Debt Relief for Poor Countries: Conditionality and Effectiveness

Almuth Scholl

Working Paper Series
2013-23

http://www.wiwi.uni-konstanz.de/econdoc/working-paper-series/
Debt Relief for Poor Countries:  
Conditionality and Effectiveness

Almuth Scholl*
University of Konstanz

Abstract

This paper studies the effectiveness of debt relief to stimulate economic growth in the most heavily indebted poor countries. We develop a neoclassical framework with a conflict of interest between the altruistic donor and the recipient government and model conditionality as an imperfectly enforceable dynamic contract. Our findings suggest that incentive-compatible conditions substantially promote fiscal reform and investment in the short- and long-run. In contrast to the recent practice of fully canceling multilateral debt, optimal debt relief is characterized by a combination of outright grants and subsidized loans. Losing loans as a policy instrument reduces welfare considerably.

Keywords: neoclassical growth, conditionality, limited enforceable dynamic contracts, foreign aid

JEL: O11, F34, F35

*This is a substantially revised version of a preliminary draft circulated as ‘Debt Relief and Incentive-Compatible Conditionality’. This research was supported by the junior professor program of the state Baden-Württemberg. Helpful comments by Matthias Döpke, Fabian Fink and seminar participants at University of Stockholm, Bonn, Karlsruhe Institute of Technology, ESEM 2011, CEF 2010 conference, EEA meetings 2010 and the German Economic Association Meeting 2010 are gratefully acknowledged. All errors are mine. Address: University of Konstanz, Department of Economics, PO Box 132, D-78457 Konstanz, Germany, Tel: +49(0)7531-88-3615, Fax: +49(0)7531-88-4101, Almuth.Scholl@uni-konstanz.de.
1 Introduction

The most heavily indebted poor countries in the world, the HIPCs, have been suffering from low income levels, stagnating economic growth and high external public debt for many years. In 1996, motivated by the concern to stimulate growth and reduce poverty, the International Monetary Fund and the World Bank founded the Initiative for Heavily Indebted Poor Countries that was supplemented by the Multilateral Debt Relief Initiative in 2005. The objective of the initiative is to provide substantial debt cancelations so that the recipient governments have free resources left to finance efficient growth-enhancing economic policies. In 2007, total costs of debt relief were estimated at $ 71 billions (IMF Factsheet, 2010).

In their seminal contributions, Krugman (1988) and Sachs (1989) show that debt relief may facilitate new lending, investment and growth if a country suffers from a debt overhang. However, as argued by Arslanalp and Henry (2004, 2006) and Eaton (1990), it is questionable whether the HIPCs are characterized by a debt overhang since they have continuously received positive net loans on concessional terms and face debt obligations to official creditors rather than commercial banks. To emphasize this point, figure 1 shows the components of public external debt as well as the components of official development assistance (ODA). Since positive ODA net loans have replaced market debt by official concessional debt, Easterly (2002) argues that debt relief has been implicitly granted to the HIPCs over the past three decades. The concern about the unsustainability of external debt burdens motivated the recommendation made by the Meltzer (2000) Commission that development assistance should be provided through outright grants rather than concessional loans.

This paper studies the question how debt relief should be provided to be an effective instrument to stimulate economic growth in the most heavily indebted poor countries. Our theoretical framework builds on the macroeconomic literature on economic development and takes into account that the main economic problem of the HIPCs is the lack of functional economic institutions rather than debt overhang. Instead of implementing efficient policies, the recipient government may divert transfers from their intended use and follow poor or wasteful economic policies. To prevent this from happening, it has become a common policy to impose conditionality on the provision of debt relief. As the sovereign recipient government may not be willing to keep the conditions, conditionality can be

---

1 Arslanalp and Henry (2005) provide empirical evidence that the debt reduction agreements under the ‘Brady Plan’ in the 1980s generated efficiency gains and stimulated investment in middle-income developing countries.
2 See also Arslanalp and Henry (2004, 2006).
3 To be considered for the HIPC Initiative countries must meet certain criteria, e.g., countries must have developed a Poverty Reduction Strategy Paper.
interpreted as an imperfectly enforceable contract between the donor and the recipient government, see e.g. Besley and Persson (2011), Cordella, Dell’Ariccia and Kletzer (2003), Kletzer (2005) and Cordella and Dell’Ariccia (2007).

Building on the recent literature on limited enforceable contracts in macroeconomic settings, this paper analyzes the dynamic properties of optimal debt relief and incentive-compatible conditionality that induce the recipient government to cooperate in the short- and long-run. We focus on the role of the concessionality level as a policy instrument to affect the recipient’s incentives to keep the conditions and differentiate between implicit and explicit debt relief. Implicit debt relief refers to loans at subsidized rates that replace market debt while explicit debt relief refers to outright grants that cancel debt. We stress the role of private international credit markets and analyze how the access to market debt influences the recipient’s incentives to fulfill conditionality.

We develop a neoclassical growth model of a small open economy in which the recipient government finances non-productive government consumption by issuing foreign debt and raising taxes. To reflect the risk of sovereign default on non-concessional debt and to highlight the role of international financial markets, we follow Chatterjee et al. (2003) and assume that market interest rates are increasing in the debt-to-capital ratio and, therefore, limit the access to international credit markets.

The donor provides costly debt relief and cares solely about the welfare of the households. In contrast, the recipient government also values non-productive government consumption and discounts the future at a higher rate, which can be interpreted as a short-hand for political economy factors that lead to overspending and debt accumulation. The conflict of interest between the donor and the recipient government raises the issue of conditionality: the donor ties the provision of debt relief to specific fiscal policy conditions that coincide with the donor’s intention. To ensure that conditionality is fulfilled, the donor threatens with aid sanctions if the recipient government does not implement the appropriate fiscal policies. The optimal design of debt relief is characterized by two policy instruments: the size of transfers and the degree of concessionality. As a benchmark, we assume that the donor is able to enforce the repayment of concessional loans, i.e., a failure on conditionality does not come along with a default on concessional debt. In a second step, we discuss the effect of relaxing this assumption.

---


5We abstract from default in equilibrium and endogenous interest rates as analyzed in e.g. Arellano (2008). Fink and Scholl (2011) analyze the impact of conditionality on sovereign risk but do not explicitly model the donor’s behavior, see also references therein.
The complex dynamic structure of the model requires numerical simulations to analyze the short and long-run properties of optimal incentive-compatible policies. Building on the methodological approach proposed by Marcet and Marimon (2011), our findings suggest that self-enforcing conditionality substantially promotes fiscal reform and stimulates investment and growth. In contrast to the recent practice of canceling 100 percent of multilateral debt under the Multilateral Debt Relief Initiative, optimal conditionality is accompanied by a degree of concessionality that implies a combination of subsidized loans and outright grants. The dynamic patterns of self-enforcing conditionality and the optimal concessionality level crucially depends on the recipient’s access to international financial markets and on the strength of the conflict of interest between the donor and the recipient. Since high market interest rates limit the access to private international credit, losing the access to conditional subsidized loans is a severe threat, and a low concessionality level is required to ensure the enforceability of the conditionality contract. Similarly, a severe conflict of interest implies that the recipient government has high incentives to dishonor the conditions. Consequently, the donor needs to make the contract more attractive by raising the degree of concessionality. If official creditors are not able to enforce concessional loans so that a failure on conditionality goes hand in hand with a default on official debt, it is optimal to provide outright grants only. Losing loans as a policy instrument reduces the effectiveness of one unit of assistance and generates a considerable welfare loss.

Our paper is related to Cordella et al. (2003) who analyze debt relief and conditionality in a stylized infinite horizon endowment economy. They show analytically that regaining access to international credit markets decreases the recipient’s incentives to cooperate. However, Cordella et al. (2003) abstract from fiscal policy and economic growth as well as the interaction of concessional loans and outright grants. Scholl (2009) contributes to this literature by analyzing the impact of incentive-compatible conditional aid on fiscal policy reform and investment in a neoclassical growth framework. However, Scholl (2009) considers a closed economy setup and abstracts from international financial markets and, therefore, does not analyze the issue of debt relief. This paper extends Cordella et al. (2003) and Scholl (2009) to provide a novel analysis of the dynamic impact of optimal incentive-compatible conditional debt relief on fiscal policy reform and growth highlighting the important role of the recipient’s access to private international credit markets and the interaction of market debt and subsidized loans. Since our framework allows us to derive the optimal degree of concessionality, we contribute to the discussion whether outright grants should be preferred to concessional loans, see

6 In similar setups, Cordella and Dell’Ariccia (2007) contrast conditional budget support and project aid while Kletzer (2005) focuses on the credibility of aid sanctions.
e.g., Bulow and Rogoff (2005), Cordella and Ulku (2004) and Cohen, Jacquet and Reisen (2007). Our paper is related to recent studies by Murshed and Sen (1995), Svensson (2000, 2003), Pedersen (1996, 2001), Azam and Laffont (2003) and Hagen (2006) who use static or two-period game-theoretic models to analyze incentive-compatibility, moral hazard and informational problems in the context of foreign aid. Moreover, the paper is connected to the literature on aid fungibility, e.g., Hagen (2006) and Pack and Pack (1990, 1993). Our neoclassical theoretical framework builds on the literature that studies the link between foreign aid and economic growth, e.g., Chenery and Strout (1966), Boone (1996), Chatterjee, Sakoulis and Turnovsky (2003) and Chatterjee and Turnovsky (2007). These studies, however, abstract from incentive compatibility issues and take aid as exogenously given. Our paper is related to Aguiar and Amador (2011) who develop a political economy model of sovereign debt and show that unconditional aid and debt relief have no long-run effects. However, since their focus is the analysis of debt default, they do not study the issue of optimal conditional aid.

