)		1.879 (0.041)		1.917 (0.034)		1.959 (0.024)	
B	1.00 .	1.187 (0.361)	.	1.108 (0.205)	.	1.050 (0.143)	.	1.022 (0.115)	.	1.006 (0.089)	.
	1.00 1.00	1.186 (0.360)	1.180 (0.251)	1.105 (0.213)	1.099 (0.162)	1.048 (0.146)	1.049 (0.128)	1.025 (0.116)	1.022 (0.107)	1.009 (0.098)	1.008 (0.093)
	1.00 -1.00	1.189 (0.371)	-1.179 (0.254)	1.103 (0.213)	-1.098 (0.168)	1.050 (0.150)	-1.049 (0.130)	1.024 (0.115)	-1.023 (0.109)	1.011 (0.093)	-1.009 (0.093)
	1.00 1.00	1.181 (0.351)	1.181 (0.257)	1.105 (0.207)	1.095 (0.162)	1.048 (0.145)	1.050 (0.127)	1.024 (0.113)	1.024 (0.108)	1.009 (0.095)	1.008 (0.091)
	1.00 -1.00	1.187 (0.357)	-1.182 (0.248)	1.105 (0.204)	-1.097 (0.158)	1.050 (0.147)	-1.048 (0.129)	1.023 (0.114)	-1.025 (0.111)	1.010 (0.094)	-1.010 (0.090)
	1.00 1.00	1.184 (0.361)	1.179 (0.247)	1.103 (0.211)	1.098 (0.159)	1.046 (0.148)	1.050 (0.125)	1.021 (0.115)	1.024 (0.110)	1.009 (0.095)	1.008 (0.093)
	1.00 -1.00	1.185 (0.367)	-1.176 (0.248)	1.106 (0.217)	-1.092 (0.168)	1.045 (0.154)	-1.046 (0.130)	1.019 (0.125)	-1.024 (0.112)	1.008 (0.107)	-1.010 (0.089)
	1.00 1.00	1.182 (0.366)	1.180 (0.250)	1.104 (0.213)	1.099 (0.163)	1.044 (0.154)	1.048 (0.126)	1.020 (0.125)	1.024 (0.107)	1.009 (0.105)	1.010 (0.093)
	1.00 -1.00	1.184 (0.360)	-1.182 (0.256)	1.105 (0.217)	-1.095 (0.160)	1.049 (0.151)	-1.048 (0.125)	1.020 (0.123)	-1.023 (0.109)	1.008 (0.102)	-1.010 (0.093)
	1.00 1.00	1.183 (0.370)	1.175 (0.254)	1.108 (0.221)	1.097 (0.167)	1.046 (0.157)	1.048 (0.128)	1.020 (0.120)	1.022 (0.109)	1.006 (0.102)	1.007 (0.089)
diag Σ	1.00	1.185 (0.822)		1.089 (0.521)		1.045 (0.360)		1.024 (0.255)		1.008 (0.174)	
	1.00	1.154 (0.874)		1.074 (0.557)		1.026 (0.377)		1.008 (0.262)		1.001 (0.175)	
	1.00	1.209 (1.028)		1.104 (0.595)		1.042 (0.395)		1.016 (0.280)		1.003 (0.187)	
	1.00	1.184 (0.948)		1.101 (0.588)		1.054 (0.406)		1.026 (0.276)		1.014 (0.183)	
	1.00	1.234 (1.006)		1.119 (0.644)		1.065 (0.447)		1.029 (0.319)		1.018 (0.201)	
	1.00	1.187 (0.968)		1.112 (0.632)		1.055 (0.445)		1.025 (0.291)		1.008 (0.187)	
	1.00	1.200 (0.961)		1.083 (0.597)		1.039 (0.418)		1.021 (0.287)		1.008 (0.191)	
	1.00	1.169 (0.954)		1.090 (0.585)		1.042 (0.403)		1.016 (0.280)		1.007 (0.187)	
	1.00	1.171 (0.929)		1.078 (0.572)		1.034 (0.411)		1.011 (0.284)		1.000 (0.186)	
	1.00	1.203 (0.987)		1.089 (0.597)		1.036 (0.391)		1.013 (0.273)		1.000 (0.185)	
ρ_j ϕ_j ϱ φ	0.93 0.83	0.900 (0.083)	0.792 (0.072)	0.915 (0.053)	0.809 (0.055)	0.924 (0.035)	0.819 (0.048)	0.927 (0.028)	0.825 (0.042)	0.928 (0.026)	0.829 (0.042)
	0.04 0.10	0.059 (0.071)	0.135 (0.072)	0.049 (0.044)	0.118 (0.050)	0.042 (0.029)	0.108 (0.042)	0.040 (0.021)	0.104 (0.036)	0.040 (0.017)	0.100 (0.031)
	0.89	0.856 (0.037)		0.874 (0.023)		0.884 (0.017)		0.887 (0.014)		0.888 (0.013)	
	0.06	0.093 (0.039)		0.076 (0.022)		0.067 (0.013)		0.063 (0.010)		0.061 (0.008)	

