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Forecasting GDP Growth Using Mixed-Frequency Models With Switching Regimes

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Forecasting GDP Growth Using Mixed-Frequency Models With Switching Regimes *

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Abstract

For modelling mixed-frequency data with business cycle pattern we introduce the Markov-switching Mixed Data Sampling model with unrestricted lag polynomial (MS-U-MIDAS). Usually models of the MIDAS-class use lag polynomials of a specific function, which impose some structure on the weights of regressors included in the model. This may deteriorate the predictive power of the model if the imposed structure differs from the data generating process. When the difference between the available data frequencies is small and there is no risk of parameter proliferation, using an unrestricted lag polynomial might not only simplify the model estimation, but also improve its forecasting performance. We allow the parameters of the MIDAS model with unrestricted lag polynomial to change according to a Markov-switching scheme in order to account for the business cycle pattern observed in many macroeconomic variables. Thus we combine the unrestricted MIDAS with a Markov-switching approach and propose a new Markov-switching MIDAS model with unrestricted lag polynomial (MS-U-MIDAS). We apply this model to a large dataset with the help of factor analysis. Monte Carlo experiments and an empirical forecasting comparison carried out for the U.S. GDP growth show that the models of the MS-U-MIDAS class exhibit similar or better nowcasting and forecasting performance than their counterparts with restricted lag polynomials.

Keywords: Markov-switching, Business cycle, Mixed-frequency data analysis, Forecasting

JEL Classification Code: C22, C53, E37

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1 Introduction

Forecasting key economic variables, such as GDP growth, is important for the decision making process both on the central administrative level (central bank, government) and for the industry. Due to the difficulties connected to the measurement of GDP, it is published with a delay of a couple of months and repeatedly revised. This creates an obstacle for the policy makers and business people who need to be ahead of or at least quickly adjust to the changes in the economy. Thus, a reliable forecasting method is badly needed, but most of the existing models do not excel in this field. Their poor forecasting performance might result from the fact that a considerable part of them does not take into account the existing non-linearities in the data (e.g. business cycle patterns) and/or fails to explore the full informational content of the data published more frequently or with a shorter lag than the dependent variable. In addition, not all models allow to exploit the informational content of large datasets due to the problem of parameter proliferation.

Many approaches do not take into account the fact that macroeconomic variables often behave differently according to the business cycle phase the economy is in, which means that a model with constant parameters might not reflect the present situation well, yet alone be useful for forecasting. Furthermore, instead of allowing for differences in frequencies of the analyzed series within the same regression, most models require the available data to be transformed (through aggregation or interpolation) so that both left- and right-hand side variables are of the same frequency. That, however, might lead to loss of the informational content of the higher-frequency data. Furthermore, many models are not suitable to deal with large datasets, which considerably limits the choice of possible explanatory variables and ignores useful information from other potential regressors. Finally, most models do not offer the possibility to include in the regression the most recent observations of the higher-frequency variable when the corresponding data on the dependent variable is not yet available. This is a major drawback, as higher-frequency variables can be useful indicators of the current state of the economy and, due to the fact that they are published relatively often and with small delay, might be very helpful in forecasting lower-frequency variables, such as GDP growth.

Possible solutions to some of the above-mentioned issues are regime-switching models, introduced by Hamilton (1989), Mixed Data Sampling Regressions (MIDAS), recently developed by Ghysels, Santa-Clara, and Valkanov (2002), and dynamic factor analysis (see e.g. Stock and Watson (2002a)). Regime-switching models allow for the change of parameters according to the state, which the economy is currently in (e.g. different parameters for the phase of expansion and recession), and accounting for business cycle patterns in modelling macroeconomic variables might improve the forecasting performance of the model. MIDAS models, on the other hand, allow for the inclusion of time series of different frequencies in the same regression, without the need of their previous transformation through aggregation or interpolation. They also make it possible to use the so-called "leads", which means that one can include in the regression the observations of higher-frequency variables even if the corresponding data on lower-frequency variables for the corresponding period is not available yet¹. Thus, they help to use the full informa-

¹This feature makes MIDAS models especially useful for dealing with ragged-edge data (out of the scope of this paper) and nowcasting (MIDAS with "leads").

tional potential of the data. Dynamic factor analysis helps to exploit the informational content of large datasets by transforming them in such a way that a substantial part of the variation of the observed variables can be explained by just a few unobserved factors. Thus, using a single factor that explains a large part of the variation, instead of a single observed variable, may capture more information from the available dataset and yet ensure the parsimony of the model.

A vast literature is available on Markov-switching models, usually in the context of modelling business cycle patterns of macroeconomic data. Anas, Billio, Ferrara, and Duca (2007) explore multivariate Markov-switching models for analyzing the relationship between the phases of the business cycle in the United States and Euro zone. Krolzig (2000) investigates the forecasting performance of the multivariate Markov-switching processes through Monte Carlo experiments and an empirical application for the United States business cycle. Also Clements and Krolzig (1998) study the forecasting performance of Markov-switching models through Monte Carlo simulations and an empirical study for the US GNP. Lahiri and Wang (1994) use the Markov-switching framework to predict the turning points in the US business cycle. Frömmel, MacDonald, and Menkhoff (2005), Cheung and Erlandsson (2005) and Engel (1994) apply Markov-switching models to explain and predict the fluctuations in exchange rates, whereas Pagliacci and Barraez (2010), Evans and Wachtel (1993) and Simon (1996) use the Markov-switching framework to analyze the past dynamics of inflation in Venezuela, the US and Australia respectively.

MIDAS models, recently suggested to the literature, have already found a number of interesting applications in both macroeconomics and finance. Kuzin, Marcellino, and Schumacher (2011) investigate the performance of a MIDAS model for nowcasting and forecasting GDP in the Euro area in comparison to a mixed-frequency VAR (with missing values of the lower frequency variables interpolated by Kalman filter). They conclude that both approaches seem to be complementary, as MIDAS performs better for short forecast horizons, whereas mixed-frequency VAR for longer ones. A similar study is performed by Marcellino and Schumacher (2010), who investigate factor MIDAS models versus state space factor models in forecasting German GDP, using different variants of static and dynamic principal components. They find that factor MIDAS models usually outperform their state-space counterparts in forecasting and the most parsimonious MIDAS regression performs best overall.

Bai, Ghysels, and Wright (2013) investigate the MIDAS regressions versus state space models through Monte Carlo simulations and an empirical exercise on predicting the GDP growth of the United States, concluding that both approaches are comparable in terms of forecasting performance. Clements and Galvão (2008), as well as Clements and Galvão (2009) use MIDAS regressions of monthly and quarterly data for forecasting the GDP growth of the United States and obtain promising results, especially for MIDAS with leads. Andreou, Ghysels, and Kourtellis (2013) test the suitability of using MIDAS factor models with leads for forecasting quarterly GDP growth of the United States with a large dataset of daily financial and quarterly macroeconomic indicators, showing relatively good performance of those kind of models, especially in the crisis periods. Barsoum (2011) carries out a similar analysis for the United Kingdom, comparing MIDAS and Factor-Augmented MIDAS (both with and without leads) with a bunch of benchmark models. He obtains mixed results on the performance of MIDAS models in general, but promising

results for MIDAS with leads.

There are also some financial applications of MIDAS models. Ghysels, Santa-Clara, and Valkanov (2006) use MIDAS to predict foreign exchange volatility. Alper, Fendoglu, and Saltoglu (2012) use MIDAS regression to forecast the stock volatility of chosen emerging and developed markets. Engle, Ghysels, and Sohn (ming) combine MIDAS regressions with GARCH models and introduce GARCH-MIDAS for predicting stock market volatility. Also Forsberg and Ghysels (2006) use MIDAS regressions in predicting realized volatility. Finally, Ghysels, Santa-Clara, and Valkanov (2005) investigate with the help of MIDAS regressions the existence of the trade-off between risk and return on stock equities.

Although useful, Markov-switching and MIDAS models can only address either the problem of business cycle patterns or the difference in frequencies of the data, which is not especially satisfying. Guérin and Marcellino (2013) combine both approaches, introducing the Markov-switching Mixed Data Sampling model (MS-MIDAS). They test it through Monte Carlo simulations and carry out empirical studies on forecasting GDP growth of the United States and the United Kingdom, showing that MS-MIDAS is a useful application. However, in their version of the model Guérin and Marcellino (2013) use the so-called restricted lag polynomial, which is based on a specific function (e.g. exponential function). Depending on this function, some structure is imposed on the weights of the regressors in the model. This prevents parameter proliferation on the one hand, but on the other hand restricts to some extent the values that those weights can take. Although quite flexible, this approach might not fully reflect the data generating process, leading to poor forecasting performance of the model. In fact Foroni, Marcellino, and Schumacher (2011) show by means of Monte Carlo simulations that for small differences in frequencies of the analyzed variables MIDAS with unrestricted lag polynomial (U-MIDAS), that is a model for which the estimated regressor weights are not restricted by any function, for most tested cases performs better than restricted MIDAS. For the rest of the cases the models perform comparably, both in the in-sample and forecasting terms. As for most macroeconomic applications quarterly and monthly data are used (thus the difference in frequencies of the variables is small and parameter proliferation is not a serious problem), the unrestricted version of MIDAS might be very useful for forecasting such variables as GDP growth. Since accounting for business cycle pattern that macroeconomic data usually exhibit might improve the forecasting performance of the model, we extend the U-MIDAS approach by incorporating it into a Markov-switching framework in order to allow for changes in parameters according to the business cycle state of the economy. Thus, we combine the approach of Guérin and Marcellino (2013) and Foroni, Marcellino, and Schumacher (2011), proposing the Markov-switching Mixed Data Sampling model with unrestricted lag polynomial (MS-U-MIDAS).

We evaluate the usefulness of the MS-U-MIDAS model by a Monte Carlo study and an empirical forecasting comparison. We first investigate the in-sample and forecasting qualities of MS-U-MIDAS through Monte Carlo experiments. For two different data generating processes (DGPs) we compare the in-sample and out-of-sample performance of the MS-U-MIDAS model to its restricted counterpart (MS-MIDAS) in terms of Root Mean Squared Error (RMSE). We find that both models perform comparably, although MS-U-MIDAS significantly beats MS-MIDAS in the case when the simulated data are

highly persistent, which is a result consistent with findings of Foroni, Marcellino, and Schumacher (2011).

In our empirical forecast comparison, we apply the MS-U-MIDAS model to forecast GDP growth of the United States using a large dataset of monthly macroeconomic and financial indicators. To reduce the dimension of the data and at the same time efficiently use the available information, we extract factors from the dataset using Principal Component Analysis (PCA). These factors are then used as regressors for forecasting the GDP growth. First, we investigate the in-sample properties of the MS-U-MIDAS model. Then the out-of-sample forecasting performance of the MS-U-MIDAS class of models is compared with the performance of a wide range of models of Markov-switching and MIDAS type, as well as benchmark models such as the random walk (RW), autoregressive (AR) and (autoregressive) distributed lag models ((A)DL). As already mentioned, the construction of the MIDAS-class of models makes it easy to include data of higher frequencies, even if the corresponding data of lower frequency is not available. Thus one can e.g. include leads of monthly variables corresponding to the quarter for which the forecast of a lower frequency variable (such as GDP growth) is made. That quality makes these models a very useful forecasting tool for policy makers and we explore this feature in our analysis, using models with leads, whenever it is possible. We find that in most analyzed cases the forecasting performance of the MS-U-MIDAS class of models is comparable or better than that of their restricted counterparts, which is probably due to the fact that the former model does not impose any structure on the weights of the regressors and thus has more flexibility in adjusting to the true data generating process.

The paper is structured as follows. Section 2 presents the class of MIDAS models used in the analysis. Section 3 describes the design and results of Monte Carlo simulations on the performance of MS-U-MIDAS. In Section 4 we apply the MS-U-MIDAS model for predicting the US GDP growth and present results of our forecasting comparison. Section 5 concludes.

2 Forecasting Models

2.1 The MIDAS model

The MIDAS model has been recently introduced to the literature by Ghysels, Santa-Clara, and Valkanov (2002). The basic version of the MIDAS regression used to obtain an h -step ahead forecast can be written using the notation based on Clements and Galvão (2008):

$$y_t^Q = \beta_0 + \beta_1 B(L^{1/m}; \boldsymbol{\theta}) x_{t-h}^M + \varepsilon_t, \quad (2.1)$$

where $B(L^{1/m}; \boldsymbol{\theta}) = \sum_{k=0}^{K-1} b(k; \boldsymbol{\theta}) L^{(k)/m}$ is the sum of weights assigned to K lags of the independent variable (the lag polynomial). $b(k; \boldsymbol{\theta})$ is the k^{th} weight of the K -lag polynomial, shaped by a certain function of $\boldsymbol{\theta}$ parameters (as e.g. exponential function described below). L denotes the lag operator such that $L^{k/m} x_{t-h}^M = x_{t-h-k/m}^M$ (see equation (2.2) for a detailed explanation). t is the time index for the lower frequency variable y ,

whereas m is the time index for the higher frequency variable x . Q describes variables observed on a quarterly and M on a monthly basis.