The paper is structured as follows. In section 2 we develop a neoclassical framework of a small open economy with a conflict of interest between the donor and the recipient government. In section 3 we analyze the quantitative properties of incentive-compatible conditional debt relief by studying transition paths and long-run properties. Finally, section 4 concludes.

2 The Model

2.1 The Environment

In the following, we consider a small open developing economy that is inhabited by a large number of infinitely-lived households who maximize lifetime utility. Preferences of the representative household are given by

$$\sum_{t=0}^{\infty} \beta_p^t u(c_t), \quad 0 < \beta_p < 1, \quad (1)$$

where $c_t$ denotes household consumption at time $t$. The utility function $u(c_t)$ satisfies $u_c(c_t) > 0$ and $u_{cc}(c_t) < 0$. $\beta_p$ denotes the private discount factor.

The household produces the consumption good and saves by investing in the capital stock $k_t$. The household’s budget constraint is described by

$$c_t + k_t = (1 - \tau_t)y_t + (1 - \delta)k_{t-1}, \quad (2)$$

The capital stock depreciates at rate $0 \leq \delta < 1$. $y_t$ denotes production at time $t$ and $\tau_t$ is the income
tax raised by the government. More broadly, one may interpret $\tau_t$ as the share of income that is lost due to inefficient economic policies.

The household produces $y_t$ by employing the production function

$$y_t = F(k_{t-1}, n_t).$$

The production function has constant returns to scale in capital $k_t$ and labor $n_t$. In the following, we normalize labor $n_t \equiv 1$, for all $t$, such that $F(k_{t-1}, 1) \equiv f(k_{t-1})$.

Preferences of the government are given by

$$\sum_{t=0}^{\infty} \beta_g^t v(c_t, g_t), \quad 0 < \beta_g < 1,$$

where the utility function $v$ satisfies $v_c(c_t, g_t) > 0$, $v_{cc}(c_t, g_t) < 0$, $v_g(c_t, g_t) > 0$ and $v_{gg}(c_t, g_t) < 0$.

We label unproductive government consumption by $g_t$ and interpret it as e.g. expenditures supporting the political elite (see also Cordella and Dell’Arriccia, 2007, and Scholl, 2009). Importantly, we allow that the government discounts the future at a different rate than the public, $\beta_g \leq \beta_p$. The higher discount rate can be interpreted as short-hand for political economy factors that lead to overspending and debt accumulation, see Easterly (2002) and Aguiar and Amador (2011).

The recipient government finances non-productive government consumption $g_t$ by raising the income tax $\tau_t$ and issuing foreign debt $d_t \geq 0$ at the market interest rate $r_{t+1}$ and by receiving transfers $\ell_t \geq 0$ from the donor. The government’s budget constraint reads as

$$g_t + (1 + r_t)d_{t-1} + (1 + r_t - q_t)\ell_{t-1} = \tau_t f(k_{t-1}) + d_t + \ell_t,$$

where $q_{t+1} \geq 0$ determines the concessionality level of the transfers $\ell_t$. If $q_{t+1} = 0$, then the donor provides non-concessional loans at the market interest rate. If $q_{t+1} = r_{t+1}$ the donor provides loans without interest. If $q_{t+1} = 1 + r_{t+1}$, transfers $\ell_t$ are given as an outright grant.\footnote{See also Cordella and Ulku (2004) who formulate the transfer scheme in a similar way.}

We follow e.g. Chatterjee et al. (2003) and assume that the market interest rate takes the functional form $r_t = \Phi \left( \frac{d_{t-1}}{k_{t-1}} \right)$ with $\Phi(0) = r^*$ and $\Phi' \left( \frac{d_{t-1}}{k_{t-1}} \right) > 0$. Thus, the market rate is at least as large as the world interest rate $r^*$ and is assumed to be strictly increasing in the debt to capital ratio reflecting the risk of sovereign default. The country risk premium is given by $r_{t+1} - r^* \geq 0$.

We assume that there is a representative altruistic donor who cares about the welfare of the households and provides costly assistance by choosing $\ell_t$ and $q_{t+1}$.\footnote{In reality, there are many other reasons for giving aid. By assuming an altruistic donor we take the most optimistic view on the effectiveness of development assistance.} The donor’s preferences take the following...
form
\[ \sum_{t=0}^{\infty} \beta^t [u(c_t) - h(q_t \ell_{t-1})]. \]  

The cost function \( h(q_t \ell_{t-1}) \) satisfies \( h(q_t \ell_{t-1}) > 0 \) if \( \ell_{t-1} \) or \( q_t \) are strictly greater than zero, and \( h(q_t \ell_{t-1}) = 0 \) if \( q_t \ell_{t-1} = 0 \). Moreover, \( h_q(q_t \ell_{t-1}) > 0, h_\ell(q_t \ell_{t-1}) > 0, h_{qq}(q_t \ell_{t-1}) \geq 0, h_{\ell\ell}(q_t \ell_{t-1}) \geq 0 \).

2.2 Conditionality as Self-Enforcing Contract

Since the recipient government discounts the future at a higher rate than the donor and, in addition, finances unproductive government consumption, there is a conflict of interest between the donor and the recipient government. The recipient government may use transfers to implement policies that do not coincide with the donor’s intention. To prevent the government from doing so, the donor imposes conditions on the provision of assistance. However, the recipient government may not be willing to fulfill these conditions.

We follow Cordella et al. (2003) and define conditionality as a dynamic contract between the donor and the recipient country. The donor provides debt relief and, in return, ties the provision of financial assistance to specific economic conditions on tax and debt policies. However, the contract is imperfectly enforceable since the sovereign recipient government can always dishonor the conditions and implement ineffective economic policies that are not in line with the donor’s preferences. We define the contract to be self-enforcing, if, at any point in time, the conditions are supportable by the threat of a permanent cutoff from any form of assistance. In addition, we assume that the donor is able to enforce the repayment of concessional loans. We impose these two assumptions to consider a severe punishment threat so that the associated self-enforceable conditionality generates a benchmark on the effectiveness of debt relief. In section 3.5 we take into account that official creditors may not be able to enforce the repayment of concessional loans and analyze the optimal design of debt relief and conditionality in such a scenario.

The value of the outside option is characterized by the optimal policy decisions of the recipient government taking as given the optimal consumption and investment choices of the household. The household’s optimality conditions are given by the usual Euler equation that connects the marginal rate of substitution between consumption today and tomorrow with the rate of return on capital
\[ u_c(c_t) = \beta_p u_c(c_{t+1})[1 - \delta + (1 - \tau_{t+1}) f_k(k_t)] \]  

(7)

together with the budget constraints of the household and the government, equations (2) and (5).
The value of the outside option $D(k_{t-1}, d_{t-1}, \ell_{t-1}, q_t)$ is determined by the solution to the following maximization problem, given $\ell_t = 0$, for all $t \geq 0$:

$$\max \left\{ v_{c_t, g_t, k_t, d_t, \tau_t} \right\} \sum_{t=0}^{\infty} \beta_t^t v(c_t, g_t)$$

subject to

$$\text{Equations (2), (5) and (7)}$$

given $k_{-1} > 0$, $d_{-1} \geq 0$, $\ell_{-1} \geq 0$, and $q_0 \geq 0$. $\ell_{-1} \geq 0$ reflects the repayment obligations of official loans that have been provided by the donor in the past. The optimality conditions associated to (8) are given in the appendix.