Table B.6: **Descriptive statistics of daily log-returns of the DJIA 30 stocks over the period 01.11.2001 – 30.09.2014** ($T = 3220$ trading days.) The log-returns are scaled up by 100.

Stock	<i>Mean</i>	<i>Median</i>	<i>Max</i>	<i>Min</i>	<i>Std. dev.</i>	<i>Skewness</i>	<i>Kurtosis</i>
AA	-0.1402	-0.1155	12.2905	-17.4431	2.1527	-0.6293	9.6854
AXP	0.0590	0.0415	13.8701	-13.0152	1.9864	0.2313	10.8759
BA	0.0056	0.0084	9.3855	-8.3093	1.5242	-0.0796	5.9745
BAC	-0.1077	-0.0214	19.2832	-24.3056	2.6600	-0.5859	21.4908
C	-0.2425	-0.1271	18.6727	-35.7600	2.8117	-2.0413	31.2911
CAT	-0.0154	-0.0220	10.9624	-10.5045	1.6677	-0.0566	6.6437
CVX	0.0132	0.0687	15.7511	-12.4858	1.3385	0.1944	16.6868
DD	0.0074	0.0200	9.8414	-10.3325	1.4668	-0.0051	9.3334
DIS	0.0767	0.0650	12.8361	-8.7905	1.5469	0.3394	8.5365
GE	-0.0724	-0.0547	11.6140	-11.2244	1.6086	-0.0879	12.3934
GS	0.0117	0.0379	17.3191	-16.7123	1.9905	-0.2022	12.8810
HD	0.0263	0.0041	11.2180	-7.6629	1.5958	0.5936	8.5457
HON	-0.0129	0.0266	10.9887	-10.0511	1.5318	-0.1542	7.7975
HPQ	0.1263	0.1270	15.8368	-14.3459	1.7783	0.2924	8.7015
IBM	0.0850	0.0890	6.3758	-6.6163	1.2021	-0.1150	6.7391
IP	-0.0508	0.0000	15.9791	-18.9470	2.0241	-0.0891	13.2514
JNJ	0.0219	0.0159	7.7170	-8.1543	0.9475	0.1872	10.2239
JPM	0.0052	0.0246	25.2835	-18.4265	2.2443	0.6163	18.6156
KO	0.0390	0.0461	8.0908	-7.1326	1.0312	0.0852	9.2492
MCD	0.0461	0.0427	10.3459	-11.3253	1.2327	-0.0315	10.0482
MMM	0.0183	0.0454	7.9747	-7.5694	1.1645	-0.1642	8.4666
MO	0.0064	0.0215	9.0816	-14.2097	1.2180	-0.8070	17.1443
MRK	0.0206	0.0196	9.7528	-11.1301	1.3961	-0.0857	10.6619
NKE	0.0699	0.0445	9.8223	-11.0729	1.4631	0.3197	8.3450
PFE	-0.0318	-0.0461	10.3769	-6.7051	1.2927	0.2863	7.5912
PG	0.0785	0.0720	7.5389	-6.9419	0.9418	-0.0247	8.9899
UTX	0.0104	0.0104	9.6287	-7.9920	1.3278	0.2187	8.7912
VZ	-0.0158	-0.0227	11.9859	-7.6583	1.3377	0.5115	9.2847
WMT	0.0145	0.0000	7.8757	-6.4151	1.0954	0.3516	8.3233
XOM	0.0496	0.0797	12.0999	-12.4545	1.3098	0.0524	14.4331