Consider an example when the dependent variable is observed at quarterly, whereas the explanatory variable at monthly frequency. In this case the MIDAS model for one-step ahead forecasts can be illustrated by noting that $m = 3$, as for each observation of y 3 observations of x are available. If we include $K = 12$ lags in the model, the MIDAS is given by:

$$y_t^Q = \beta_0 + \beta_1[b(1; \boldsymbol{\theta})x_{t-1}^M + b(2; \boldsymbol{\theta})x_{t-4/3}^M + \dots + b(12; \boldsymbol{\theta})x_{t-14/3}^M] + \varepsilon_t. \quad (2.2)$$

The dependent variable y is explained by an equation, consisting of an intercept β_0 and a lag polynomial weighted by parameter β_1 . Each lag k of the independent variable x is introduced to the regression with a weight $b(k; \boldsymbol{\theta})$. The weights of the lag polynomial sum up to 1 and their values are restricted by a specific function. The lags of the explanatory variable are specified by the measure k/m , where $m = 3$ indicates the number of observations of the higher-frequency variable (e.g. monthly variable x) for each observation of the lower-frequency variable (e.g. quarterly variable y). That is, if e.g. y_t is the observation of the dependent variable for the first quarter of 2011 (March 2011), then x_{t-1} denotes the observation of the explanatory variable for December 2010 (1 quarter before), $x_{t-4/3}$ for November 2010 (4 months before), whereas $x_{t-14/3}$ for January 2010 (14 months before) and so forth.

MIDAS models offer the possibility to include leads in the regression. Leads enable forecasters to use available data of higher-frequency when the corresponding observations of the lower-frequency variable are yet unknown. In this case one can include the most recent observations of the explanatory variable in the lag polynomial of equation (2.2).

The MIDAS model enables the use of the available data in an efficient way and at the same time prevents the proliferation of parameters to be estimated. This is due to the fact that the lags of the higher-frequency variable are weighted according to a certain function that depends only on a small number of parameters $\boldsymbol{\theta}$. That is why, one does not need to estimate the slope parameters of all lags included in the model, but through the estimation of $\boldsymbol{\theta}$ the presumed shape of the lag coefficients $b(k; \boldsymbol{\theta})$ is obtained.

There are a couple of different lag polynomials used in MIDAS regressions (e.g. Beta and Almon lag polynomials). We focus on the most commonly used function for the lag polynomial in MIDAS models - the exponential function. The so-called Exponential Almon lag polynomial parametrizes $b(k; \boldsymbol{\theta})$ according to the following scheme:

$$b(k; \boldsymbol{\theta}) = \frac{\exp(\theta_1 k + \theta_2 k^2 + \dots + \theta_Q k^Q)}{\sum_{k=1}^K \exp(\theta_1 k + \theta_2 k^2 + \dots + \theta_Q k^Q)}. \quad (2.3)$$

This specification ensures that the lag coefficients are positive and sum up to one, which is the necessary condition for the identification of the parameters of the model. The parameter estimation is done by non-linear least squares method. Empirical applications are usually based on two parameters of the above-described function, thus $\boldsymbol{\theta} = (\theta_1, \theta_2)'$, which simplifies the model, but still ensures flexibility in the specification of the shape of the polynomial. A wide variety of shapes of lag coefficients can be obtained, ranging from equal weights for all lags, through weights declining at a given pace, to weights forming a hump shape.

2.2 Unrestricted MIDAS

In some cases the shape of the lag polynomial determined by e.g. the function (2.3) may be too restrictive compared to the underlying data generating process. Therefore a model without restrictions on the weights of the lag polynomial was introduced by Foroni, Marcellino, and Schumacher (2011) and denoted as the unrestricted MIDAS (U-MIDAS):

$$y_t^Q = \beta_0 + \sum_{k=0}^{K-1} \beta_{k+1} x_{t-h-k/m}^M + \varepsilon_t.$$

The notation is consistent with equation (2.1).

What distinguishes the above regression from equation (2.1) is the fact that no structure is imposed on the shape of the weights of the lag polynomial. That means that all the parameters β_{k+1} of this polynomial need to be estimated, whereas in the case of the MIDAS model the number of the parameters to be estimated was by construction limited to four: β_0 , β_1 , θ_1 and θ_1 . However, when the difference in frequencies of the analyzed data is small, as is the case in many macroeconomic applications, the issue of parameter proliferation due to the use of the U-MIDAS model is not especially problematic. In fact Foroni, Marcellino, and Schumacher (2011) show through Monte Carlo simulations that, when the difference in frequencies between the dependent and independent variable is small (e.g. quarterly vs. monthly data), the U-MIDAS model performs similarly or better than MIDAS with the exponential Almon lag polynomial in both in-sample and forecasting terms. In addition, the U-MIDAS model can be estimated by ordinary least squares, which simplifies the estimation in comparison to the MIDAS model. The drawback of the U-MIDAS is that when the difference in frequencies between the variables in the model is large, its performance declines dramatically due to the proliferation of the parameters, so this approach is not suitable for all kinds of analyses. However, for many macroeconomic applications the use of functional lag polynomials in MIDAS does not seem to be necessary and the use of the U-MIDAS model instead may be beneficial.

2.3 Markov-switching U-MIDAS

Macroeconomic data often exhibit a business cycle pattern. Therefore, it is reasonable to assume that the parameters of an econometric model, which is supposed to reflect the behaviour of the variables during different business cycle phases, change according to the phase which the economy is currently in. One possible solution to that problem is the Markov-switching model introduced by Hamilton (1989) where parameters of the model depend on the current economic regime (e.g. parameters differ in the recession and expansion phase). Guérin and Marcellino (2013) combined Markov-switching approach with the MIDAS framework and introduced Markov-switching MIDAS regression (MS-MIDAS):

$$y_t^Q = \beta_0(S_t) + \beta_1(S_t)B(L^{1/m}; \boldsymbol{\theta})x_{t-h}^M + \varepsilon_t(S_t),$$

where $\varepsilon_t|S_t \sim NID(0, \sigma^2(S_t))$, that is, the error terms are normally and identically distributed with mean zero and variance $\sigma^2(S_t)$, which varies with changing states of the

world. $S_t = \{1, \dots, R\}$ denotes different states (regimes) of the world present in the data generating process. The MS-MIDAS model is in a way similar to equation (2.1). However, S_t in the brackets indicates the parameters that change according to different regimes. In the above regression the intercept β_0 , the slope parameter β_1 and the variance of the error term $\sigma^2(S_t)$ are allowed to change.

The probability of transition from the current regime a to regime b is defined as follows:

$$p_{ab} = Pr(S_{t+1} = b | S_t = a).$$

All possible transition probabilities form a matrix P with probabilities of remaining in the same regime in the next period at the diagonal and the probabilities of switching to another state in the next period below and above the diagonal. E.g. for two regimes a and b :

$$P = \begin{pmatrix} p_{aa} & p_{ab} \\ p_{ba} & p_{bb} \end{pmatrix}. \quad (2.4)$$

p_{ab} is the probability of switching from state a to state b in the next period, p_{ba} the probability of changing from regime b to a in the following period, whereas p_{aa} and p_{bb} are the probabilities of staying in the same regime in the next period. The sums of probabilities in each row add up to one. Thus in the case of two regimes it is sufficient to determine e.g. p_{aa} and p_{bb} to obtain the whole matrix. We assume that the transition probabilities stay constant over time, which is a standard approach in the Markov-switching applications.

As explained above, MIDAS models with lag polynomials restricted by some specific function might not be flexible enough to reflect well the true data generating process. That applies also to the Markov-switching version of the model that was introduced by Guérin and Marcellino (2013). As Forni, Marcellino, and Schumacher (2011) found that using the unrestricted version of the model might improve its forecasting performance, we incorporate the unrestricted lag polynomial into the Markov-switching framework and introduce the unrestricted Markov-switching MIDAS model (MS-U-MIDAS):

$$y_t^Q = \beta_0(S_t) + \sum_{k=0}^{K-1} \beta_{k+1}(S_t) x_{t-h-k/m}^M + \varepsilon_t(S_t).$$

The parameters of the above equation, that is, the intercept β_0 , the slope parameter β_{k+1} and the variance of the error term σ_ε^2 can change according to different regimes in order to account for the business cycle pattern of the data. Note that while in the MS-MIDAS of Guérin and Marcellino (2013) the parameters θ stay fixed at their estimated values, all the parameters in the MS-U-MIDAS may switch, giving the latter model more flexibility. With the help of information criteria one can decide on the number of regimes present in the data generating process and on the parameters that are allowed to switch. One can e.g. take into consideration a model with all the above-mentioned parameters switching, but one can also consider a model with e.g. only the intercept and/or the variance of the error term switching. Thus the above presented model offers great flexibility in modelling the available data and may be very useful for forecasting purposes.

All variations of the Markov-switching models presented in this paper, are estimated by the Maximum-Likelihood method. Thus, an assumption about the normality of the error

terms is required. Following the procedure described by Hamilton (1994) we maximize the following log-likelihood function:

$$L = \sum_{t=1}^T \log f(y_t^Q | \Omega_{t-1}),$$

where $f(y_t^Q | \Omega_{t-1})$ denotes the density of y_t^Q conditional on Ω_{t-1} - the information given up to time $t - 1$. The conditional density $f(y_t^Q | \Omega_{t-1})$ can be rewritten as:

$$f(y_t^Q | \Omega_{t-1}) = \sum_{i=1}^R P(S_t = i | \Omega_{t-1}) f(y_t^Q | S_t = i, \Omega_{t-1}).$$

The maximization of the log-likelihood function is carried out with the help of the Expectation Maximization algorithm, as described in Hamilton (1994). We use MATLAB for all computations².

2.4 MIDAS models with autoregressive dynamics

Many empirical studies show that adding an autoregressive term to a model significantly improves its forecasting performance. Therefore, we also introduce autoregressive dynamics into the models considered in this paper. The most straightforward way to do this in the basic version of MIDAS would be by using:

$$y_t^Q = \beta_0 + \lambda y_{t-1}^Q + \beta_1 B(L^{1/m}; \boldsymbol{\theta}) x_{t-1}^M + \varepsilon_t.$$

The above equation can, however, be rewritten as:

$$y_t^Q = \beta_0(1 - \lambda)^{-1} + \beta_1(1 - \lambda L)^{-1} B(L^{1/m}; \boldsymbol{\theta}) x_{t-1}^M + (1 - \lambda L)^{-1} \varepsilon_t.$$

Ghysels, Sinko, and Valkanov (2007) argue that this formulation is problematic, as it results in the weights of the lag polynomial being a product of not only a polynomial in $L^{1/m}$, $B(L^{1/m}; \boldsymbol{\theta})$, but also a polynomial in L , which imposes a seasonal response of the dependent variable to the regressor, even if the seasonal pattern is not present in the explanatory variable itself. Thus the above specification should only be used for data showing clear seasonal patterns. If this is not the case Clements and Galvão (2008) propose a solution for the problem by introducing autoregressive dynamics into MIDAS (MIDAS-AR) as a common factor so that the response of y to x remains non-seasonal:

$$y_t^Q = \beta_0 + \lambda y_{t-1}^Q + \beta_1 B(L^{1/m}; \boldsymbol{\theta})(1 - \lambda L)x_{t-1}^M + \varepsilon_t.$$

A multi-step version of that model can be presented as follows:

$$y_t^Q = \beta_0 + \lambda y_{t-p}^Q + \beta_1 B(L^{1/m}; \boldsymbol{\theta})(1 - \lambda L^p)x_{t-h}^M + \varepsilon_t. \quad (2.5)$$

²We gratefully acknowledge the help of Pierre Guérin who provided us with his GAUSS code for the MS-MIDAS estimation as a robustness check for our code. For the estimation of MS-U-MIDAS class of models we modified the Toolbox for Markov-switching models of Perlin (2011), available at the website: <http://www.mathworks.com/matlabcentral/fileexchange/authors/21596>.

The introduction of the autoregressive dynamics into the U-MIDAS model (U-MIDAS-AR) is straightforward and the model is given by:

$$y_t^Q = \beta_0 + \lambda y_{t-1}^Q + \sum_{k=0}^{K-1} \beta_{k+1} x_{t-h-k/m}^M + \varepsilon_t.$$

The fact that one can introduce the autoregressive dynamics into the U-MIDAS in a more intuitive way than in the case of the MIDAS regression is a big advantage of the former model, as the issue of autoregressive dynamics in MIDAS has not yet been fully resolved in the literature and the solution of Clements and Galvão (2008) has been considered simply a recommendation. However, as the problem addressed both by Clements and Galvão (2008) and Ghysels, Sinko, and Valkanov (2007) does not seem to be negligible, we implement the solution proposed in (2.5) incorporated into the Markov-switching framework.

The Markov-switching versions of MIDAS-AR and U-MIDAS-AR used for h -step ahead forecasts are given by:

$$\text{MS-MIDAS-AR: } y_t^Q = \beta_0(S_t) + \lambda y_{t-p}^Q + \beta_1(S_t) B(L^{1/m}; \boldsymbol{\theta}) (1 - \lambda L^p) x_{t-h}^M + \varepsilon_t(S_t) \quad (2.6)$$

and

$$\text{MS-U-MIDAS-AR: } y_t^Q = \beta_0(S_t) + \sum_{i=1}^p \lambda_i y_{t-i}^Q + \sum_{j=0}^{J-1} \beta_{j+1}(S_t) x_{t-h-j/m}^M + \varepsilon_t(S_t) \quad (2.7)$$

3 Monte Carlo Experiment

We investigate the in-sample fit and forecasting qualities of MS-U-MIDAS with autoregressive dynamics through Monte Carlo experiments for two different data generating processes (DGPs). The first DGP is an extended version of the process used by Foroni, Marcellino, and Schumacher (2011) and is given as a bivariate Markov-switching VAR(1):

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} \rho & \delta_l(S_t) \\ \delta_h & \rho \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} e_{y,t}(S_t) \\ e_{x,t}(S_t) \end{pmatrix}. \quad (3.1)$$

We assume a business cycle pattern in the DGP by allowing some of the parameters of the above-described model to switch between regimes. In our simulations we allow for two regimes. We assume that y_t depends on x_t , but y_t has no influence on x_t . Thus we set the parameter δ_h to zero. For the sake of comparison, we use similar set of possible parameter values as Foroni, Marcellino, and Schumacher (2011). We run the simulations for various values of the parameter ρ to account for different possible degrees of persistence of the variable y_t : $\rho = \{0.1; 0.5; 0.9\}$. However, we assume that the degree of persistence of variable y_t stays constant across regimes, which is an assumption consistent with the models presented in Section 2.4. Unlike ρ , the parameter δ_l , determining how strong x_t influences y_t , can take different values for different regimes: $\delta_l(S_t = 1) = \{0.1; 0.5\}$ and $\delta_l(S_t = 2) = 1$. The error terms also change their characteristics across the

regimes. We assume that $e_{y,t}$ and $e_{x,t}$ are independently and normally distributed with mean zero. Their variances switch between regimes and are chosen in such a way that the unconditional variance of y_t in the first regime equals to 1, whereas in the second regime to 2.