The donor designs a contract that offers a transfer scheme $\{\ell_t, q_{t+1}\}_{t=0}^{\infty}$ and, in return, expects the recipient government to implement tax and debt policies $\{\tau_t, d_t\}_{t=0}^{\infty}$ that are associated with the allocation $\{c_t, g_t, k_t\}_{t=0}^{\infty}$. The optimal self-enforcing contract is given by the solution to the following maximization problem:

$$\max \left\{ v_{c_t, g_t, k_t, d_t, \tau_t, \ell_t, q_t+1} \right\} \sum_{t=0}^{\infty} \beta_t^t [u(c_t) - h(q_{t+1})]$$

subject to

$$\sum_{j=0}^{\infty} \beta_t^j v(c_{t+j}, g_{t+j}) \geq D(k_{t-1}, d_{t-1}, \ell_{t-1}, q_t)$$

$$\text{Equations (2), (5) and (7)}$$

given $k_{-1} > 0$, $d_{-1} \geq 0$ and $\ell_{-1} \geq 0$. Equation (10) is the enforcement constraint of the contract: the contract is self-enforcing as long as the value of fulfilling the conditions, $\sum_{j=0}^{\infty} \beta_t^j v(c_{t+j}, g_{t+j})$, is at least as large as the value of defecting, $D(k_{t-1}, d_{t-1}, \ell_{t-1}, q_t)$.

As the enforcement constraint (10) contains future realizations of the decision variables, we follow Marcet and Marimon (2011, 1992) and introduce an additional co-state variable $\mu_t$ to solve the donor’s maximization problem. Let $\gamma_t \geq 0$ be the Lagrange-multiplier on the enforcement constraint (10). In the appendix it is shown that the donor’s maximization problem (9) can be transformed into the following saddle-point formulation:

---

9This method has been frequently applied in the macroeconomic literature on limited commitment, see e.g. Kehoe and Perri (2002) and Cooley, Marimon and Quadrini (2004).
The optimal contract is characterized by the following first order conditions:

\[ \zeta_t = \beta_p \left[ \lambda_t u_c(c_{t+1})(1 - \tau_{t+1})f_{k,k}(k_t) + \zeta_{t+1}((1 - \tau_{t+1})f_{k}(k_t) + 1 - \delta) 
+ \mu_{t+1}v_g(c_{t+1}, g_{t+1})\tau_{t+1}f_{k}(k_t) - \gamma_{t+1}D_{k}(k_t, d_t, \ell_t, q_{t+1}) \right] \]  
(13)

\[ \mu_t v_g(c_t, g_t) = \beta_p \left( \mu_{t+1}v_g(c_{t+1}, g_{t+1})(1 + r_{t+1}) + \gamma_{t+1}D_{d}(k_t, d_t, \ell_t, q_{t+1}) \right) \]  
(14)

\[ \mu_t v_g(c_t, g_t) = \beta_p \left( \mu_{t+1}v_g(c_{t+1}, g_{t+1})(1 + r_{t+1} - q_{t+1}) + h_{\ell}(q_{t+1})\ell_t \right) 
+ \gamma_{t+1}D_{\ell}(k_t, d_t, \ell_t, q_{t+1}) \]  
(15)

\[ \mu_{t+1}v_g(c_{t+1}, g_{t+1})\ell_t = h_{\ell}(q_{t+1})\ell_t + \gamma_{t+1}D_{q}(k_t, d_t, \ell_t, q_{t+1}) \]  
(16)

Equation (13) relates the marginal costs and benefits of investing one additional unit in the capital stock. If capital is raised by one unit, the recipient’s incentive to default on the contract increases, reflected by the term \( \gamma_{t+1}D_{k}(k_t, d_t, \ell_t, q_{t+1}) > 0 \). Equation (14) can be interpreted as the government’s
Euler equation since it relates the marginal costs and marginal benefits of issuing one additional unit non-concessional debt. Here, the increasing market debt implies higher repayment obligations decreasing the recipient values of dishonoring the contract conditions, $\gamma_{t+1}D_d(k_t, d_t, \ell_t, q_{t+1}) < 0$. Equation (15) determines the optimal level of transfers $\ell_t$. The recipient government faces marginal benefits and costs equal to $\mu_tv_g(c_{t+1}, g_{t+1})$ and $\mu_{t+1}v_g(c_{t+1}, g_{t+1})(1 + r_{t+1} - q_{t+1})$, respectively, when getting access to one additional unit of $\ell_t$ while the donor is confronted with marginal costs equal to $h_\ell(q_{t+1}\ell_t)$. However, at the same time, higher official loans imply higher repayment obligations decreasing the government’s outside option, $\gamma_{t+1}D_\ell(k_t, d_t, q_{t+1}\ell_t) \leq 0$, and increasing the donor’s leverage over the recipient government’s policies. Equation (16) characterizes the optimal degree of concessionality $q_{t+1}$ by equalizing the marginal benefits, $\mu_{t+1}v_g(c_{t+1}, g_{t+1})$, and the marginal costs $h_q(q_{t+1}\ell_t) + \gamma_{t+1}D_q(k_t, d_t, \ell_t, q_{t+1})$. Note that a higher degree of concessionality decreases the repayment obligations and, thus, increases the recipient government’s value of dishonoring the contract, $\gamma_{t+1}D_q(k_t, d_t, \ell_t, q_{t+1}) \geq 0$. Equation (17) describes the optimal tax rate and equation (18) determines the optimal choice of household’s consumption. Equation (19) is the complementary slackness condition.

The equilibrium conditions form a system of highly nonlinear equations that depend on the state variables $k_t, d_t, \ell_t, \lambda_t$ and $\mu_t$. Because no analytical closed-form solution can be derived, we solve the model numerically to study transition paths and steady states. Since the model assumes no exogenous growth, we suppose that the economy converges to a steady state that is characterized by a constant weight $\bar{\mu}$, a constant allocation $(\bar{c}, \bar{g}, \bar{k})$, a constant transfer scheme $(\bar{\ell}, \bar{q})$ and constant policies $(\bar{\tau}, \bar{d})$ that fulfill the optimality conditions associated with the donor’s maximization problem (9). The absence of uncertainty allows us to apply a backward procedure to solve for the transitional dynamics. The details of the numerical algorithm are described in the appendix.

3 Quantitative Results and Discussion

3.1 Parametrization

Table 1 provides some details on the history of debt relief shown in figure 1 and summarizes data on annual per capita income measured in constant 2000 US $, non-concessional and concessional debt and Official Development Assistance (ODA) as shares of GDP in percent. We consider a subset of the HIPCs for which data from 1975 to 2005 are available and report 10-year-averages. We sort the countries with respect to their average level of non-concessional debt between 1975 and 1985 and
divide them into quartiles. The statistics refer to the median country of each quartile and reveal that
there is substantial heterogeneity across HIPCs: between 1975 and 1985 average non-concessional
debt shares of the median countries vary between approximately 4 and 46 percent. The data show
that countries with very low levels of debt are also extremely poor in terms of annual per capita
income indicating a fairly limited access to international financial markets. All countries received
considerable amounts of ODA in form of grants and concessional loans. However, the impact on per
capita income seems to be limited, calling the effectiveness of aid into question.\textsuperscript{10} Average ODA
net loans appear to be positive for the entire time period so that substantial levels of concessional
debt have been accumulated in the HIPCs. All countries received substantial debt relief under the
HIPC-Initiative and the Multilateral Debt Relief Initiative; e.g., in 2006 Burkina Faso and Cameroon
received debt relief equal to 24 and 22 percent of GDP, respectively, while Haiti and Congo Republic
were granted 29 and 34 percent of GDP in the years 2005 to 2007.

To parameterize the model on an annual basis we first suppose that the recipient economy does not
receive any form of assistance, i.e., $\ell_t = 0$, for all $t$. The optimal choices of the recipient government
are characterized by the solution to the government’s maximization problem (8). We consider this
as our initial situation and choose the functional forms and parameters of the model as to mimic the
heterogeneity of average non-concessional debt shares between 1975 and 1985.

We assume that the household and the government have logarithmic utilities:

\[
\begin{align*}
    u(c_t) &= \ln c_t \\
    v(c_t, g_t) &= \ln c_t + \alpha \ln g_t
\end{align*}
\]

where $\alpha \in \{0.2; 0.4; 0.6\}$ is the weight that the recipient government puts on the utility of non-
productive consumption $g_t$. It measures the benevolence of the government and determines the gov-
ernment consumption share as well as the tax rate. We set the private discount rate $\beta_p = 0.95$ and
define $r^* = 1/\beta_p - 1$ so that the world interest rate equals 5.26 percent. We follow Chatterjee et
al. (2003) and assume that the market interest rate is strictly increasing in the debt-to-capital ratio

\[
r_t = r^* + e^{\phi \epsilon_{t-1}/\beta_p} - 1 \quad \text{with } \phi = 0.4.
\]

We suppose that the recipient government discounts the future at a higher rate reflecting political instability leading to overspending and debt accumulation (East-
ery, 2002). Since $\beta_g$ determines the non-concessional debt share in the economy, we analyze its

\textsuperscript{10}There is a large empirical literature on aid and economic growth, e.g., Burnside and Dollar (2000, 2004), Hansen and
and Subramanian (2008) there seems to be no robust evidence concerning the interaction of foreign aid, sound economic
policies and growth.
impact by assuming the values 0.85, 0.88, 0.91 and 0.94. The production function is specified as 
\[ f(k_{t-1}) = k_{t-1}^\theta \] with \( \theta = 0.3 \). The capital stock depreciates at the rate \( \delta = 0.1 \).