Table B.7: **Estimation results for the StatStabF and GARCHStabF model by Indirect Inference:** estimates and standard errors (in parentheses) for $n = 10$ series of daily returns over the period 01.11.2001 – 30.09.2014 ($T = 3220$ trading days), with $k = 1$ and $k = 2$ and $H = 100$ simulation paths.

Model	StatStabF			GARCHStabF		
Parameters	k=1	k=2		k=1	k=2	
α	1.597 (0.015)	1.612 (0.015)		1.834 (0.056)	1.860 (0.053)	
B	0.582 (0.213)	0.606 (0.020)	.	0.409 (0.065)	0.481 (0.068)	.
	0.299 (0.012)	0.380 (0.014)	0.216 (0.015)	0.210 (0.034)	0.300 (0.043)	0.179 (0.032)
	0.407 (0.017)	0.516 (0.020)	0.303 (0.021)	0.286 (0.045)	0.409 (0.059)	0.253 (0.044)
	0.451 (0.017)	0.558 (0.020)	0.282 (0.021)	0.316 (0.051)	0.443 (0.063)	0.237 (0.041)
	0.290 (0.012)	0.339 (0.013)	0.133 (0.014)	0.204 (0.033)	0.269 (0.039)	0.111 (0.021)
	0.830 (0.024)	0.819 (0.025)	-0.167 (0.026)	0.583 (0.092)	0.651 (0.092)	-0.139 (0.031)
	1.032 (0.029)	0.982 (0.031)	-0.481 (0.031)	0.726 (0.115)	0.780 (0.111)	-0.402 (0.069)
	1.079 (0.029)	1.029 (0.031)	-0.453 (0.032)	0.758 (0.120)	0.816 (0.115)	-0.377 (0.066)
	0.920 (0.026)	0.878 (0.027)	-0.306 (0.027)	0.646 (0.102)	0.698 (0.100)	-0.254 (0.047)
	1.023 (0.027)	0.980 (0.029)	-0.346 (0.028)	0.719 (0.113)	0.778 (0.110)	-0.286 (0.053)
$diag \Sigma$	0.550 (0.026)	0.547 (0.026)		0.372 (0.127)	0.400 (0.136)	
	0.310 (0.015)	0.212 (0.012)		0.228 (0.071)	0.152 (0.057)	
	0.661 (0.031)	0.466 (0.025)		0.475 (0.140)	0.312 (0.117)	
	0.555 (0.026)	0.370 (0.022)		0.378 (0.129)	0.270 (0.101)	
	0.330 (0.015)	0.290 (0.014)		0.244 (0.078)	0.210 (0.070)	
	0.580 (0.029)	0.607 (0.028)		0.423 (0.139)	0.430 (0.146)	
	0.556 (0.030)	0.476 (0.029)		0.396 (0.143)	0.343 (0.124)	
	0.550 (0.030)	0.503 (0.029)		0.399 (0.147)	0.374 (0.138)	
	0.631 (0.032)	0.637 (0.032)		0.468 (0.151)	0.446 (0.158)	
	0.369 (0.021)	0.373 (0.021)		0.264 (0.095)	0.272 (0.099)	
				0.926 (0.025)	0.913 (0.027)	0.887 (0.046)
				0.045 (0.023)	0.054 (0.025)	0.064 (0.036)
				0.914 (0.024)	0.897 (0.020)	
				0.042 (0.016)	0.059 (0.016)	
ρ_j						
ϕ_j						
ϱ						
φ						

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