The second DGP we consider for the out-of-sample analysis is an autoregressive Markov-switching MIDAS process (MS-MIDAS-AR) given by equation (2.6). We use an exponential Almon lag polynomial specified in (2.3) with $\theta = (\theta_1, \theta_2)'$. We account for two regimes and we allow the intercept β_0 , the slope parameter β_1 and the variance of the error term σ^2 to switch between the regimes. The parameters θ and the autoregressive parameter λ stay constant across the regimes. We carry out the experiment for two different sets of parameters, whose exact values are given in Table 3. The parameter values were chosen according to Foroni, Marcellino, and Schumacher (2011) and the estimation results for our dataset. We investigate the out-of-sample performance of MS-U-MIDAS-AR versus MS-MIDAS-AR for the case when the latter model is favoured by the DGP.

For both DGPs we assume two different possible matrices of transition probabilities. For the first case $p_{11} = 0.95$ and $p_{22} = 0.85$, whereas for the second case $p_{11} = p_{22} = 0.95$. The high probabilities of staying in the same regime in the next period reflect the high persistence of the business cycle regimes observed in reality. As for both DGPs we only allow for two regimes, the above-described probabilities fully specify the matrix P given in (2.4).

We compare the in-sample and forecasting performance of the MS-U-MIDAS-AR relative to MS-MIDAS-AR to see which of them performs better for both types of generated DGPs. In all types of simulations we account for the start-up effect, that is, we delete the first 100 simulated values of the variables y_t and x_t .

3.1 In-sample analysis

For the in-sample analysis we first generate 600 observations of y_t and x_t , following a bivariate Markov-switching VAR (MS-VAR), described in (3.1). This can be thought of as using 600 monthly observations. Then we assume that y_t is observed only every third period, which would correspond to observing y_t on a quarterly frequency. Thus, we obtain 200 low-frequency observations. Then for each replication we estimate MS-U-MIDAS-AR and MS-MIDAS-AR and investigate the in-sample fit of these models by means of Root Mean Squared Error (RMSE). RMSE is defined as a square root of the average squared differences between the estimated values of the dependent variable $\hat{y}_t$ and the actually observed values of y_t :

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (\hat{y}_t - y_t)^2}{n}},$$

where n is the number of periods that are taken into account by evaluation of the in-sample fit of the model.

We replicate the above-described process 1000 times and obtain results that can be found in Table 1. The table shows the mean over 1000 simulations of the ratio of RMSE of MS-

Table 1: In-sample Root Mean Squared Error of MS-U-MIDAS-AR relative to MS-MIDAS-AR (DGP is a bivariate MS-VAR(1) with two regimes).

ρ	δ_l		transition probabilities		RMSE (MS-U-MIDAS-AR vs. MS-MIDAS-AR)					
					Percentiles					
	$S_t = 1$	$S_t = 2$	p_{11}	p_{22}	mean	10 th	25 th	50 th	75 th	90 th
0.1	0.1	1	0.95	0.85	0.98	0.92	0.95	0.98	1.01	1.03
0.1	0.5	1	0.95	0.85	0.98	0.92	0.95	0.98	1.01	1.03
0.5	0.1	1	0.95	0.85	0.98	0.92	0.95	0.98	1.01	1.04
0.5	0.5	1	0.95	0.85	0.96	0.90	0.93	0.96	0.99	1.02
0.9	0.1	1	0.95	0.85	0.85	0.74	0.79	0.85	0.91	0.95
0.9	0.5	1	0.95	0.85	0.49	0.37	0.43	0.50	0.55	0.59
0.1	0.1	1	0.95	0.95	0.98	0.92	0.96	0.99	1.01	1.04
0.1	0.5	1	0.95	0.95	0.97	0.91	0.95	0.98	1.01	1.03
0.5	0.1	1	0.95	0.95	0.97	0.90	0.94	0.97	1.00	1.03
0.5	0.5	1	0.95	0.95	0.96	0.89	0.92	0.96	0.99	1.02
0.9	0.1	1	0.95	0.95	0.78	0.65	0.71	0.78	0.85	0.93
0.9	0.5	1	0.95	0.95	0.44	0.32	0.37	0.44	0.51	0.56

The table presents the summary of results of 1000 Monte Carlo simulations. For each replication 600 observations of a higher-frequency variable x_t and 200 observations of a lower-frequency variable y_t are generated according to a bivariate MS-VAR(1) model with two regimes. Then the MS-U-MIDAS-AR and MS-MIDAS-AR models are estimated and their in-sample fit is measured by Root Mean Squared Error. The table presents the mean, as well as 25th, 50th, 75th and 90th percentiles of the ratio of the RMSE of the MS-U-MIDAS-AR to the RMSE of MS-MIDAS-AR over all 1000 simulations. The values below 1 indicate a better in-sample performance of the MS-U-MIDAS-AR model, the values above 1 mean that MS-MIDAS-AR outperforms its unrestricted counterpart. The analysis is done for different transition probabilities between the two regimes and for different values of the parameters of the MS-VAR model (see equation (3.1)).

U-MIDAS-AR to the RMSE of MS-MIDAS-AR, as well as its 10th, 25th, 50th, 75th and 90th percentiles to provide an overview of the distribution of the results for all replications. The results in Table 1 are presented as a ratio of the RMSE of MS-U-MIDAS-AR to the RMSE of MS-MIDAS-AR, so the values below 1 mean that MS-U-MIDAS-AR beats MS-MIDAS-AR in terms of the in-sample performance, whereas values above 1 indicate that the latter model is relatively better.

Simulations carried out for different sets of parameters, according to which the DGP was generated, show clearly that in all considered cases, the MS-U-MIDAS-AR on average outperforms the corresponding model with a restricted lag polynomial. However, for most combinations of the parameters the difference in the performance of the two models is small. The only exception is the case when $\rho = 0.9$. Therefore, it seems that the biggest advantage of MS-U-MIDAS-AR over MS-MIDAS-AR is the fact that the former model shows a particularly good relative performance in the situations when the persistence of the variable y_t is high and the analyzed case is close to the unit root.

3.2 Out-of-sample analysis

For the study of the out-of-sample performance of the MS-U-MIDAS-AR model versus MS-MIDAS-AR, we generate additional 30 out-of-sample lower-frequency observations from a specific DGP. In other words, we consider 200 low-frequency observations for the in-sample estimation and 30 for forecasting evaluation. Then we compare the forecasting power of MS-U-MIDAS-AR relative to MS-MIDAS-AR for one-step ahead forecasts by means of Root Mean Squared Error (RMSE). We replicate the procedure 1000 times for two different DGPs that were described in the earlier sections: MS-VAR(1) and MS-MIDAS-AR.

3.2.1 DGP generated as a two-regime MS-VAR

The first out-of-sample experiment is carried out for the DGP generated as a MS-VAR(1) (see equation (3.1)) according to the procedure described above. The results of the forecasting evaluation of the MS-U-MIDAS-AR model versus MS-MIDAS-AR can be found in Table 2. For different combinations of the parameters of the DGP, the table contains the mean and 25th, 50th, 75th and 90th percentiles of the ratio of the RMSE of MS-U-MIDAS-AR relative to the RMSE of MS-MIDAS-AR over all 1000 replications. Values below 1 indicate that MS-U-MIDAS-AR outperforms MS-MIDAS-AR in the out-of-sample analysis, whereas values above 1 indicate that MS-MIDAS-AR is relatively better.

Table 2: Out-of-sample Root Mean Squared Error of MS-U-MIDAS-AR relative to MS-MIDAS-AR (DGP is a bivariate MS-VAR(1) with two regimes).

ρ	δ_i		transition probabilities		RMSE (MS-U-MIDAS-AR vs. MS-MIDAS-AR)					
					Percentiles					
	$S_t = 1$	$S_t = 2$	p_{11}	p_{22}	mean	10 th	25 th	50 th	75 th	90 th
0.1	0.1	1	0.95	0.85	1.04	0.92	0.98	1.03	1.09	1.17
0.1	0.5	1	0.95	0.85	1.04	0.93	0.98	1.03	1.10	1.17
0.5	0.1	1	0.95	0.85	1.04	0.92	0.97	1.03	1.09	1.17
0.5	0.5	1	0.95	0.85	1.01	0.88	0.93	1.00	1.08	1.17
0.9	0.1	1	0.95	0.85	0.91	0.65	0.76	0.90	1.03	1.19
0.9	0.5	1	0.95	0.85	0.58	0.41	0.47	0.56	0.67	0.79
0.1	0.1	1	0.95	0.95	1.05	0.93	0.98	1.03	1.10	1.18
0.1	0.5	1	0.95	0.95	1.04	0.93	0.98	1.04	1.10	1.18
0.5	0.1	1	0.95	0.95	1.02	0.88	0.94	1.01	1.09	1.17
0.5	0.5	1	0.95	0.95	1.01	0.85	0.92	1.00	1.08	1.16
0.9	0.1	1	0.95	0.95	0.83	0.49	0.63	0.79	0.98	1.23
0.9	0.5	1	0.95	0.95	0.54	0.35	0.42	0.51	0.63	0.77

The table presents the summary of results of 1000 Monte Carlo simulations. For each replication 600 observations of a higher-frequency variable x_t and 200 observations of a lower-frequency variable y_t are generated for the in-sample period, whereas 30 lower-frequency (90 higher-frequency) observations are generated for the out-of-sample evaluation according to a bivariate MS-VAR(1) model with two regimes. Then the MS-U-MIDAS-AR and MS-MIDAS-AR models are estimated and their out-of-sample performance is measured by Root Mean Squared Error. The table presents the mean, as well as 25th, 50th, 75th and 90th percentiles of the ratio of the RMSE of the MS-U-MIDAS-AR to the RMSE of MS-MIDAS-AR over all 1000 simulations. The values below 1 indicate a better out-of-sample performance of the MS-U-MIDAS-AR model, the values above 1 mean that MS-MIDAS-AR outperforms its unrestricted counterpart. The analysis is done for different transition probabilities between the two regimes and for different values of the parameters of the MS-VAR model (see equation (3.1)).

For the out-of-sample analysis, when the true DGP is MS-VAR(1), there is no clear winner. For most of the combinations of the parameters, MS-U-MIDAS-AR and MS-MIDAS-AR perform on average comparably, although the latter model seems to be slightly better. However, for the cases, when $\rho = 0.9$, MS-U-MIDAS-AR clearly outperforms MS-MIDAS-AR, which confirms the results obtained for the in-sample analysis. It seems that in the case when the persistence in the dependent variable is high and thus when more information relevant for the future can be exploited from the past data, the unrestricted version of the model performs considerably better in both in-sample and out-of-sample evaluation.

3.2.2 DGP generated as a two-regime MS-MIDAS-AR

In order to investigate the out-of-sample performance of MS-U-MIDAS-AR vs. MS-MIDAS-AR in the case when MS-MIDAS-AR is the true DGP, we generate data that

follow the MS-MIDAS-AR model with an exponential Almon lag and two regimes (see equation (2.6)). We allow the intercept β_0 , the slope parameter β_1 and the variance of the error term σ^2 to switch. We consider two possible sets of parameters of the DGP and the transition probabilities that are also used for the first DGP. The details of the parametrization of the model can be found in Table 3. This table also contains results of the out-of-sample performance evaluation of the MS-U-MIDAS-AR model versus MS-MIDAS-AR. The evaluation is done using the mean and 25th, 50th, 75th and 90th percentiles of the ratio of the RMSE of MS-U-MIDAS-AR relative to the RMSE of MS-MIDAS-AR over all 1000 replications. Values below 1 indicate that MS-U-MIDAS-AR outperforms MS-MIDAS-AR, whereas values above 1 mean that the latter model performs better.

For all considered combinations of parameters the MS-MIDAS-AR model on average slightly outperforms the MS-U-MIDAS-AR, although the differences between them are small. Therefore, it seems that even in the case when MS-MIDAS-AR is favoured (as this is the true DGP), MS-U-MIDAS-AR performs similarly well in the out-of-sample analysis.

Table 3: Out-of-sample Root Mean Squared Error of MS-U-MIDAS-AR relative to MS-MIDAS-AR (DGP is a MS-MIDAS-AR with two regimes).