Table 2 summarizes the steady state properties for these functional forms and parameter values. In the following we refer to this steady state as the ‘No Aid’ steady state. Our parametrization generates non-concessional debt shares between 3 and 43 percent of GDP which square well with average debt shares observed between 1975 and 1985. Country risk premia vary between 1 and 12 percent which seems to be reasonable in comparison with other low-income countries.\(^{11}\) Private consumption shares vary between 52 and 69 percent while government consumption shares range from 11 to 32 percent, depending on the parameter values for \( \alpha \) and \( \beta_g \). The associated steady state tax rates vary between 13 and 36 percent. Thus, we consider economies in which up to 36 percent of GDP are lost due to inefficient economic policies. Investment shares vary between 12 and 17 percent, respectively.

It is evident that the larger \( \alpha \), i.e., the less benevolent the recipient government, the larger the income tax rate, the lower the investment share and, thus, the poorer the economy in terms of capital and output levels. The government consumption share is increasing while the private consumption share is decreasing in \( \alpha \). Moreover, non-concessional debt as a share of total output is decreasing in \( \alpha \) reflecting the fact that poor countries have limited access to international debt markets.

With respect to \( \beta_g \), our theoretical economy implies that recipient governments that discount the future at higher rates accumulate more debt leading to higher risk premia issued by international credit markets. The more indebted countries face considerable debt service obligations and suffer from low capital levels and high tax rates as well as high government consumption and low private consumption shares.

In our analysis of optimal debt relief we assume that the donor’s preferences are described by a linear cost function: 
\[ h(q_t \ell_{t-1}) = \kappa q_t \ell_{t-1} \] with \( \kappa = 1.5 \) such that the steady state value of outright grants under self-enforcing conditionality mimics the properties of the data.

### 3.2 Long-Run Properties of Self-Enforcing Conditionality

Table 3 summarizes the long-run properties of the optimal self-enforcing contract between the altruistic donor and the recipient government. The steady state depends on the parameters \( \alpha \) and \( \beta_g \) that determine the strength of the conflict of interest between the donor and the recipient government.

It turns out that incentive-compatibility requires permanent assistance \( \ell \) between 13 to 21 percent of GDP. Define 
\[ \pi_{t+1} = 1 + r_{t+1} - q_{t+1} \] as the share of transfers \( \ell_t \) that is provided in form of

\(^{11}\)Country risk premia on sovereign debt are measured by the EMBI global which is, however, not available for the HIPC.
interest-free loans. The steady state value $\pi$ is between 53 and 78 percent indicating that the optimal transfer scheme is characterized by a combination of loans and grants. The donor imposes conditions that require substantial tax cuts and considerable reductions in government consumption and non-concessional debt shares. These economic policies provide incentives to invest and generate a substantial increase in the long-run capital stock as well as in the private consumption share. Note that the implication of our theoretical model is in contrast to the recommendation made by the Meltzer (2000) Commission that development assistance should be provided through outright grants rather than concessional loans. The intuition is that loans, unlike grants, imply repayment obligations that decrease the value of the outside option of the recipient government. Thus, the donor’s leverage over the recipient government’s policies is stronger and makes conditionality easier to enforce. This is true as long as official creditors are able to enforce the repayment of international loans. We relax this assumption in section 3.5.

Let $w_t \equiv \mu_{t} \pi_{t}^{\alpha_{t}}$ be the relative weight that the donor needs to put on the utility of government consumption $\ln g_t$ to induce the recipient government to cooperate and to implement efficient fiscal policies. Recall that $\mu_{t}$ is the additional costate-variable that measures the binding pattern of the enforcement constraint. The steady state value $\pi_t$ and, therefore, the relative weight $w_t$, is increasing in the strength of conflict of interest, i.e., increasing in $\alpha$ and decreasing in $\beta_g$. Hence, the weight making the recipient government indifferent between fulfilling the contract and choosing the outside option is increasing in the non-benevolence and in the impatience of the recipient government. This is reflected by weaker conditions that are imposed on the provision of assistance: tax cuts are lower whereas government consumption and non-concessional debt shares are higher. $\pi$ is decreasing in $\alpha$, i.e., the share of subsidized loans is decreasing in the non-benevolence of the recipient government. Since high values of $\alpha$ imply severe conflicts of interest between the altruistic donor and the recipient government, the government has high incentives to dishonor the conditions. It turns out to be efficient to improve the attractiveness of the contract by decreasing the share of loans and providing more outright grants.

Interestingly, $\pi$ is decreasing in $\beta_g$. Although high values of $\beta_g$ imply only weak conflicts of interest between the donor and the recipient government, more outright grants are provided to more patient governments. The intuition is that patient governments face market interest rates that are not too high and do not limit the access to private international credit markets. The small wedge between the concessional and the market interest rate makes a permanent cutoff from subsidized loans a less threatening outside option. As a consequence, to make the contract more attractive so that the recip-
ient fulfills conditionality, the donor needs to increase the concessional level and to provide more assistance in form of outright grants.

3.3 Implicit Debt Relief

Figure 1 and table 1 have shown that between 1975 and 1985 the HIPCs accumulated considerable levels of non-concessional debt at high interest rates. To reduce the debt burden, debt relief was implicitly granted to the HIPCs by replacing market debt by concessional debt (Easterly, 2002). This led to a substantial accumulation of concessional debt in the mid-’90s. The objective of this section is to analyze the properties of optimal debt relief in a situation where the recipient country faces non-concessional debt at high market interest rates but limited concessional debt as it was the case between 1975 and 1985. To simplify our analysis, we consider the ‘No Aid’ steady state as initial situation. Recall that we have chosen the parameters of the model in such a way that the ‘No Aid’ steady state values of non-concessional debt mimic the empirical ones between 1975 and 1985. In the following, we analyze the short- and long-run effects of optimal debt relief as well as the dynamic properties of self-enforcing conditionality.

Figure 2 considers various values of $\alpha$ and $\beta_g$ and plots the transition paths to the steady state that are the outcome of the donor’s maximization problem (9) given $k_{-1} = k_{\text{no aid}}$, $d_{-1} = d_{\text{no aid}}$ and $\ell_{-1} = 0$. All variables are normalized by their respective ‘No Aid’ steady state values except for $w_t$ and $\pi_{t+1}$ that are given in levels. Transfers $\ell_t$ are normalized by the ‘No Aid’ steady state value of output. The figure shows that it is optimal to give high transfers in the initial situation since the recipient country suffers from a low capital stock and non-concessional debt at high interest rates. In return, the recipient government has to reduce government consumption and to decrease the tax rate substantially. In fact, in the initial period, the tax rate may be negative meaning that the recipient government subsidizes production to improve the incentives to invest in the capital stock. Over time, as the capital stock grows and non-concessional debt shrinks, the recipient government becomes richer and benefits from an improved access to private international credit markets. As a permanent cutoff from any form of assistance becomes less threatening, the relative weight $w_t$ increases over time, i.e., the donor needs to raise the weight on the utility of government consumption to ensure the enforceability of the contract. This implies that the tax rate as well as government consumption rise over time. In the short run, the relative weight is higher for more patient governments while in the long-run the opposite is true. This is due to the fact that patient governments initially face lower market interest rates and, thus, have better access to international private credit and are less
dependent on the provision of assistance. In the long-run, the impatient recipient governments have higher incentives to defect on conditionality since the conflict of interest is more severe. This is reflected by the pattern of government consumption: in the short-run (long-run), the reduction in government consumption is higher (lower) for the more impatient governments.

The dynamic pattern of the optimal transfer scheme and the degree of concessionality crucially depends on the degree of impatience. First suppose that the recipient government is impatient, $\beta_g = 0.85$. Total transfers $\ell_t$ decrease as the capital stock grows and non-concessional debt decreases over time. Note, however, that transfers do not converge to zero since the recipient government would return to the initial inefficient economic policies in that case. Over time, as the recipient government becomes richer, the incentives to breach the contract and to dishonor the conditions increase. It turns out to be optimal to provide a rising fraction $\pi_{t+1}$ in form of subsidized loans so that the repayment obligations decrease the value of the recipient’s outside option. Now suppose that the recipient government is patient, $\beta_g = 0.91$, but quite non-benevolent, $\alpha = 0.6$. In this case, the fraction $\pi_{t+1}$ of total transfers that is provided as subsidized loans is falling over time. Moreover, after a sharp short-run decrease, total transfers continuously increase until the steady state level is reached. The transition path of this transfer scheme can be explained by the strong incentives to defect on conditionality. The outside option is less threatening in this setup for three reasons. First, there is a severe conflict of interest between the altruistic donor and the non-benevolent recipient government lowering the government’s value of the conditionality contract. Second, as patient recipient governments are mildly indebted and face moderate country risk premia, the wedge between the market and the subsidized interest rate is low making the recipient less dependent on assistance. Third, the growing capital stock increases the value of the outside option over time. To prevent that the recipient government fails to meet the conditions and takes the outside option, the donor needs to make the contract more attractive by providing higher transfers and, in addition, raising the concessionality level, i.e., the fraction of outright grants over time.