Choice of parameters for the Data Generating Process							
set	regime	β_0	θ_1	θ_2	β_1	λ	σ^2
1	$S_t = 1$	-1	$2 \cdot 10^{-1}$	$3 \cdot 10^{-2}$	0.6	0.2	1
	$S_t = 2$	1	$2 \cdot 10^{-1}$	$3 \cdot 10^{-2}$	0.2	0.2	0.67
2	$S_t = 1$	-0.5	0.7	-0.5	0.8	0.3	1
	$S_t = 2$	0.5	0.7	-0.5	0.1	0.3	0.67

set	transition probabilities		RMSE (MS-U-MIDAS-AR vs. MS-MIDAS-AR)					
	p_{11}	p_{22}	Percentiles					
			mean	10 th	25 th	50 th	75 th	90 th
1	0.95	0.85	1.04	0.91	0.97	1.03	1.09	1.17
1	0.95	0.95	1.02	0.89	0.96	1.02	1.08	1.17
2	0.95	0.85	1.05	0.91	0.98	1.04	1.11	1.20
2	0.95	0.95	1.04	0.90	0.98	1.03	1.09	1.17

The table presents the summary of results of 1000 Monte Carlo simulations. For each replication 600 observations of a higher-frequency variable x_t and 200 observations of a lower-frequency variable y_t are generated for the in-sample period, whereas 30 lower-frequency (90 higher-frequency) observations are generated for the out-of-sample evaluation according to a MS-MIDAS-AR model with two regimes. Then the MS-U-MIDAS-AR and MS-MIDAS-AR models are estimated and their out-of-sample performance is measured by Root Mean Squared Error. The table presents the mean, as well as 25th, 50th, 75th and 90th percentiles of the ratio of the RMSE of the MS-U-MIDAS-AR to the RMSE of MS-MIDAS-AR over all 1000 simulations. The values below 1 indicate a better out-of-sample performance of the MS-U-MIDAS-AR model, the values above 1 mean that MS-MIDAS-AR outperforms its unrestricted counterpart. The analysis is done for different transition probabilities between the two regimes and for different values of the parameters of the MS-MIDAS-AR model (see equation (2.6)).

4 Forecasting GDP growth of the United States

In our empirical study we investigate the in-sample and forecasting performance of the class of MS-U-MIDAS models in forecasting quarterly GDP growth of the United States with the help of monthly macroeconomic and financial variables. We compare the performance of the MS-U-MIDAS models with the corresponding models of the MS-MIDAS class, as well as some other Markov-switching models: Markov-Switching Distributed Lag model (MS-DL) and Markov-Switching Autoregressive Distributed Lag model (MS-ADL).

The MS-DL and MS-ADL models are equivalent to MS-MIDAS and MS-MIDAS-AR correspondingly with all lags of the explanatory variable equally weighted. That is why, it is sensible to compare these simple models with MS-MIDAS regressions in order to investigate whether the use of models with estimated lag polynomial weights is necessary. In addition, in our comparison we include benchmark models without Markov-switching, in particular MIDAS and U-MIDAS (with and without autoregressive dynamics), Distributed Lag model (DL), Autoregressive Distributed Lag model (ADL), AR(2) and random walk (RW). These models are simpler equivalents of the models with Markov-switching, so it is sensible to use them for the sake of comparison of the model performance. An overview of the models we use in the analysis can be found in Table A.1 in the Appendix.

The first part of the empirical study concentrates on the description and the introductory analysis of the data set, including the extraction of factors from the data. Further parts include the analysis of the in-sample fit of the MS-U-MIDAS-AR model, as well as its forecasting performance in comparison to a wide spectrum of other models mentioned above.

4.1 Data

For the empirical exercise on forecasting quarterly GDP growth of the United States, we use a set of 156 monthly macroeconomic and financial variables, yielding information on i.a.: industrial production, unemployment, balance of payments, inflation, federal debt, bank assets, interest rates, government bonds, stock prices and some leading indicators. The data on output covers the period from 1959:Q1 to 2011:Q3 (211 quarters). The monthly data used in the analysis cover the period from June 1958 to September 2011. As Markov-switching models usually involve the estimation of a large number of parameters, the use of such models requires a long data sample. Thus, although in our sample we include a wide spectrum of macroeconomic and financial variables, our data selection is limited by the availability of time series throughout the analyzed period. The data we use come from Datastream. The information on the business cycles in the United States comes from the National Bureau of Economic Research³ and is used to assess the performance of the model in predicting regime changes in the economy.

We carried out Augmented Dickey-Fuller and Philips-Perron unit root tests for all time series included in our data set to investigate their order of integration. Based on the results we transformed the data to ensure stationarity. Depending on the characteristics and behaviour of the data we either used first/second differences or first/second differences of logarithms of the time series to stationarize them. The GDP data was transformed according to the formula commonly used in these type of applications: $100(\ln(GDP_t) - \ln(GDP_{t-1}))$. Thus, we obtained a stationary GDP growth series, used for further analysis. The details of the transformation for specific variables and the exact description of all time series can be found in Table A.2 in the Appendix.

³Data available at: <http://www.nber.org/cycles/cyclesmain.html>

4.2 Extracting factors from the dataset

MIDAS models are usually applied within a univariate framework, that is, only one explaining variable is included in the model. That ensures parsimony and simplifies the estimation, but does not seem to be an optimal solution, as it is difficult to pick up a single variable that summarizes the overall economic situation accurately and preserves the same level of relevance for the economic activity over a long time span. A sensible approach should explore the informational content of a wide range of available macroeconomic and financial time series, which only as a whole, not as single variables, can reflect the current state of economy. However, the inclusion of many regressors in the model results in parameter proliferation and therefore is very problematic. A possible solution to that issue is the use of factor analysis. There is a rich, recent literature available on techniques of factor extraction, e.g. Forni, Hallin, Lippi, and Reichlin (2000), Bai and Ng (2002), Stock and Watson (2002a), Bai (2003) or Forni, Hallin, Lippi, and Reichlin (2005). It is assumed that an N -dimensional set of explanatory variables $\mathbf{X}_t$ can be represented by a few common latent factors $\mathbf{F}_t$, which are then used as regressors in the main analysis:

$$\mathbf{X}_t = \Delta \mathbf{F}_t + \mathbf{e}_t,$$

where $\mathbf{e}_t$ is a vector of idiosyncratic disturbances.

The main idea behind factor analysis is to transform the available dataset in such a way that a substantial part of the variation of the observed variables can be explained by just a few unobserved factors. Thus, using a single factor that explains a large part of the variation, instead of a single observed variable, may capture more information from the available dataset and yet ensure the parsimony of the model. However, as useful as this method is, one should use it with some caution. The factor analysis is a purely technical way of summarizing the variation of the data set in the most efficient way in order to reduce the dimension of the system. The factors which explain the biggest part of the overall variation in the data set, are not always the best predictors of the dependent variable. Consequently, the choice of factors may play a crucial role in the analysis and the forecasting performance of the model might depend heavily on it. To achieve stable forecasting results and at the same time maintain the parsimony of the model, one may use forecast combinations of single-factor regressions. We follow this approach in further parts of the paper.

Following the above-described way of reducing the dimension of the dataset, we use Principal Component Analysis to extract factors from our set of monthly macroeconomic and financial variables for the United States. For the introductory analysis, we extract factors using the whole available sample period of the monthly data, that is, from June 1958 to September 2011. However, for further analysis of the forecasting performance of the models, we first extract factors for the in-sample period only and then for the out-of-sample period factors are extracted recursively, that is, they are updated with each forecast calculation. For the whole available sample period, we find that the first factor explains about 10% of the variation of the dataset, whereas the first five factors together account for about 27% of the variation. These numbers might not sound impressive, but they are not uncommon for large datasets which include a wide spectrum of different variables (see e.g. Stock and Watson (2002b)). Figure A.1 in the Appendix illustrates graphically the percentage of the whole variation of the dataset, explained by the first ten factors.

The choice of the optimal number of factors for further analysis is not a trivial issue. We follow the approach of Bai and Ng (2002), who treat the selection of the number of factors as a problem of optimizing the trade-off between the goodness-of-fit of the model and its parsimony. We apply different information criteria proposed by the authors, finding that, when the maximum considered number of factors is equal to the dimension of the data (which is a standard approach), all of the criteria indicate that one should use the number of factors equal to the number of variables in the data set. This is not an uncommon result (see Barsoum (2011) or Andreou, Ghysels, and Kourtellis (2013)), but using the maximum number of factors is not a reasonable thing to do, as the main goal of factor analysis is to reduce the dimension of the analyzed system. The alternative is to restrict the maximum number of factors, as one of the reasons for the above-mentioned result might be the fact that when the data set is large and encompasses many different time series, a small number of factors can explain only a relatively small part of the variation of this data set. This results in a large number of factors being chosen by the procedure to satisfy the goodness-of-fit criterion. However, when the maximum possible number of factors is restricted (in our case e.g. to be below 100), the first and second criterion by Bai and Ng (2002) consistently point to four and five factors respectively. Thus, considering five factors for further analysis seems to be a reasonable approach. One can make use of the informational potential in each of these factors by considering them together as regressors within one model or by combining forecasts obtained through models which use them as single explanatory variables. To avoid parameter proliferation we focus on the latter option and investigate in further parts of the paper the accuracy of forecast combinations made with single-factor models. The graphical representation of those factors together with the GDP growth can be found in Figure A.2 in the Appendix.

4.3 Further issues concerning the MS-MIDAS class of models

An issue of utmost importance for the models of Markov-switching type is the choice of the optimal number of regimes and the determination of the parameters that are allowed to switch in the model. For the model specification we use the Akaike (AIC) and Bayesian (BIC) Information Criteria. We test the cases when the number of regimes equals two (recession and expansion) and three (recession, stable growth and rapid expansion). However, the information criteria do not deliver unanimous results on the optimal number of regimes. AIC points towards three regimes (three regimes were also used in a similar study for the US GDP by Guérin and Marcellino (2013), although that result was obtained through BIC), whereas BIC, which penalizes the growth of the number of parameters in the model more strongly, points towards a model with two regimes. Thus, for the sake of robustness we present results for models with two and three regimes. In addition, we find it important to see how the performance of the MS-U-MIDAS model changes in comparison with its restricted counterpart in the case when the number of parameters grows (two vs. three regimes).

Furthermore, with the help of AIC and BIC we investigate models that allow for a switch only in the intercept β_0 , in the intercept β_0 and variance of the error term σ^2 , as well as in the intercept β_0 , variance σ^2 and slope parameter β_1 (the coefficient of the autoregressive term λ and the parameters of the lag polynomial θ do not switch between regimes). Both

criteria favour models with all parameters switching. As the volatility of the analyzed time series of GDP growth changes over time, and increases especially dramatically during the crisis periods, we do not think it is reasonable to consider a model without a switch in the variance of the error term and this case is not further examined in our paper. However, we do consider the other cases, that is, for the sake of completeness we investigate the performance of models with a switch in intercept and variance only, as well as with all the parameters allowed to switch between the regimes.

We also use the above-mentioned information criteria to determine the optimal number of lags of the explanatory variables included in the analyzed models, setting the maximum number of autoregressive lags to 4 (four quarters can be taken into account) and the maximum number of lags of the other explanatory variables to 6 (six for the monthly data, that is the information from the last two quarters can be included) in order to avoid the proliferation of parameters. In most cases the information criteria opt for a different number of lags. Thus, we include results obtained according to both AIC and BIC as a robustness check of the forecasting performance of the analyzed models.

We include leads in order to fully make use of all the available information from the monthly variables, whenever a model allows for this possibility (MIDAS class of models with and without Markov-switching). In addition, we present results for models without leads in order to compare the performance of MIDAS regressions with simpler models such as ADL or DL for exactly the same data sample (leads enable the use of additional available observations which simpler models cannot take into account) and to assess improvement in forecasting performance of MIDAS class of models due to the inclusion of leads.

4.4 In-sample fit of the MS-U-MIDAS model

In this paper we consider many different models with a lot of parameters, so it is difficult to present the estimation results for all of them. Generally, for regime-switching models, one can observe that the parameters differ between the states of the world - small intercept β_0 and high variance σ^2 indicate a recession, whereas high intercept β_0 and low variance σ^2 point to expansion. A useful feature of Markov-switching models is that, apart from the estimated parameters and fitted values of the dependent variable, one can also obtain the probabilities of being in a specific regime in a given point in time. Thus, one can calculate the probabilities of being in a recessionary period. As this feature is particularly important for the policymakers and gives the opportunity to show the in-sample performance of the model in a graphical way, we present the smoothed probabilities of being in a recessionary regime in relation to the actual crisis periods (taken from NBER database).

We consider the performance of the five factors, taken for further analysis, separately and present the results for the MS-U-MIDAS model with two and three regimes (with intercept, slope and variance of the error term switching). For the case of two regimes all of the five factors perform quite well in detecting the recent periods of crisis (after 1990). However, most of them (especially factors 1, 4 and 5) face substantial problems in matching the earlier recessions, which is mainly due to the fact that the overall volatility

Figure 1: Smoothed probabilities (dark line) of a crisis (left) and expansion (right), calculated for the period from 1959:Q1 to 2011:Q3 for the MS-U-MIDAS model with two regimes (all parameters allowed to switch). The probabilities are compared with the actual recessionary periods according to NBER (bars).

of the series before 1990 is considerably higher than after 1990. Factor 2, on the other hand, seems to match the crisis periods before 1990 especially well, whereas the spike

Figure 2: Smoothed probabilities (dark line) of a crisis (left), stable growth (middle) and expansion (right), calculated for the period from 1959:Q1 to 2011:Q3 for the MS-U-MIDAS model with three regimes (all parameters allowed to switch). The probabilities are compared with the actual recessionary periods according to NBER (bars).

representing the dot-com bubble from 2000-2001 is in its case not so conspicuous (see Figure 1). This points to the fact that some of the factors (factor 2 and 3) perform better in matching the recessions before 1990, while others excel in detecting the more recent

crises. This might be an argument in favour of using multiple factors within the same model or combining results obtained from single-factor regressions. The latter option is used in further parts of the paper.