Overall, the impact of incentive-compatible conditional debt relief is substantial. Relative to the initial ‘No Aid’ situation, capital and private consumption is substantially increased. Thereby, the relative impact is increasing in $\alpha$ while the differences for the various values of $\beta_g$ are minor. Although the relative impact is increasing in $\alpha$, the countries that suffer from non-benevolent governments are still the poorest in terms of per capita income levels, see table 3.

The transitional dynamics show that optimal debt relief is characterized by a combination of concessional loans and outright grants in the short- and long-run. Thus, non-concessional debt is partly
replaced by subsidized loans and partly repaid by using outright grants. In contrast to the historical experience, optimal concessional debt levels are quite moderate.

3.4 Explicit Debt Relief

Figure 1 and table 1 have shown that the HIPCs accumulated large amounts of concessional debt between 1980 and the early 2000. This pattern motivated the foundation of the HIPC Initiative and the Multilateral Debt Relief Initiative that allow for a 100 percent cancelation of official multilateral debt. The objective of this section is to analyze the properties of optimal explicit debt relief in form of outright grants that directly delete official debt.

As initial situation we assume that the recipient economy faces a substantial, non-optimal level of concessional debt and, at the same time, receives considerable amounts of outright grants. We assume that the initial concessional debt level equals 45 percent of GDP at a subsidized interest rate equal to 0.5 percent and that grants amount to 5 percent of GDP. This is in line with what happened on average between 1995 and 2005 in many HIPCs, see table 1. The interest rate of 0.5 percent covers the administrative costs and is commonly imposed on loans provided to the poorest countries. In terms of our model these values correspond to $\pi = 0.8939$ and $\ell_y = 0.45$. Since the impact of aid on per capita income levels seems to have been rather limited, see Rajan and Subramanian (2008) for econometric evidence, we suppose that in the initial situation, total transfers are provided unconditionally, i.e., the recipient government takes them as given and chooses optimal fiscal policies by solving its maximization problem (8). As the findings are qualitatively similar for the various values of $\alpha$, we consider $\alpha = 0.4$ and $\beta_g = 0.85$ and $\beta_g = 0.91$. Figure 3 shows that the initial situation is characterized by a low effectiveness of unconditional assistance: relative to the ‘No Aid’ steady state, the capital stock and private consumption are raised by only 5 to 6 percent. Instead of using transfers to finance efficient policies, the recipient government increases its consumption by 5 to 6 percent relative to the ‘No Aid’ steady state.

Now suppose that the donor imposes incentive-compatible conditionality and designs its optimal debt relief policy according to the maximization problem (9) given $k_{-1} = k^{\text{uncond}}_{-1}$, $d_{-1} = d^{\text{uncond}}_{-1}$ and $\ell_{-1} = \ell^{\text{uncond}}_{-1}$. Figure 3 shows the dynamic properties of the incentive-compatible contract and plots the transition paths to the steady state associated with self-enforcing conditional debt relief. All variables are normalized by their respective ‘No Aid’ steady state values except for $w_t$ and $\pi_{t+1}$

12To save space we omit the table that summarizes all properties of the steady state if assistance is unconditionally provided.
that are given in levels. Transfers $\ell_t$ are normalized by the ‘No Aid’ steady state value of output. If the recipient economy faces high, non-optimal levels of concessional debt, optimal debt relief is characterized by a jump decrease in $\pi$ from 0.89 to a value between 0.55 and 0.6 indicating that part of the debt is explicitly canceled by an outright grant. At the same time, optimal transfers $\ell_t$ show a fall from 45 percent as a share of ‘No Aid’ steady state output to 28 and 23 percent for $\beta_g = 0.85$ and $\beta_g = 0.91$, respectively. The negative net loans are financed by a large reduction in government consumption and by a moderate increase in non-concessional debt. Moreover, initially, the tax rate remains nearly unchanged. Thus, on the one hand, optimal debt relief is characterized by outright grants, but, on the other hand, conditionality forces the recipient government to repay part of its concessional debt. In contrast to recent policies implemented by the Multilateral Debt Relief Initiative, a 100 percent cancelation of official debt is not optimal. This, however, is true only if the donor is able to enforce the repayment of concessional debt.

3.5 Outright Grants Only

In the previous sections we have assumed that official creditors are able to enforce the fulfillment of concessional debt obligations. This assumption is in line with the view that official creditors have better enforcement technologies than private creditors. Moreover, the historical experience indicates that debtors rarely default on official debt, see Jeanne and Zettelmeyer (2001). On the other hand one might argue that, over the past decades, official creditors continued to provide positive net loans just to prevent default on concessional debt. Therefore, in this section, we take into account that recipient governments may not be willing to repay official loans and assume that a failure on conditionality goes hand in hand with a default on concessional debt. In such a scenario, in the absence of uncertainty, the donor loses loans as a policy instrument and provides outright grants only, as recommended by the Meltzer (2000) Commission and by Bulow and Rogoff (2005).

In figure 4 we consider the ‘No Aid’ steady state as initial situation and plot the transition paths to the steady state. Since the dynamic patterns are qualitatively similar for different constellations of $\alpha$ and $\beta_g$ we focus on $\alpha = 0.4$ and $\beta_g = 0.91$ to save space. To facilitate a comparison with the previous scenario, we plot the transition paths of the optimal combination of grants and loans (dotted line) together with the transition paths associated with outright grants in isolation (solid line). Moreover, to make the findings interpretable, we analyze the impact of one unit of development assistance and show non-concessional debt, capital, government consumption and consumption per unit of assistance $q_{t+1} \ell_t$, where one unit is normalized to 0.01. The relative weight, transfers, the tax
rate and the interest rate are given in levels.

Overall, the general pattern of the transition paths associated with optimal grants look very similar to those of section 3.3. The donor provides grants to implement tax cuts and to reduce government consumption so that private incentives to invest in capital increase. Over time, the growing capital stock and the shrinking level of non-concessional debt make fulfilling the conditions less attractive for the recipient government. To ensure that the contract is self-enforcing, conditionality becomes less severe, i.e., government consumption and the tax rate increase over time.

In the medium- and long-run, if concessional loans are not enforceable and assistance takes the form of outright grants only, the relative weight on the utility of government consumption is larger compared to the weight that would be optimal otherwise. As a consequence, the size of assistance \( q_{t+1} \ell_t \) as well as government consumption and market debt per unit of assistance are permanently higher. It is evident that the impact of one unit of assistance on capital, consumption and non-concessional debt is lower if the donor loses loans as a policy instrument.

In the short-run, if giving grants is the only policy instrument, the tax rate is higher compared to the tax rate associated with the optimal combination of loans and grants. The intuition for this finding is that loans allow larger transfers that substantially relax the government’s budget constraint so that considerable tax cuts can be implemented. For this reason, the short-run effect of one unit of assistance on capital, consumption, debt and interest rate is substantial if the donor is able to implement a combination of loans and grants. However, the lower market interest rate improves the access to international financial markets making the outside option less severe for the recipient government. For this reason, in the short-run, self-enforcing conditionality requires a higher relative weight and larger government consumption per unit of assistance if optimal policy is characterized by a combination of loans and grants instead of being restricted to outright grants only. The opposite is true in the long-run.

In table 4 we consider the ‘No Aid’ steady state as initial situation and analyze household’s welfare per unit of assistance \( q_{t+1} \ell_t \). We compare the welfare impact of the optimal combination of loans and grants, ‘LG’, with the welfare impact of grants in isolation, ‘G’. As benchmark we take the steady state associated with the optimal combination of loans as grants. We use compensating variations to
formulate differences in lifetime utility and express the change in welfare $\Delta^{LG}$ and $\Delta^{G}$ as follows:

\[
\sum_{t=0}^{\infty} \beta^t u \left( (1 + \Delta^{LG}) \left( \frac{c_t}{q_t} \right)^{LG} \right) = \sum_{t=0}^{\infty} \beta^t u \left( \left( \frac{c_t}{q_t+1} \right)^{LG} \right) \quad (20)
\]

\[
\sum_{t=0}^{\infty} \beta^t u \left( (1 + \Delta^{G}) \left( \frac{c_t}{q_t} \right)^{LG} \right) = \sum_{t=0}^{\infty} \beta^t u \left( \left( \frac{c_t}{q_t+1} \right)^{G} \right) \quad (21)
\]

If concessional debt is enforceable and the donor implements the optimal combination of loans and grants, the impact on welfare $\Delta^{LG}$ varies between $-13.78$ and $-1.89$ percent of steady state consumption per unit of assistance, depending on the parameter values $\alpha$ and $\beta_g$. The welfare gain of providing the optimal combination of loans and grants is increasing in the non-benevolence and decreasing in the impatience of the recipient government. In the absence of any assistance non-benevolent governments implement extremely wasteful economic policies so that the capital stock is very low making one unit of development assistance quite effective.