However, the unsatisfactory performance of the model in identifying recessionary periods before 1990 could also be a point in favour of a model with three regimes, which due to higher parameter flexibility might be able to deal with the change in volatility of the GDP growth of the United States across the analyzed period better than the model with two regimes. The estimation of the MS-U-MIDAS model with three regimes and all parameters switching yields slightly better in-sample results for the period before 1990 in terms of matching the actual crisis periods in comparison to the model with two regimes (see Figure 2), although the results for factors 1,4 and 5 are still not satisfactory for this period of time. The recent recessions are again matched more accurately. However, a relative in-sample performance improvement for some factors in the case of three regimes does not guarantee that models with three regimes will perform better than those with two regimes in the out-of-sample analysis, which is the main purpose of this work.

4.5 Out-of-sample performance of the MS-U-MIDAS model

In order to investigate the forecasting performance of the MS-U-MIDAS model, we divide our original sample into two parts: the time between 1959:Q1 and 1999:Q4 is considered the in-sample period (160 quarters), whereas the period from 2000:Q1 to 2011:Q3 (47 quarters) is used for the forecasting exercise. The choice of the in- and out-of-sample periods is arbitrary. However, the in-sample period should be long enough to ensure precise parameter estimation, whereas the out-of-sample period, which encompasses both some expansion and recession periods, is chosen to be long enough to help to reliably assess the forecasting performance of the model.

We calculate one, two and four-step ahead forecasts recursively. We also extract factors from the explanatory dataset in a recursive way. The forecasting exercise is carried out not only for the MS-U-MIDAS class of models, but also for a wide range of other models: MS-MIDAS models with restricted lag polynomial, Markov-switching Distributed Lag model (MS-DL), Markov-switching Autoregressive Distributed Lag model (MS-ADL), and some models without Markov-switching: MIDAS and U-MIDAS (with and without autoregressive dynamics), Distributed Lag model (DL), Autoregressive Distributed Lag model (ADL), AR(2) and random walk (RW). The optimal number of lags of the explanatory variables included in the models are determined through information criteria (AIC and BIC) and there are hardly any differences between the out-of-sample performance of the models estimated according to AIC or BIC. However, in the main part of the paper we focus on the results only for models with the number of lags chosen according to AIC. AIC does not penalize the number of parameters as much as the BIC does, and so creates a potentially more difficult test for the U-MIDAS class of models, which are more prone to parameter proliferation than their restricted counterparts. For the sake of robustness, all results are replicated for BIC and presented in the Appendix (Tables A.6-A.8). Leads are used in the models whenever it is possible (all MIDAS class of models, both restricted and unrestricted), although in the Appendix one can also find the corresponding results

without leads for models with the number of lags chosen according to AIC and BIC (Tables A.9-A.14). The comparison between the results with and without leads shows that adding the most recent available information to the model can considerably improve its forecasting performance.

The forecasts are first calculated for models that use each of the five single factors, considered in the analysis, separately. Then those forecasts are combined according to five different schemes, in order to enhance the forecasting performance of the single factors. We first consider the mean (Mean) and median (Med.) of all single-factor models' forecasts. Further we use a scheme, proposed by Stock and Watson (2004), that assigns weights to single-factor forecasts according to their Squared Discounted Mean Forecast Error (S.D.E.), thus accounting for past forecasting performance of the model and at the same time giving more weight to more recent observations. The forecast weights w_{it} in this method are calculated as follows:

$$w_{it} = \frac{m_{it}^{-1}}{\sum_{j=1}^n m_{jt}^{-1}}, \quad \text{where} \quad m_{it} = \sum_{s=T_0}^{t-h} \delta^{t-h-s} (Y_{s+h}^h - \hat{Y}_{i,s+h|s}^h)^2.$$

δ denotes the discount factor, which in our analysis is equal to 0.9, h is the forecast horizon, Y the actual and $\hat{Y}$ the estimated GDP growth, n the number of forecasts combined, t is the time index and i is the index denoting the forecast of i -th model considered for the combination.

Finally, we also consider a forecast combination scheme which trims the recently worst model - we take account for the recent quarter (Tr.(1)) and recent four quarters (Tr.(4)).

The forecasts obtained through the above-described combination methods are then compared in terms of Root Mean Squared Error (RMSE). We also compare the forecasting performance of the combination schemes with the performance of single-factor models. The results of such exercise for different forecasting horizons can be found in Tables A.3-A.5 in the Appendix. To save space, in the main part of the paper we present results for different forecasting horizons, but only for one forecast combination scheme (Mean) and only for selected models (Table 4). The RMSEs are presented relative to the benchmark model, which is an AR(2). Thus, values below 1 indicate better performance of the specific model than a simple AR(2), whereas values above 1 indicate worse performance.

Our results confirm the findings of many forecasting exercises, which lead to the conclusion that combining forecasts gives more robust and in many cases also better results than using forecasts of individual single-variable models (Timmermann (2006)). Forecast combinations do not only usually display more forecasting accuracy, but also show more reliability than individual forecasts, as the out-of-sample performance of individual forecasts for most analyzed models depends strongly on the single factor used for forecasting. Thus, the same model may display very good or very poor performance depending on which of the five single factors is used for forecasting. There is also no visible pattern which could enable us to rank the factors in terms of their forecasting suitability. The forecasting combination schemes, on the other hand, perform in general better than single forecasts and their performance is considerably more robust across the models. Moreover, the differences in the average performance of the five combination schemes (Mean, Med., S.D.E., Tr.(1) and Tr.(4)) are small for each of the considered regressions, so they are more

Table 4: Root Mean Squared Errors relative to AR(2) model. Calculations for one-step ahead forecasts (number of lags in the models chosen according to AIC, leads used when possible).

Benchmarks	RMSE			
RW	1.11			
AR(2)	1.00			

Model	Switch	1-step ahead	2-step ahead	4-step ahead
3 regimes				
MS-U-MIDAS	I,S,V	0.92	1.04	0.96
MS-U-MIDAS-AR	I,S,V	0.81	0.94	0.95
MS-MIDAS	I,S,V	0.86	1.00	0.93
MS-MIDAS-AR	I,S,V	0.84	0.96	0.94
MS-DL	I,S,V	0.98	0.96	0.94
MS-ADL	I,S,V	0.93	0.97	0.94
Models without Markov-switching				
U-MIDAS	—	0.95	1.03	1.02
U-MIDAS-AR	—	0.85	0.96	1.00
MIDAS	—	0.91	1.06	1.00
MIDAS-AR	—	0.85	0.97	0.99
DL	—	1.00	0.97	1.08
ADL	—	0.94	0.98	0.98

Column "Switch" gives information on the parameters that are allowed to switch in the model: I-intercept β_0 , V-variance of the error term σ^2 , S-slope β_1 .

The table presents the out-of-sample RMSEs for chosen models compared in this paper. The original sample is divided into the in-sample period between 1959:Q1 and 1999:Q4 (160 quarters) and the out-of-sample period between 2000:Q1 and 2011:Q3 (47 quarters). For the out-of-sample period one-, two- and four-step ahead forecasts are calculated recursively. In order to evaluate the forecasting performance of the models we calculate the RMSE for each model for the whole out-of-sample period. The RMSEs are calculated for the forecast combination scheme that involves taking the mean of the forecasts from single factor models.

suitable for the comparison of the out-of-sample performance of the analyzed models and the following conclusions are made on their basis.

The results show that for short forecasting horizons (one-step ahead forecast corresponding to a horizon of one quarter) the best performing model among all investigated regressions is MS-U-MIDAS-AR and this model also beats the benchmarks AR(2) and random walk. This result is robust, whether a model with two or three regimes is considered. However, its restricted counterpart, MS-MIDAS-AR also performs well and the differences between both are in many cases statistically insignificant according to the Diebold-Mariano test⁴. Both types of models beat the benchmark models significantly for one-step ahead forecasts. For two- and four-step-ahead forecasts the performance of all considered models deteriorates, but MS-U-MIDAS-AR still performs relatively well and manages to outperform the benchmarks. It is worth noticing that the performance of the MS-U-MIDAS models for all forecast horizons is robust to the number of regimes, although the number of parameters to be estimated grows fast with the number of considered states of the world. Furthermore, the comparison between the results with and without leads shows that adding the most recent available information to the model can considerably improve its forecasting performance.

⁴Results of the Diebold-Mariano test are available at request.

Moreover, MS-MIDAS models without the autoregressive part, regardless of the kind of the lag polynomial used, perform worse than the corresponding models with autoregressive dynamics. In addition, the version of unrestricted MIDAS model without Markov-switching performs also well, slightly beating the corresponding MIDAS models with the restricted lag polynomial in some of the cases. That confirms the finding that using the restricted version of MIDAS is not always necessary. Other models, investigated in this paper, which perform relatively well and beat the benchmark AR(2) for the five above-described combination schemes are MS-ADL class of models. However, the differences between those models and the AR(2) model are in most cases smaller than for the best performing MS-U-MIDAS models, especially for one-step ahead forecasts.

5 Conclusion

The class of MIDAS models, recently introduced into the literature, opens new possibilities for researchers to use the available data of different frequencies in the most efficient way in forecasting and deal with the problem of different publication lags of many macroeconomic variables. The Markov-switching MIDAS additionally introduces the possibility of modelling the business cycle pattern present in many macroeconomic time series. However, one of the issues still open for the research on MIDAS is the optimal choice of the shape of weights in the lag polynomial used in this class of models. In most applications this choice is restricted by a specific function shaping the lag polynomial and might not fully reflect the true data generating process.

In this paper we investigate the usefulness of factor MIDAS models with unrestricted lag polynomial and Markov-switching component for modelling large datasets. We show that it can be implemented with good results for forecasting tasks, when the difference between the frequencies of the analyzed variables is small (e.g. quarterly - monthly data). We compare the MS-U-MIDAS-AR and MS-MIDAS-AR through Monte Carlo simulations for two different DGPs and through an empirical forecasting exercise for the GDP growth of the United States. In both cases, the MS-U-MIDAS-AR model performs comparably or outperforms the MS-MIDAS-AR regression. This makes the unrestricted Markov-switching MIDAS a useful alternative to the standard restricted MS-MIDAS model for many macroeconomic applications, especially when a researcher does not know much about the pattern that the weights of the lag polynomial exhibit. Furthermore, we show that using forecast combinations of models with different factors provides significantly better results than using forecasts from single factor models. To sum up, the methods presented and applied in this paper help to make better use of the informational content of the available datasets and thus obtain better forecasts for the variables of interest.

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A Appendix

Table A.1: Regression models used in the analysis.

Models without Markov-switching	
RW	$y_t^Q = \beta_0 + y_{t-1}^Q + \varepsilon_t$
AR(p)	$y_t^Q = \beta_0 + \sum_{i=1}^p \lambda_i y_{t-i}^Q + \varepsilon_t$
ADL(p,q)	$y_t^Q = \beta_0 + \sum_{i=1}^p \lambda_i y_{t-i}^Q + \sum_{j=1}^q \beta_j x_{t-j}^Q + \varepsilon_t$
MIDAS	$y_t^Q = \beta_0 + \beta_1 B(L^{1/m}; \boldsymbol{\theta}) x_{t-h}^M + \varepsilon_t$
MIDAS-AR	$y_t^Q = \beta_0 + \lambda y_{t-p}^Q + \beta_1 B(L^{1/m}; \boldsymbol{\theta}) (1 - \lambda L^p) x_{t-h}^M + \varepsilon_t$
U-MIDAS	$y_t^Q = \beta_0 + \sum_{k=0}^{K-1} \beta_{k+1} x_{t-h-k/m}^M + \varepsilon_t$
U-MIDAS-AR	$y_t^Q = \beta_0 + \sum_{i=1}^p \lambda_i y_{t-i}^Q + \sum_{k=0}^{K-1} \beta_{k+1} x_{t-h-k/m}^M + \varepsilon_t$
Markov-switching Models	
MS-U-MIDAS (I,V)	$y_t^Q = \beta_0(S_t) + \sum_{k=0}^{K-1} \beta_{k+1} x_{t-h-k/m}^M + \varepsilon_t(S_t)$
MS-U-MIDAS (I,V,S)	$y_t^Q = \beta_0(S_t) + \sum_{k=0}^{K-1} \beta_{k+1}(S_t) x_{t-h-k/m}^M + \varepsilon_t(S_t)$
MS-U-MIDAS-AR (I,V)	$y_t^Q = \beta_0(S_t) + \sum_{i=1}^p \lambda_i y_{t-i}^Q + \sum_{k=0}^{K-1} \beta_{k+1} x_{t-h-k/m}^M + \varepsilon_t(S_t)$
MS-U-MIDAS-AR (I,V,S)	$y_t^Q = \beta_0(S_t) + \sum_{i=1}^p \lambda_i y_{t-i}^Q + \sum_{k=0}^{K-1} \beta_{k+1}(S_t) x_{t-h-k/m}^M + \varepsilon_t(S_t)$
MS-MIDAS (I,V)	$y_t^Q = \beta_0(S_t) + \beta_1 B(L^{1/m}; \boldsymbol{\theta}) x_{t-h}^M + \varepsilon_t(S_t)$
MS-MIDAS (I,V,S)	$y_t^Q = \beta_0(S_t) + \beta_1(S_t) B(L^{1/m}; \boldsymbol{\theta}) x_{t-h}^M + \varepsilon_t(S_t)$
MS-MIDAS-AR (I,V)	$y_t^Q = \beta_0(S_t) + \lambda y_{t-p}^Q + \beta_1 B(L^{1/m}; \boldsymbol{\theta}) (1 - \lambda L^p) x_{t-h}^M + \varepsilon_t(S_t)$
MS-MIDAS-AR (I,V,S)	$y_t^Q = \beta_0(S_t) + \lambda y_{t-p}^Q + \beta_1(S_t) B(L^{1/m}; \boldsymbol{\theta}) (1 - \lambda L^p) x_{t-h}^M + \varepsilon_t(S_t)$
MS-DL(q) (I,V)	$y_t^Q = \beta_0(S_t) + \sum_{j=1}^q \beta_j x_{t-j}^Q + \varepsilon_t(S_t)$
MS-DL(q) (I,V,S)	$y_t^Q = \beta_0(S_t) + \sum_{j=1}^q \beta_j(S_t) x_{t-j}^Q + \varepsilon_t(S_t)$
MS-ADL(p,q) (I,V)	$y_t^Q = \beta_0(S_t) + \sum_{i=1}^p \lambda_i y_{t-i}^Q + \sum_{j=1}^q \beta_j x_{t-j}^Q + \varepsilon_t(S_t)$
MS-ADL(p,q) (I,V,S)	$y_t^Q = \beta_0(S_t) + \sum_{i=1}^p \lambda_i y_{t-i}^Q + \sum_{j=1}^q \beta_j(S_t) x_{t-j}^Q + \varepsilon_t(S_t)$

Notation: Q-quarterly data; M-monthly data; $S_t = \{1, \dots, R\}$ and S_t -regime in period t ; I-switch in the intercept; V-switch in the variance of the error term; S-switch in the slope

Table A.2: Data used in the empirical exercise for the United States.