If concessional loans are not enforceable and the donor provides assistance in form of outright grants only, the impact on welfare $\Delta^{G}$ follows qualitatively the same pattern as before. However, losing loans as a policy instrument generates welfare losses up to 5 percent of steady state consumption per unit of assistance. In line with our findings of the previous sections, the importance of loans as a policy instrument is decreasing in the non-benevolence and the patience of the recipient government.

4 Conclusions

This paper has studied the effectiveness of debt relief as a policy instrument to stimulate economic growth in the most heavily indebted poor countries. We have developed a neoclassical growth framework with a conflict of interest between the altruistic donor and the recipient government. Following the recent literature on incentive-compatibility in the context of foreign aid, conditionality has been modeled as an imperfectly enforceable dynamic contract: the donor offers to provide debt relief and, in return, expects the government to implement fiscal as well as debt policies that coincide with the donor’s intention.

Our findings have suggested that imposing incentive-compatible conditions on the provision of debt relief substantially promotes fiscal reform and investment. If official creditors are able to enforce the repayment of concessional loans, optimal policy is characterized by a combination of loans and outright grants. Unlike grants, loans imply repayment obligations that increase the donor’s leverage over the recipient government’s policies and make conditionality easier to enforce. If official loans are limited enforceable such that a failure on conditionality goes hand in hand with a default on
concessional debt, optimal policy is characterized by outright grants only. Losing loans as a policy instrument reduces the effectiveness of one unit of development assistance and lowers welfare. The dynamic patterns of the optimal transfer scheme and self-enforcing conditionality have been shown to depend critically on the degree of the recipient’s non-benevolence and impatience since these parameters affect the access to international credit markets and the strength of the conflict of interest. Our analysis has been based on the assumption that there is full commitment on the donor’s side and that the punishment threat is fully credible. However, this may not be the case since the donor is altruistic and might gain by relaxing the sanctions. In an endowment economy Cordella et al. (2003) and Kletzer (2005) show that a renegotiation-proof equilibrium is characterized by aid flows in punishment that are smaller than those made in equilibrium. If we allow for reduced transfers in punishment, the value of breaching the conditionality contract becomes larger so that the recipient government’s incentives to defect increase. As a result, conditionality is less severe and the effectiveness of debt relief is lower. Similarly, allowing for limited commitment on the donor’s side introduces additional sources for ineffectiveness of development assistance. Since our theoretical framework considers a dynamic setting with capital and two types of debt, the analysis of two-sided limited commitment and the specification of renegotiation-proof equilibria are beyond the scope of this paper and left for future research. Instead, we interpret our findings as a benchmark on the effectiveness of debt relief.
A  Optimality Conditions

A.1  The Default Value

The default value is given by the solution to the recipient government’s maximization problem (8). The optimality conditions are:

$$
\zeta_t = v_c(c_t, g_t) + \frac{\beta_p}{\beta_g} u_{cc}(c_t) \lambda_{t-1} \left[ (1 - \tau_t) f_k(k_{t-1}) + 1 - \delta \right] - \lambda_t u_{cc}(c_t) \tag{22}
$$

$$
\zeta_t = \frac{\beta_p}{\beta_g} \lambda_t u_c(c_{t+1}) (1 - \tau_{t+1}) f_k(k_t) + \zeta_{t+1} (1 - \tau_{t+1}) f_k(k_t) + 1 - \delta \\
+ v_g(c_{t+1}, g_{t+1}) \tau_{t+1} f_k(k_t) \tag{23}
$$

$$
v_g(c_t, g_t) = \beta_g v_g(c_{t+1}, g_{t+1})(1 + r_{t+1}) \tag{24}
$$

$$
\zeta_t f(k_{t-1}) = v_g(c_t, g_t) f(k_{t-1}) - \frac{\beta_p}{\beta_g} \lambda_{t-1} u_c(c_t) f_k(k_{t-1}), \tag{25}
$$

where $\zeta_t$ and $\lambda_t$ are the Lagrange multipliers associated to the household’s budget constraint and the Euler equation, respectively. Equation (22) determines the optimal choice of households’ consumption. Equation (23) relates the marginal costs and benefits of investing one additional unit in the capital stock. Equation (24) can be interpreted as the government’s Euler equation since it relates the marginal costs and marginal benefits of issuing one additional unit foreign debt. Finally, equation (25) determines the optimal tax choice.

A.2  Self-Enforcing Conditionality

The Lagrangian associated to the donor’s maximization problem (9) is given by

$$
L = \sum_{t=0}^{\infty} \beta_p^t \left[ u(c_t) - h(q_t \ell_{t-1}) + \gamma_t \left( \sum_{j=0}^{\infty} \beta_g^j v(c_{t+j}, g_{t+j}) - D(k_{t-1}, d_{t-1}, \ell_{t-1}, q_t) \right) \right] 
$$

subject to (2), (5) and (7). It is straightforward to show that the following equality holds:

$$
\sum_{t=0}^{\infty} \beta_p^t \gamma_t \sum_{j=0}^{\infty} \beta_g^j v(c_{t+j}, g_{t+j}) = \sum_{t=0}^{\infty} \beta_p^t \mu_t v(c_t, g_t) \\
\text{s.t.} \\
\mu_t = \left( \frac{\beta_g}{\beta_p} \right) \mu_{t-1} + \gamma_t, \quad \mu_{-1} = 0.
$$

Thus, the Lagrangian becomes

$$
L = \sum_{t=0}^{\infty} \beta_p^t \left[ u(c_t) - h(q_t \ell_{t-1}) + \mu_t v(c_t, g_t) - \gamma_t D(k_{t-1}, d_{t-1}, \ell_{t-1}, q_t) \right] 
$$

subject to

$$
\mu_t = \left( \frac{\beta_g}{\beta_p} \right) \mu_{t-1} + \gamma_t, \quad \mu_{-1} = 0.
$$

21
B Numerical Algorithm

Due to the complexity of the model, we rely on numerical simulations to analyze the properties of different debt relief policies. Since there is no uncertainty we use a backward procedure to solve for the transitional dynamics, see also Scholl (2009) and Trabandt (2009).

Considering the optimal value of default, the equilibrium is characterized by the equations (2), (5), (7), (22) to (25). To make the system of equations finite dimensional we assume that the economy converges to the steady state in finitely many periods $T + 1$. Hence, as time starts in $t = 0$, in period $T$ the state variables are given by their steady state values, $k_T = \bar{k}$, $d_T = \bar{d}$ and $\lambda_T = \bar{\lambda}$. Given the initial values $k_{-1}, d_{-1}$, and $\lambda_{-1}$, we need to solve for $\{c_t, g_t, \tau_t, \zeta_t\}_{t=0}^T$ and $\{k_t, d_t, \lambda_t\}_{t=0}^{T-1}$. To do so, we consider the equilibrium conditions (2), (5), (22) and (25) for $t = 0, \ldots, T$ and the equilibrium conditions (7), (23) to (24) that look forward to $t + 1$ for $t = 0, \ldots, T - 1$. Since we have as many unknowns as equations the system of nonlinear equations can be solved by employing a numerical nonlinear equation solver.

Considering the case of self-enforcing conditionality, the absence of uncertainty implies that the enforcement is always binding until the steady state is reached. Hence, we can employ the same solution strategy as above. Note that the enforcement constraint requires the calculation of the default value $D(k_{t-1}, d_{t-1}, \ell_{t-1}, q_t)$ that includes the transitional dynamics to the steady state that occurs if no development assistance is provided to the recipient government. As before, to make the system of equations finite dimensional we assume that the economy converges to the steady state in finitely many periods $T + 1$. Hence, as time starts in $t = 0$, in period $T$ the state variables are given by their steady state values, $k_T = \bar{k}$, $d_T = \bar{d}$, $\ell_T = \bar{\ell}$, $\mu_T = \bar{\mu}$ and $\lambda_T = \bar{\lambda}$. Given the initial values $k_{-1}, d_{-1}$, $\ell_{-1}, q_0, \mu_{-1}$ and $\lambda_{-1}$, we need to solve for $\{c_t, g_t, \tau_t, q_{t+1}, \zeta_t\}_{t=0}^T$ and $\{k_t, d_t, \ell_t, \mu_t, \lambda_t\}_{t=0}^{T-1}$. To do so, we consider the equilibrium conditions (2), (5), (16) and (18) for $t = 0, \ldots, T$ and the equilibrium conditions (7), (13) to (15) and (19) that look forward to $t + 1$ for $t = 0, \ldots, T - 1$. Since we have as many unknowns as equations the system of nonlinear equations can be solved by employing a numerical nonlinear equation solver.
References


Kletzer, K., 2005. Aid and Sanctions, mimeo, University of California, Santa Cruz.


Sachs, J., 1989. The Debt Overhang in Developing Countries, in Calvo, G. A., Kouri, R. F. P.,

Dynamics 12(2), 377-391.


Svensson, J., 2000. When is Foreign Aid Policy Credible? Aid Dependence and Conditionality,

of Development Economics 70(2), 381-402.

Tables and Figures
Figure 1: The History of Debt Relief in HIPC countries; Annual data on debt are taken from the World Bank, World Development Indicators. Annual data on Official Development Assistance (ODA) are taken from the OECD. We consider a subset of the HIPC countries for which data for the entire time period is available. These countries are: Benin, Bolivia, Burkina Faso, Burundi, Cameroon, Central African Republic, Chad, Cote d’Ivoire, Ghana, Guyana, Haiti, Honduras, Liberia, Madagascar, Malawi, Mali, Mauritania, Nicaragua, Niger, Rwanda, Senegal, Sudan, Togo, Uganda and Zambia.

Table 1: Income, Debt and Official Development Assistance in HIPC countries

<table>
<thead>
<tr>
<th>Percentile of d/y: 0 – 25</th>
<th>Median: Burkina Faso</th>
<th>Per capita income</th>
<th>Debt as share of GDP</th>
<th>ODA as share of GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>percent of d/y</td>
<td>non-concess.</td>
<td>concess.</td>
</tr>
<tr>
<td>1975 - 1985</td>
<td></td>
<td>163</td>
<td>3.70</td>
<td>12.52</td>
</tr>
<tr>
<td>1985 - 1995</td>
<td></td>
<td>181</td>
<td>6.30</td>
<td>28.93</td>
</tr>
<tr>
<td>1995 - 2005</td>
<td></td>
<td>225</td>
<td>2.06</td>
<td>41.10</td>
</tr>
<tr>
<td>Percentile of d/y: 25 – 50</td>
<td>Median: Cameroon</td>
<td>Per capita income</td>
<td>Debt as share of GDP</td>
<td>ODA as share of GDP</td>
</tr>
<tr>
<td>----------------------------</td>
<td>---------------------</td>
<td>------------------</td>
<td>---------------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>1975 - 1985</td>
<td></td>
<td>746</td>
<td>13.51</td>
<td>11.84</td>
</tr>
<tr>
<td>1985 - 1995</td>
<td></td>
<td>747</td>
<td>28.32</td>
<td>20.89</td>
</tr>
<tr>
<td>1995 - 2005</td>
<td></td>
<td>632</td>
<td>34.58</td>
<td>46.57</td>
</tr>
<tr>
<td>Percentile of d/y: 50 – 75</td>
<td>Median: Haiti</td>
<td>Per capita income</td>
<td>Debt as share of GDP</td>
<td>ODA as share of GDP</td>
</tr>
<tr>
<td>1975 - 1985</td>
<td></td>
<td>1054</td>
<td>24.88</td>
<td>14.84</td>
</tr>
<tr>
<td>1985 - 1995</td>
<td></td>
<td>1066</td>
<td>50.55</td>
<td>39.95</td>
</tr>
<tr>
<td>1995 - 2005</td>
<td></td>
<td>1161</td>
<td>23.93</td>
<td>45.84</td>
</tr>
<tr>
<td>Percentile of d/y: 75 – 100</td>
<td>Median: Congo Republic</td>
<td>Per capita income</td>
<td>Debt as share of GDP</td>
<td>ODA as share of GDP</td>
</tr>
<tr>
<td>1975 - 1985</td>
<td></td>
<td>1057</td>
<td>46.19</td>
<td>25.36</td>
</tr>
<tr>
<td>1985 - 1995</td>
<td></td>
<td>1146</td>
<td>105.73</td>
<td>61.09</td>
</tr>
<tr>
<td>1995 - 2005</td>
<td></td>
<td>1065</td>
<td>83.71</td>
<td>67.67</td>
</tr>
</tbody>
</table>

Notes: Annual data on per capita income and concessional and non-concessional debt are taken from the World Bank, World Development Indicators. Annual data on Official Development Assistance (ODA) are taken from the OECD. Per capita income is measured in constant 2000 US $; the shares are given in percent. We consider a subset of the HIPC countries for which data for the entire time period is available. These countries are: Benin, Bolivia, Burkina Faso, Burundi, Cameroon, Central African Republic, Chad, Cote d’Ivoire, Ghana, Guyana, Haiti, Honduras, Liberia, Madagascar, Malawi, Mali, Mauritania, Nicaragua, Niger, Rwanda, Senegal, Sudan, Togo, Uganda and Zambia. All entries are averages for the time periods 1975 – 1985, 1985 – 1995 and 1995 – 2005. We consider the period 1975 – 1985 and sort the countries with respect to their level of non-concessional debt as share of GDP. Statistics refer to the median country of each quartile.
Table 2: Steady State Properties, No Aid

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\bar{c}$</th>
<th>$\bar{x}$</th>
<th>$\bar{y}$</th>
<th>$\bar{d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>$\beta_g = 0.85$</td>
<td>1.94</td>
<td>19.15</td>
<td>12.38</td>
</tr>
<tr>
<td></td>
<td>$\beta_g = 0.88$</td>
<td>2.00</td>
<td>17.40</td>
<td>8.37</td>
</tr>
<tr>
<td></td>
<td>$\beta_g = 0.91$</td>
<td>2.06</td>
<td>15.55</td>
<td>4.63</td>
</tr>
<tr>
<td></td>
<td>$\beta_g = 0.94$</td>
<td>2.12</td>
<td>13.88</td>
<td>1.12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\bar{c}$</th>
<th>$\bar{x}$</th>
<th>$\bar{y}$</th>
<th>$\bar{d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>$\beta_g = 0.85$</td>
<td>1.64</td>
<td>28.02</td>
<td>12.38</td>
</tr>
<tr>
<td></td>
<td>$\beta_g = 0.88$</td>
<td>1.67</td>
<td>27.08</td>
<td>8.37</td>
</tr>
<tr>
<td></td>
<td>$\beta_g = 0.91$</td>
<td>1.73</td>
<td>25.44</td>
<td>4.63</td>
</tr>
<tr>
<td></td>
<td>$\beta_g = 0.94$</td>
<td>1.79</td>
<td>23.58</td>
<td>1.12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\bar{c}$</th>
<th>$\bar{x}$</th>
<th>$\bar{y}$</th>
<th>$\bar{d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>$\beta_g = 0.85$</td>
<td>1.40</td>
<td>35.57</td>
<td>12.38</td>
</tr>
<tr>
<td></td>
<td>$\beta_g = 0.88$</td>
<td>1.43</td>
<td>34.76</td>
<td>8.37</td>
</tr>
<tr>
<td></td>
<td>$\beta_g = 0.91$</td>
<td>1.48</td>
<td>33.02</td>
<td>4.63</td>
</tr>
<tr>
<td></td>
<td>$\beta_g = 0.94$</td>
<td>1.55</td>
<td>30.89</td>
<td>1.12</td>
</tr>
</tbody>
</table>

Notes: $\bar{k}$, $\bar{\tau}$, $\bar{\tau} - r^*$, $\bar{c}$, $\bar{x}$, $\bar{y}$, $\bar{d}$, and $\bar{y}$ denote the steady state values of capital, the tax rate, household consumption, investment, government consumption, non-concessional debt and output, respectively. $\bar{\tau} - r^*$ is the country risk premium. The tax rate, the country risk premium and the shares are given in percent.
### Table 3: Steady State Properties, Self-Enforcing Conditionality