Nr.	Data description	Transf. code
	REAL GROSS DOMESTIC PRODUCT	5
1	AVERAGE DURATION OF UNEMPLOYMENT (WEEKS)	5
2	AVERAGE PRIME RATE CHARGED BY BANKS	2
3	AVERAGE WEEKLY INITIAL CLAIMS - UNEMPLOYMENT INSURANCE	5
4	AVG HOURLY REAL EARNINGS - CONSTRUCTION	6
5	AVG HOURLY REAL EARNINGS - GOODS-PRODUCING	5
6	AVG HOURLY REAL EARNINGS - MANUFACTURING	5
7	AVG HOURLY REAL EARNINGS - NATURAL RESOURCES & MINING	5
8	AVG OVERTIME HOURS - DURABLE GOODS	5
9	AVG OVERTIME HOURS - MANUFACTURING	5
10	AVG OVERTIME HOURS - NONDURABLE GOODS	5
11	AVG WKLY HOURS - CONSTRUCTION	5
12	AVG WKLY HOURS - DURABLE GOODS	5
13	AVG WKLY HOURS - GOODS-PRODUCING	5
14	AVG WKLY HOURS - MANUFACTURING	5
15	AVG WKLY HOURS - NATURAL RESOURCES & MINING	5
16	AVG WKLY HOURS - NONDURABLE GOODS	5
17	AVG WKLY HOURS - NONMETALLIC MINERAL PRODUCTS	5
18	CHANGE IN EMPLMT. - NONFARM INDUSTRIES	1
19	CHANGE IN PRIVATE EMPLOYMENT - NONFARM INDUSTRIES	1
20	CHICAGO PRCHSG MNGR DIFFUSION INDEX: ORDERS BACKLOG - SMALLER	5
21	CHICAGO PRCHSG MNGR DIFFUSION INDEX: SUPPLIER DELIVERIES - SAME	5
22	CHICAGO PRCHSG MNGR DIFFUSION INDEX: SUPPLIER DELIVERIES, FASTER	2
23	CHICAGO PRCHSG MNGR DIFFUSION INDEX: SUPPLIER DELIVERIES, SLOWER	2
24	CHICAGO PURCHASING MANAGER DIFFUSION INDEX - BACKLOG	5
25	CHICAGO PURCHASING MANAGER DIFFUSION INDEX - DELIVERIES	5
26	CHICAGO PURCHASING MANAGER DIFFUSION INDEX - EMPLOYMENT	5
27	CHICAGO PURCHASING MANAGER DIFFUSION INDEX - INVENTORIES	5
28	CHICAGO PURCHASING MANAGER DIFFUSION INDEX - PRICES PAID	5
29	CHICAGO PURCHASING MANAGER DIFFUSION INDEX - PRODUCTION	5
30	CHICAGO PURCHASING MANAGER DIFFUSION INDEX: EMPLOYMENT - FEWER	5
31	CHICAGO PURCHASING MANAGER DIFFUSION INDEX: EMPLOYMENT - MORE	2
32	CHICAGO PURCHASING MANAGER DIFFUSION INDEX: EMPLOYMENT - SAME	5
33	CHICAGO PURCHASING MANAGER DIFFUSION INDEX: INVENTORIES - LARGER	5
34	CHICAGO PURCHASING MANAGER DIFFUSION INDEX: INVENTORIES - SAME	5
35	CHICAGO PURCHASING MANAGER DIFFUSION INDEX: PRICES PAID - HIGHER	5
36	CHICAGO PURCHASING MANAGER DIFFUSION INDEX: PRICES PAID - LOWER	2
37	CHICAGO PURCHASING MANAGER DIFFUSION INDEX: PRICES PAID - SAME	2
38	CHICAGO PURCHASING MANAGER DIFFUSION INDEX: PRODUCTION - HIGHER	5
39	CHICAGO PURCHASING MANAGER DIFFUSION INDEX: PRODUCTION - LOWER	5
40	CHICAGO PURCHASING MANAGER DIFFUSION INDEX: PRODUCTION - SAME	5
41	CHICAGO PURCHASING MANAGER: BUYING POLICY - 16-30 DAYS	5
42	CHICAGO PURCHASING MANAGER: BUYING POLICY - 31-60 DAYS	5
43	CHICAGO PURCHASING MANAGER: BUYING POLICY - AVERAGE DAYS	5
44	CHICAGO PURCHASING MANAGER: BUYING POLICY - MORE THAN 60 DAYS	5
45	CHICAGO PURCHASING MNGR DIFFUSION INDEX: INVENTORIES - SMALLER	5
46	CHICAGO PURCHASING MNGR DIFFUSION INDEX: ORDERS BACKLOG - LARGER	5
47	CHICAGO PURCHASING MNGR DIFFUSION INDEX: ORDERS BACKLOG - SAME	5
48	CIVILIAN UNEMPLOYMENT RATE	2
49	COML BANK ASSETS - COMMERCIAL & INDL LOANS	1
50	COMMERCIAL BANK ASSETS - BANK CREDIT	2
51	COMMERCIAL BANK ASSETS - CONSUMER LOANS	2
52	COMMERCIAL BANK ASSETS - LOANS & LEASES IN BANK CREDIT	6
53	COMMERCIAL BANK ASSETS - OTHER LOANS & LEASES	1
54	COMMERCIAL BANK ASSETS - OTHER SECURITIES	2
55	COMMERCIAL BANK ASSETS - REAL ESTATE LOANS	2
56	COMMERCIAL BANK ASSETS - SECURITIES IN BANK CREDIT	5
57	COMMERCIAL BANK ASSETS - U.S. GOVERNMENT SECURITIES	5
58	COMMERCIAL BANK LIABILITIES - OTHER	2
59	COMMERCIAL BANK RESIDUAL (ASSETS LESS LIABILITIES)	3
60	CONSUMER CONFIDENCE - EXPECTATIONS	5
61	CONSUMER CREDIT OUTSTANDING	6
62	CONSUMER CREDIT: TOTAL PERCENT CHANGE	2
63	CONSUMER INSTALLMENT CREDIT NET CHANGE	3
64	CORPORATE BOND YIELD - MOODY'S AAA SEASONED ISSUES	2
65	CORPORATE BOND YIELD - MOODY'S BAA SEASONED ISSUES	2
66	CPI - ALL ITEMS LESS FOOD & ENERGY (CORE)	6
67	CPI - ALL URBAN: ALL ITEMS	6
68	CPI ENERGY	5

Continued on Next Page ...

Table A.2: Data used in the empirical exercise for the US. – Continued

Nr.	Data description	Transf. code
69	CPI: PURCHASING POWER OF THE CONSUMER DOLLAR	6
70	CRB SPOT INDEX (1967=100)	5
71	CRB SPOT INDEX FATS & OILS	5
72	CRB SPOT INDEX FOODSTUFFS	5
73	CRB SPOT INDEX LIVESTOCK	5
74	CRB SPOT INDEX METALS	5
75	CRB SPOT INDEX RAW INDUSTRIALS	5
76	CRB SPOT INDEX TEXTILES	5
77	DOW JONES INDUSTRIALS	5
78	ECONOMIC CYCLE RESEARCH INSTITUTE LEADING INDEX	5
79	EURO TO US \$	5
80	EXPORTS	5
81	FEDERAL DEBT - MARKETABLE SEC. TREASURY BILLS	5
82	FEDERAL DEBT - MARKETABLE SEC. TREASURY BONDS	6
83	FEDERAL DEBT - MARKETABLE SEC. TREASURY NOTES	6
84	GOLD HOLDINGS	5
85	GOODS AND SERVICES BALANCE ON A BALANCE OF PAYMENTS BASIS	2
86	GOVERNMENT BENCHMARK BOND - REAL - 10Y YIELD	2
87	HOURLY WAGES	6
88	IMPORTS - PETROLEUM PRODUCTS - CRUDE OIL	6
89	IMPORTS	5
90	INDUSTRIAL PRODUCTION	5
91	INDUSTRIAL PRODUCTION - TOTAL INDEX	5
92	INTERNATIONAL RESERVES	5
93	ISM MANUFACTURERS SURVEY RESULTS: EMPLOYMENT - HIGHER	5
94	ISM MANUFACTURERS SURVEY RESULTS: EMPLOYMENT - LOWER	5
95	ISM MANUFACTURERS SURVEY RESULTS: EMPLOYMENT - SAME	5
96	ISM MANUFACTURERS SURVEY RESULTS: INVENTORIES - HIGHER	5
97	ISM MANUFACTURERS SURVEY RESULTS: INVENTORIES - LOWER	5
98	ISM MANUFACTURERS SURVEY RESULTS: INVENTORIES - SAME	5
99	ISM MANUFACTURERS SURVEY RESULTS: NEW ORDERS - BETTER	5
100	ISM MANUFACTURERS SURVEY RESULTS: NEW ORDERS - SAME	5
101	ISM MANUFACTURERS SURVEY RESULTS: NEW ORDERS - WORSE	5
102	ISM MANUFACTURERS SURVEY RESULTS: PRICES PAID - HIGHER	5
103	ISM MANUFACTURERS SURVEY RESULTS: PRICES PAID - LOWER	2
104	ISM MANUFACTURERS SURVEY RESULTS: PRICES PAID - SAME	5
105	ISM MANUFACTURERS SURVEY RESULTS: PRODUCTION - BETTER	5
106	ISM MANUFACTURERS SURVEY RESULTS: PRODUCTION - SAME	5
107	ISM MANUFACTURERS SURVEY RESULTS: PRODUCTION - WORSE	5
108	ISM MANUFACTURERS SURVEY: EMPLOYMENT INDEX	5
109	ISM MANUFACTURERS SURVEY: INVENTORIES INDEX	5
110	ISM MANUFACTURERS SURVEY: NEW ORDERS INDEX	5
111	ISM MANUFACTURERS SURVEY: PRICES PAID INDEX	5
112	ISM MANUFACTURERS SURVEY: PRODUCTION INDEX	5
113	ISM MANUFACTURERS SURVEY: SUPPLIER DELIVERY INDEX	5
114	ISM MANUFACTURERS SVY RESULTS: EMPLOYMENT - NET	1
115	ISM MANUFACTURERS SVY RESULTS: NEW ORDERS - NET	1
116	ISM MANUFACTURERS SVY RESULTS: PRICES - NET	2
117	ISM MANUFACTURERS SVY RESULTS: PRODUCTION - NET	1
118	ISM PURCHASING MANAGERS INDEX	5
119	LEADING INDEX - COPPER INDUSTRY	5
120	LEADING INDEX - METAL PRICES	5
121	LEADING INDEX - PRIMARY METALS	5
122	LEADING INDEX - STEEL INDUSTRY	5
123	LEADING INDEX - VENDOR PERFORMANCE, SLOWER DELIVERIES	5
124	MFG - INDUSTRIAL CONFIDENCE INDICATOR	5
125	MONEY MARKET RATE (FEDERAL FUNDS)	2
126	NEW PRIVATE HOUSING UNITS STARTED	5
127	NIKKEI 225 STOCK AVERAGE	5
128	PPI - FINISHED GOODS	6
129	PRODUCER PRICE INDEX - CAPITAL EQUIPMENT	6
130	PRODUCER PRICE INDEX - CRUDE MATERIALS	5
131	PRODUCER PRICE INDEX - FINISHED CONSUMER GOODS	6
132	PRODUCER PRICE INDEX - INTERMEDIATE MATERIALS	6
133	S&P/TSX COMPOSITE INDEX	5
134	SDR TO US \$	5
135	STANDARD & POOR'S INDEX OF 500 COMMON STOCKS	5
136	STOCK PRICES - US	5
137	STOCK PRICES - CANADA	5
138	STOCK PRICES - FEDERAL REPUBLIC OF GERMANY	5
139	STOCK PRICES - FRANCE	5

Continued on Next Page ...