<table>
<thead>
<tr>
<th></th>
<th>(\tilde{k})</th>
<th>(\tau)</th>
<th>(\tau - r^*)</th>
<th>(\tilde{c}/\tilde{y})</th>
<th>(\tilde{c}/\tilde{y})</th>
<th>(\tilde{d}/\tilde{y})</th>
<th>(\ell/\tilde{y})</th>
<th>(\pi/\tilde{y})</th>
<th>(\pi/\tilde{y})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha = 0.2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta_g = 0.85)</td>
<td>2.41</td>
<td>5.79</td>
<td>4.99</td>
<td>75.70</td>
<td>18.52</td>
<td>6.38</td>
<td>21.77</td>
<td>13.00</td>
<td>78.22</td>
</tr>
<tr>
<td>(\beta_g = 0.88)</td>
<td>2.47</td>
<td>4.16</td>
<td>3.40</td>
<td>77.00</td>
<td>18.84</td>
<td>6.25</td>
<td>15.38</td>
<td>13.33</td>
<td>74.35</td>
</tr>
<tr>
<td>(\beta_g = 0.91)</td>
<td>2.52</td>
<td>2.87</td>
<td>1.92</td>
<td>78.04</td>
<td>19.09</td>
<td>6.05</td>
<td>8.94</td>
<td>13.50</td>
<td>71.71</td>
</tr>
<tr>
<td>(\beta_g = 0.94)</td>
<td>2.55</td>
<td>1.94</td>
<td>0.48</td>
<td>78.78</td>
<td>19.27</td>
<td>5.78</td>
<td>2.30</td>
<td>13.46</td>
<td>70.54</td>
</tr>
<tr>
<td>(\alpha = 0.4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta_g = 0.85)</td>
<td>2.33</td>
<td>8.02</td>
<td>5.45</td>
<td>73.90</td>
<td>18.08</td>
<td>11.77</td>
<td>23.07</td>
<td>17.61</td>
<td>64.70</td>
</tr>
<tr>
<td>(\beta_g = 0.88)</td>
<td>2.39</td>
<td>6.28</td>
<td>3.71</td>
<td>75.30</td>
<td>18.42</td>
<td>11.53</td>
<td>16.33</td>
<td>17.93</td>
<td>62.55</td>
</tr>
<tr>
<td>(\beta_g = 0.91)</td>
<td>2.44</td>
<td>4.89</td>
<td>2.08</td>
<td>76.42</td>
<td>18.69</td>
<td>11.14</td>
<td>9.49</td>
<td>18.31</td>
<td>62.04</td>
</tr>
<tr>
<td>(\beta_g = 0.94)</td>
<td>2.48</td>
<td>3.82</td>
<td>0.52</td>
<td>77.28</td>
<td>18.91</td>
<td>10.64</td>
<td>2.43</td>
<td>17.51</td>
<td>60.19</td>
</tr>
<tr>
<td>(\alpha = 0.6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta_g = 0.85)</td>
<td>2.26</td>
<td>10.02</td>
<td>5.76</td>
<td>72.29</td>
<td>17.68</td>
<td>16.24</td>
<td>23.80</td>
<td>21.01</td>
<td>57.95</td>
</tr>
<tr>
<td>(\beta_g = 0.88)</td>
<td>2.32</td>
<td>8.17</td>
<td>3.92</td>
<td>73.78</td>
<td>18.05</td>
<td>15.94</td>
<td>16.88</td>
<td>20.44</td>
<td>54.41</td>
</tr>
<tr>
<td>(\beta_g = 0.91)</td>
<td>2.38</td>
<td>6.72</td>
<td>2.20</td>
<td>74.95</td>
<td>18.33</td>
<td>15.39</td>
<td>9.83</td>
<td>20.97</td>
<td>55.16</td>
</tr>
<tr>
<td>(\beta_g = 0.94)</td>
<td>2.42</td>
<td>5.56</td>
<td>0.55</td>
<td>75.87</td>
<td>18.56</td>
<td>14.68</td>
<td>2.52</td>
<td>20.01</td>
<td>53.70</td>
</tr>
</tbody>
</table>

**Notes:** \(\tilde{k}, \tau, \bar{c}, \bar{y}, \bar{d}, \bar{y}, \bar{\pi}\) and \(\ell\) denote the steady state values of capital, the tax rate, household consumption, investment, government consumption, non-concessional debt, output and transfers, respectively. \(\tau - r^*\) is the country risk premium. \(\bar{\pi} = 1 + \frac{\tau}{\bar{\pi}}\) denotes the degree of concessionality. \(\bar{\pi} = -\frac{\tau - \bar{\pi}}{\ell}\) denotes outright grants. \(w = \frac{\alpha}{1 + \beta_g}\) denotes the relative weight that the donor puts on the utility of government consumption to ensure enforceability. The tax rate, the country risk premium, the shares and the relative weight are given in percent.
### Table 4: Welfare Analysis

<table>
<thead>
<tr>
<th></th>
<th>$\Delta LG$</th>
<th>$\Delta G$</th>
<th>$\Delta G - \Delta LG$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 0.2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_g = 0.85$</td>
<td>-13.78</td>
<td>-18.85</td>
<td>-5.06</td>
</tr>
<tr>
<td>$\beta_g = 0.88$</td>
<td>-7.48</td>
<td>-11.56</td>
<td>-4.08</td>
</tr>
<tr>
<td>$\beta_g = 0.91$</td>
<td>-4.24</td>
<td>-6.74</td>
<td>-2.50</td>
</tr>
<tr>
<td>$\beta_g = 0.94$</td>
<td>-1.89</td>
<td>-2.44</td>
<td>-0.55</td>
</tr>
<tr>
<td>$\alpha = 0.4$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_g = 0.85$</td>
<td>-9.49</td>
<td>-11.99</td>
<td>-2.50</td>
</tr>
<tr>
<td>$\beta_g = 0.88$</td>
<td>-6.00</td>
<td>-8.11</td>
<td>-2.11</td>
</tr>
<tr>
<td>$\beta_g = 0.91$</td>
<td>-4.00</td>
<td>-5.37</td>
<td>-1.37</td>
</tr>
<tr>
<td>$\beta_g = 0.94$</td>
<td>-2.52</td>
<td>-2.75</td>
<td>-0.23</td>
</tr>
<tr>
<td>$\alpha = 0.6$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_g = 0.85$</td>
<td>-7.12</td>
<td>-8.96</td>
<td>-1.84</td>
</tr>
<tr>
<td>$\beta_g = 0.88$</td>
<td>-5.02</td>
<td>-6.48</td>
<td>-1.46</td>
</tr>
<tr>
<td>$\beta_g = 0.91$</td>
<td>-3.73</td>
<td>-4.71</td>
<td>-0.98</td>
</tr>
<tr>
<td>$\beta_g = 0.94$</td>
<td>-2.70</td>
<td>-2.90</td>
<td>-0.20</td>
</tr>
</tbody>
</table>

**Notes:** The initial situation of the welfare analysis is given by the ‘No Aid’ steady state. ‘LG’ refers to the optimal combination of loans and grants while ‘G’ refers to optimal grants in isolation. The benchmark is the steady state that would occur if the donor implements the optimal combination of loans as grants. Welfare gains $\Delta LG$ and $\Delta G$ are given in percent of steady state consumption and calculated per unit of assistance $q_{t+1} \ell_t$, see equations (20) and (21).
Figure 2: Dynamic Effects of Implicit Debt Relief; The initial situation is given by the ‘No Aid’ steady state. The weight \( w_t \) and \( \pi_{t+1} = 1 + r_{t+1} - q_{t+1} \) are given in levels. Transfers \( \ell_t \) are normalized by the ‘No Aid’ steady state value of output. Non-concessional debt \( d_t \) and capital \( k_t \) are normalized by their respective ‘No Aid’ steady state values. The solid line refers to \( \beta_g = 0.85 \) while the dotted line refers to \( \beta_g = 0.91 \).
Figure 2 continued: Tax rates $\tau_t$, government consumption $g_t$, consumption $c_t$ and the market interest rate $r_{t+1}$ are normalized by their respective ‘No Aid’ steady state values. The solid line refers to $\beta_g = 0.85$ while the dotted line refers to $\beta_g = 0.91$. 
Figure 3: Dynamic Effects of Explicit Debt Relief; The initial situation is given by the steady state that is associated with unconditionally provided transfers with $\pi = 0.45$ and $\pi = 0.8939$. The relative weight $w_t$ and $\pi_{t+1} = 1 + r_{t+1} - q_{t+1}$ are given in levels. Transfers $\ell_t$ are normalized by the ‘No Aid’ steady state value of output. Non-concessional debt $d_t$, capital $k_t$, tax rates $\tau_t$, government consumption $g_t$, consumption $c_t$ and interest rate $r_{t+1}$ are normalized by their respective ‘No Aid’ steady state values. The figure refers to $\alpha = 0.4$. The solid line refers to $\beta_g = 0.85$ while the dotted line refers to $\beta_g = 0.91$. 
Figure 4: Dynamic Effects of Grants Only; The initial situation is given by the ‘No Aid’ steady state. The figure refers to $\alpha = 0.4$ and $\beta_g = 0.85$. The solid line refers to optimal grants while the dotted line refers to the optimal combination of grants and loans. The relative weight $w_t$, $q_{t+1} \ell_t$, the tax rate $\tau_t$ and the interest rate $r_{t+1}$ are given in levels. Non-concessional debt $d_t$, capital $k_t$, government consumption $g_t$, consumption $c_t$ are given per unit of $q_{t+1} \ell_t$, where one unit is normalized to 0.01.