Table A.2: Data used in the empirical exercise for the US. – Continued

Nr.	Data description	Transf. code
140	STOCK PRICES - ITALY	5
141	STOCK PRICES - JAPAN	5
142	STOCK PRICES - UNITED KINGDOM	5
143	TOPIX	5
144	TOTAL RETAIL TRADE	5
145	TOTAL TREASURY SECURITIES OUTSTANDING (PUBLIC DEBT)	6
146	TREASURY YIELD ADJUSTED TO CONSTANT MATURITY - 1 YEAR	2
147	TREASURY YIELD ADJUSTED TO CONSTANT MATURITY - 10 YEAR	2
148	TREASURY YIELD ADJUSTED TO CONSTANT MATURITY - 3 YEAR	2
149	TREASURY YIELD ADJUSTED TO CONSTANT MATURITY - 5 YEAR	2
150	UK £ TO US \$	5
151	UNITED STATES GOLD RESERVES	5
152	US FED FUNDS EFF RATE	2
153	US STATE & LOCAL BOND MIDDLE RATE	2
154	US T-BILL SEC MARKET 3 MONTH	2
155	WORKERS ON INVOLUNTARY PARTTIME - ALL INDUSTRIES	5
156	WTI CRUDE OIL SPOT PRICE	5

The above-described data are taken from Datastream.

Transformation codes used for stationarizing the data:

- 1 - stationary data, no transformation needed
- 2 - data stationarized by taking first differences
- 3 - data stationarized by taking second differences
- 5 - data stationarized by taking first differences of logarithms
- 6 - data stationarized by taking second differences of logarithms

Figure A.1: Variation of the monthly dataset explained by the first ten factors. The bars show the percentage of overall variation of the data set explained by each of the first ten factors obtained by Principal Component Analysis. The line shows the cumulated percentage of explained variation for the first ten factors. The maximum number of factors is equal to the dimension of the data set, which is 156.

Figure A.2: GDP growth of the United States and the first five factors extracted from the monthly database by Principal Component Analysis.

Table A.3: Root Mean Squared Errors relative to AR(2) model. Calculations for one-step ahead forecasts (number of lags in the models chosen according to AIC, leads used when possible).

Benchmarks	RMSE										
RW	1.11										
AR(2)	1.00										
Model	Switch	Fac.1	Fac.2	Fac.3	Fac.4	Fac.5	Mean	Med.	S.D.E.	Tr.(1)	Tr.(4)
2 regimes											
MS-U-MIDAS	I,V	0.95	1.24	0.95	1.16	0.87	0.93	0.92	0.92	0.91	0.89
MS-U-MIDAS	I,S,V	0.92	1.35	1.14	1.24	0.88	0.96	0.90	0.94	0.92	0.91
MS-U-MIDAS-AR	I,V	0.89	1.05	0.80	<i>0.97</i>	0.88	0.83	0.86	0.83	<i>0.80</i>	0.82
MS-U-MIDAS-AR	I,S,V	<i>0.80</i>	1.12	0.94	1.00	<i>0.86</i>	<i>0.81</i>	<i>0.78</i>	<i>0.81</i>	0.81	<i>0.81</i>
MS-MIDAS	I,V	0.94	1.21	0.95	1.14	0.87	0.91	0.92	0.90	0.89	0.88
MS-MIDAS	I,S,V	0.91	1.27	0.90	1.20	0.87	0.90	0.90	0.87	0.85	0.85
MS-MIDAS-AR	I,V	0.95	1.07	0.80	1.03	0.90	0.86	0.89	0.84	0.84	0.83
MS-MIDAS-AR	I,S,V	0.92	1.11	<i>0.79</i>	1.11	0.87	0.85	0.84	0.82	0.82	<i>0.81</i>
MS-DL	I,V	1.03	1.19	1.34	1.14	1.04	1.05	1.08	1.06	1.03	1.03
MS-DL	I,S,V	1.03	1.18	1.65	1.15	1.01	1.02	1.01	1.07	1.02	1.02
MS-ADL	I,V	0.92	<i>0.94</i>	1.07	<i>0.97</i>	0.97	0.93	0.94	0.93	0.94	0.92
MS-ADL	I,S,V	0.92	<i>0.94</i>	1.30	0.98	0.96	0.93	0.93	0.93	0.94	0.94
Average		0.93	1.14	1.05	1.09	0.92	0.92	0.91	0.91	0.90	0.89
3 regimes											
MS-U-MIDAS	I,V	0.94	1.12	0.91	1.12	0.89	0.91	0.89	0.90	0.88	0.88
MS-U-MIDAS	I,S,V	1.07	1.21	1.01	1.21	0.98	0.92	0.91	0.92	0.92	0.90
MS-U-MIDAS-AR	I,V	0.91	1.21	0.81	1.07	0.90	0.87	0.88	0.85	0.83	0.86
MS-U-MIDAS-AR	I,S,V	<i>0.88</i>	1.07	1.01	1.10	<i>0.84</i>	<i>0.81</i>	<i>0.82</i>	<i>0.81</i>	<i>0.78</i>	<i>0.78</i>
MS-MIDAS	I,V	0.93	1.08	0.91	1.06	0.93	0.90	0.90	0.89	0.88	0.88
MS-MIDAS	I,S,V	0.93	1.09	0.88	1.12	0.94	0.86	0.89	0.85	0.85	0.85
MS-MIDAS-AR	I,V	0.92	1.02	0.91	1.03	0.91	0.87	0.91	0.87	0.86	0.85
MS-MIDAS-AR	I,S,V	0.98	1.10	<i>0.80</i>	1.09	0.86	0.84	0.84	<i>0.81</i>	0.81	0.82
MS-DL	I,V	0.95	1.03	1.11	1.08	1.04	1.00	1.02	0.99	0.99	0.98
MS-DL	I,S,V	1.00	1.10	1.63	1.02	0.99	0.98	0.98	1.01	0.98	0.98
MS-ADL	I,V	0.93	<i>0.93</i>	1.16	<i>0.97</i>	0.97	0.93	0.92	0.93	0.94	0.92
MS-ADL	I,S,V	0.96	0.98	1.22	1.05	0.94	0.93	0.95	0.93	0.95	0.94
Average		0.95	1.08	1.03	1.08	0.93	0.90	0.91	0.90	0.89	0.89
Models without Markov-switching											
U-MIDAS	—	0.99	1.32	1.07	1.17	0.86	0.95	0.95	0.94	0.90	0.90
U-MIDAS-AR	—	<i>0.94</i>	1.13	0.91	1.03	<i>0.85</i>	<i>0.85</i>	<i>0.84</i>	0.84	0.82	0.82
MIDAS	—	1.00	1.23	1.06	1.16	0.87	0.91	0.90	0.91	0.87	0.86
MIDAS-AR	—	1.00	1.09	<i>0.84</i>	1.06	<i>0.85</i>	<i>0.85</i>	0.85	<i>0.83</i>	<i>0.81</i>	<i>0.81</i>
DL	—	1.12	1.55	1.01	1.11	1.00	1.00	0.96	0.96	0.93	0.97
ADL	—	0.97	<i>1.01</i>	1.09	<i>1.00</i>	1.00	0.94	0.96	0.94	0.93	0.93
Average		1.00	1.22	1.00	1.09	0.91	0.92	0.91	0.90	0.88	0.88

Column "Switch" gives information on the parameters that are allowed to switch in the model: I-intercept β_0 , V-variance of the error term σ^2 , S-slope β_1 .

The table presents the out-of-sample RMSEs for different models compared in this paper. The original sample is divided into the in-sample period between 1959:Q1 and 1999:Q4 (160 quarters) and the out-of-sample period between 2000:Q1 and 2011:Q3 (47 quarters). For the out-of-sample period one-step ahead forecasts are calculated recursively. In order to evaluate the forecasting performance of the models we calculate the RMSE for each model for the whole out-of-sample period. The RMSEs are calculated for the cases when only a forecast from a single factor is taken into account (columns 3-8), as well as for different forecast combination schemes (in this case the forecasts from single-factor-models are combined). The forecast combination schemes include: simple averaging (Mean), median (Med.), forecasts weighted according to Squared Discounted Mean Forecast Error (S.D.E.) and forecast combinations which trim the recently worst model - when last period is considered (Tr.(1)) and when the last four quarters are considered (Tr.(4)). The row "Average" contains the average RMSE of a particular forecast combination scheme over all considered models. Numbers in italics indicate the best performing model within a class of models without Markov-switching, with two regimes or with three regimes.

Table A.4: Root Mean Squared Errors relative to AR(2) model. Calculations for two-step ahead forecasts (number of lags in the models chosen according to AIC, leads used when possible).

Benchmarks	RMSE										
RW	1.09										
AR(2)	1.00										
Model	Switch	Fac.1	Fac.2	Fac.3	Fac.4	Fac.5	Mean	Med.	S.D.E.	Tr.(1)	Tr.(4)
2 regimes											
MS-U-MIDAS	I,V	1.04	1.12	1.10	1.12	1.02	1.04	1.07	1.05	1.04	1.03
MS-U-MIDAS	I,S,V	0.98	1.11	1.15	1.09	1.02	1.01	1.03	1.02	1.01	1.02
MS-U-MIDAS-AR	I,V	0.97	1.00	1.03	1.03	0.98	0.98	0.98	0.98	0.97	0.98
MS-U-MIDAS-AR	I,S,V	0.94	0.99	1.02	1.03	0.98	0.95	0.95	0.95	0.95	0.96
MS-MIDAS	I,V	1.01	1.12	1.15	1.11	1.02	1.06	1.07	1.06	1.04	1.05
MS-MIDAS	I,S,V	1.00	1.16	1.16	1.11	1.04	1.04	1.07	1.05	1.06	1.07
MS-MIDAS-AR	I,V	0.96	1.00	1.03	0.99	0.96	0.97	0.98	0.97	0.98	0.96
MS-MIDAS-AR	I,S,V	0.92	1.06	1.01	1.03	0.97	0.96	0.98	0.96	0.95	0.95
MS-DL	I,V	1.04	1.09	1.09	1.13	1.03	1.05	1.06	1.05	1.04	1.03
MS-DL	I,S,V	1.05	1.17	1.25	1.15	1.02	1.05	1.05	1.06	1.05	1.03
MS-ADL	I,V	0.98	0.98	0.97	0.99	0.98	0.96	0.97	0.96	0.96	0.96
MS-ADL	I,S,V	0.98	1.02	1.11	1.03	0.97	0.96	0.98	0.97	0.96	0.97
Average		0.99	1.07	1.09	1.07	1.00	1.00	1.02	1.01	1.00	1.00
3 regimes											
MS-U-MIDAS	I,V	1.04	1.03	1.09	1.04	0.98	1.02	1.04	1.02	1.02	1.03
MS-U-MIDAS	I,S,V	1.21	1.05	1.10	1.04	1.00	1.04	1.03	1.04	1.05	1.05
MS-U-MIDAS-AR	I,V	1.00	0.99	1.08	0.99	0.99	0.98	1.00	0.98	0.99	0.97
MS-U-MIDAS-AR	I,S,V	0.97	0.98	1.08	1.02	0.92	0.94	0.96	0.94	0.93	0.94
MS-MIDAS	I,V	0.97	1.04	1.08	1.06	1.00	1.00	1.03	1.00	1.01	1.01
MS-MIDAS	I,S,V	1.00	1.04	1.07	1.02	1.03	1.00	1.02	1.00	1.01	0.99
MS-MIDAS-AR	I,V	0.98	1.04	1.06	1.00	1.00	1.00	1.00	1.00	1.02	1.00
MS-MIDAS-AR	I,S,V	0.97	1.04	1.06	0.95	0.93	0.96	0.93	0.96	0.95	0.95
MS-DL	I,V	0.98	1.01	1.02	1.05	1.02	1.00	1.00	1.00	0.99	1.00
MS-DL	I,S,V	0.96	1.05	1.10	1.08	0.96	0.96	0.97	0.98	0.95	0.95
MS-ADL	I,V	1.00	0.97	0.99	1.06	0.95	0.98	0.98	0.97	0.98	0.97
MS-ADL	I,S,V	1.00	1.04	1.07	1.03	0.98	0.97	0.98	0.98	0.98	0.98
Average		1.01	1.02	1.07	1.03	0.98	0.99	0.99	0.99	0.99	0.99
Models without Markov-switching											
U-MIDAS	—	1.06	1.14	1.12	1.10	1.01	1.03	1.05	1.04	1.05	1.05
U-MIDAS-AR	—	1.00	1.00	1.03	0.99	0.97	0.96	0.97	0.96	0.97	0.97
MIDAS	—	1.04	1.18	1.12	1.16	1.05	1.06	1.05	1.07	1.04	1.05
MIDAS-AR	—	0.98	1.05	1.01	1.02	1.00	0.97	0.98	0.98	0.99	0.98
DL	—	1.16	1.32	1.04	1.13	1.09	0.97	0.98	0.97	0.95	1.00
ADL	—	0.99	1.00	0.98	1.03	0.98	0.98	0.99	0.97	0.98	0.96
Average		1.04	1.12	1.05	1.07	1.02	0.99	1.00	1.00	1.00	1.00

Column "Switch" gives information on the parameters that are allowed to switch in the model: I-intercept β_0 , V-variance of the error term σ^2 , S-slope β_1 .

The table presents the out-of-sample RMSEs for different models compared in this paper. The original sample is divided into the in-sample period between 1959:Q1 and 1999:Q4 (160 quarters) and the out-of-sample period between 2000:Q1 and 2011:Q3 (47 quarters). For the out-of-sample period two-step ahead forecasts are calculated recursively. In order to evaluate the forecasting performance of the models we calculate the RMSE for each model for the whole out-of-sample period. The RMSEs are calculated for the cases when only a forecast from a single factor is taken into account (columns 3-8), as well as for different forecast combination schemes (in this case the forecasts from single-factor-models are combined). The forecast combination schemes include: simple averaging (Mean), median (Med.), forecasts weighted according to Squared Discounted Mean Forecast Error (S.D.E.) and forecast combinations which trim the recently worst model - when last period is considered (Tr.(1)) and when the last four quarters are considered (Tr.(4)). The row "Average" contains the average RMSE of a particular forecast combination scheme over all considered models. Numbers in italics indicate the best performing model within a class of models without Markov-switching, with two regimes or with three regimes.

Table A.5: Root Mean Squared Errors relative to AR(2) model. Calculations for four-step ahead forecasts (number of lags in the models chosen according to AIC, leads used when possible).

Benchmarks	RMSE										
RW	1.23										
AR(2)	1.00										
Model	Switch	Fac.1	Fac.2	Fac.3	Fac.4	Fac.5	Mean	Med.	S.D.E.	Tr.(1)	Tr.(4)
2 regimes											
MS-U-MIDAS	I,V	1.06	1.08	0.98	1.02	1.01	1.01	1.01	1.01	1.02	1.00
MS-U-MIDAS	I,S,V	0.97	1.04	1.03	1.01	1.01	0.98	0.99	0.98	0.98	0.97
MS-U-MIDAS-AR	I,V	0.99	1.03	0.96	0.99	1.01	0.98	0.99	0.98	0.98	0.98
MS-U-MIDAS-AR	I,S,V	0.98	1.07	1.01	1.04	1.03	0.99	0.99	0.99	0.99	0.97
MS-MIDAS	I,V	1.06	1.02	0.97	0.99	1.01	0.99	1.00	0.99	0.99	0.99
MS-MIDAS	I,S,V	1.02	1.04	1.02	1.01	0.99	0.99	0.99	0.99	1.00	0.99
MS-MIDAS-AR	I,V	1.01	1.00	0.95	0.98	0.99	0.97	0.98	0.97	0.97	0.97
MS-MIDAS-AR	I,S,V	1.03	1.00	1.02	0.97	1.00	0.99	0.99	0.99	0.99	0.99
MS-DL	I,V	1.03	1.01	0.99	1.01	1.00	0.99	1.00	0.99	1.00	1.00
MS-DL	I,S,V	1.00	1.01	1.04	0.95	1.01	0.96	0.99	0.96	0.97	0.97
MS-ADL	I,V	1.01	0.99	0.97	0.96	1.00	0.97	0.98	0.97	0.98	0.98
MS-ADL	I,S,V	1.01	1.00	1.02	0.95	1.00	0.96	0.98	0.95	0.96	0.96
Average		1.01	1.02	1.00	0.99	1.01	0.98	0.99	0.98	0.99	0.98
3 regimes											
MS-U-MIDAS	I,V	0.97	0.97	0.97	0.96	0.96	0.95	0.96	0.95	0.96	0.97
MS-U-MIDAS	I,S,V	1.06	0.97	0.98	1.00	0.97	0.96	0.96	0.96	0.96	0.95
MS-U-MIDAS-AR	I,V	1.05	1.00	0.97	0.95	0.97	0.96	0.98	0.96	0.96	0.97
MS-U-MIDAS-AR	I,S,V	0.97	0.99	0.96	1.00	0.98	0.95	0.95	0.95	0.95	0.96
MS-MIDAS	I,V	0.97	1.00	0.96	0.96	0.97	0.96	0.96	0.96	0.96	0.96
MS-MIDAS	I,S,V	0.96	0.99	0.94	0.90	0.98	0.93	0.94	0.93	0.93	0.94
MS-MIDAS-AR	I,V	0.96	0.97	0.95	1.00	0.94	0.95	0.95	0.95	0.95	0.95
MS-MIDAS-AR	I,S,V	0.94	0.99	0.95	0.92	0.95	0.94	0.95	0.94	0.94	0.94
MS-DL	I,V	0.95	0.95	0.93	0.97	0.96	0.95	0.95	0.95	0.94	0.94
MS-DL	I,S,V	0.98	1.00	0.92	0.93	0.96	0.94	0.97	0.93	0.94	0.94
MS-ADL	I,V	0.98	0.95	0.95	0.98	0.98	0.96	0.95	0.96	0.96	0.96
MS-ADL	I,S,V	0.99	0.97	0.91	0.92	1.01	0.94	0.95	0.93	0.94	0.94
Average		0.98	0.98	0.95	0.96	0.97	0.95	0.96	0.95	0.95	0.95
Models without Markov-switching											
U-MIDAS	—	1.07	1.12	1.03	1.03	1.02	1.02	1.02	1.02	1.01	1.01
U-MIDAS-AR	—	1.04	1.10	1.06	1.01	1.01	1.00	1.00	1.00	1.00	1.00
MIDAS	—	1.07	1.07	1.00	1.02	1.01	1.00	1.02	1.00	1.00	1.01
MIDAS-AR	—	1.04	1.03	0.99	1.04	1.00	0.99	0.99	0.99	0.99	1.00
DL	—	1.22	1.02	1.08	1.14	1.10	1.08	1.06	1.08	1.07	1.07
ADL	—	1.00	1.02	1.01	0.97	1.00	0.98	0.99	0.98	0.98	0.98
Average		1.08	1.06	1.03	1.04	1.02	1.01	1.01	1.01	1.01	1.01

Column "Switch" gives information on the parameters that are allowed to switch in the model: I-intercept β_0 , V-variance of the error term σ^2 , S-slope β_1 .

The table presents the out-of-sample RMSEs for different models compared in this paper. The original sample is divided into the in-sample period between 1959:Q1 and 1999:Q4 (160 quarters) and the out-of-sample period between 2000:Q1 and 2011:Q3 (47 quarters). For the out-of-sample period four-step ahead forecasts are calculated recursively. In order to evaluate the forecasting performance of the models we calculate the RMSE for each model for the whole out-of-sample period. The RMSEs are calculated for the cases when only a forecast from a single factor is taken into account (columns 3-8), as well as for different forecast combination schemes (in this case the forecasts from single-factor-models are combined). The forecast combination schemes include: simple averaging (Mean), median (Med.), forecasts weighted according to Squared Discounted Mean Forecast Error (S.D.E.) and forecast combinations which trim the recently worst model - when last period is considered (Tr.(1)) and when the last four quarters are considered (Tr.(4)). The row "Average" contains the average RMSE of a particular forecast combination scheme over all considered models. Numbers in italics indicate the best performing model within a class of models without Markov-switching, with two regimes or with three regimes.

Table A.6: Root Mean Squared Errors relative to AR(2) model. Calculations for one-step ahead forecasts (number of lags in the models chosen according to BIC, leads used when possible).

Benchmarks	RMSE										
RW	1.11										
AR(2)	1.00										
Model	Switch	Fac.1	Fac.2	Fac.3	Fac.4	Fac.5	Mean	Med.	S.D.E.	Tr.(1)	Tr.(4)
2 regimes											
MS-U-MIDAS	I,V	0.93	1.20	0.95	1.15	<i>0.87</i>	0.92	0.94	0.91	0.89	0.89
MS-U-MIDAS	I,S,V	0.95	1.24	1.00	1.22	0.88	0.94	0.92	0.92	0.90	0.89
MS-U-MIDAS-AR	I,V	0.87	1.02	0.82	<i>0.97</i>	<i>0.87</i>	0.83	0.85	0.83	<i>0.80</i>	<i>0.81</i>
MS-U-MIDAS-AR	I,S,V	<i>0.80</i>	1.12	0.93	1.00	<i>0.87</i>	<i>0.82</i>	<i>0.78</i>	<i>0.81</i>	0.81	<i>0.81</i>
MS-MIDAS	I,V	0.94	1.21	0.95	1.14	<i>0.87</i>	0.91	0.92	0.90	0.89	0.88
MS-MIDAS	I,S,V	0.91	1.27	0.90	1.20	<i>0.87</i>	0.90	0.90	0.87	0.85	0.85
MS-MIDAS-AR	I,V	0.95	1.07	0.80	1.03	0.90	0.86	0.89	0.84	0.84	0.83
MS-MIDAS-AR	I,S,V	0.92	1.11	<i>0.79</i>	1.11	<i>0.87</i>	0.85	0.84	0.82	0.82	<i>0.81</i>
MS-DL	I,V	1.03	1.19	1.34	1.13	1.04	1.05	1.09	1.06	1.04	1.04
MS-DL	I,S,V	1.01	1.18	1.65	1.12	1.01	1.05	1.05	1.10	1.03	1.04
MS-ADL	I,V	0.92	<i>0.94</i>	1.09	<i>0.97</i>	0.98	0.93	0.94	0.93	0.94	0.93
MS-ADL	I,S,V	0.98	<i>0.94</i>	1.42	0.98	0.98	0.94	0.94	0.96	0.95	0.95
Average		0.93	1.12	1.05	1.09	0.92	0.92	0.92	0.91	0.90	0.89
3 regimes											
MS-U-MIDAS	I,V	0.94	1.12	0.91	1.09	0.89	0.91	0.91	0.90	0.88	0.88
MS-U-MIDAS	I,S,V	0.91	1.08	1.01	1.21	0.98	0.94	0.92	0.93	0.93	0.92
MS-U-MIDAS-AR	I,V	0.92	1.21	0.83	1.07	0.91	0.88	0.89	0.86	0.85	0.87
MS-U-MIDAS-AR	I,S,V	<i>0.90</i>	1.06	0.93	1.10	<i>0.83</i>	<i>0.81</i>	<i>0.84</i>	<i>0.80</i>	<i>0.80</i>	<i>0.79</i>
MS-MIDAS	I,V	0.93	1.08	0.91	1.06	0.93	0.90	0.90	0.89	0.88	0.88
MS-MIDAS	I,S,V	0.93	1.09	0.88	1.12	0.94	0.86	0.89	0.85	0.85	0.85
MS-MIDAS-AR	I,V	0.92	1.02	0.91	1.03	0.91	0.87	0.91	0.87	0.86	0.85
MS-MIDAS-AR	I,S,V	0.98	1.10	<i>0.80</i>	1.09	0.86	0.84	<i>0.84</i>	0.81	0.81	0.82
MS-DL	I,V	0.95	1.03	1.11	1.08	1.04	1.00	1.02	1.00	0.99	0.98
MS-DL	I,S,V	1.00	1.04	1.54	1.02	0.99	0.97	0.98	1.00	0.96	0.96
MS-ADL	I,V	0.93	<i>0.95</i>	1.09	<i>0.97</i>	0.97	0.92	0.93	0.92	0.93	0.91
MS-ADL	I,S,V	0.97	1.04	1.41	1.03	0.94	0.95	0.97	0.98	0.95	0.96
Average		0.94	1.07	1.03	1.07	0.93	0.90	0.92	0.90	0.89	0.89
Models without Markov-switching											
U-MIDAS	—	0.99	1.32	1.07	1.17	0.86	0.95	0.95	0.94	0.90	0.90
U-MIDAS-AR	—	<i>0.93</i>	1.17	<i>0.84</i>	1.05	<i>0.85</i>	0.86	<i>0.83</i>	0.84	0.82	0.83
MIDAS	—	1.00	1.23	1.06	1.16	0.87	0.91	0.90	0.91	0.87	0.86
MIDAS-AR	—	1.00	1.09	<i>0.84</i>	1.06	<i>0.85</i>	<i>0.85</i>	0.85	<i>0.83</i>	<i>0.81</i>	<i>0.81</i>
DL	—	1.12	1.55	1.01	1.11	1.00	1.00	0.96	0.96	0.93	0.97
ADL	—	1.01	<i>1.05</i>	1.09	<i>1.04</i>	1.00	0.96	0.99	0.96	0.97	0.96
Average		1.01	1.24	0.98	1.10	0.91	0.92	0.91	0.91	0.88	0.89

Column "Switch" gives information on the parameters that are allowed to switch in the model: I-intercept β_0 , V-variance of the error term σ^2 , S-slope β_1 .

The table presents the out-of-sample RMSEs for different models compared in this paper. The original sample is divided into the in-sample period between 1959:Q1 and 1999:Q4 (160 quarters) and the out-of-sample period between 2000:Q1 and 2011:Q3 (47 quarters). For the out-of-sample period one-step ahead forecasts are calculated recursively. In order to evaluate the forecasting performance of the models we calculate the RMSE for each model for the whole out-of-sample period. The RMSEs are calculated for the cases when only a forecast from a single factor is taken into account (columns 3-8), as well as for different forecast combination schemes (in this case the forecasts from single-factor-models are combined). The forecast combination schemes include: simple averaging (Mean), median (Med.), forecasts weighted according to Squared Discounted Mean Forecast Error (S.D.E.) and forecast combinations which trim the recently worst model - when last period is considered (Tr.(1)) and when the last four quarters are considered (Tr.(4)). The row "Average" contains the average RMSE of a particular forecast combination scheme over all considered models. Numbers in italics indicate the best performing model within a class of models without Markov-switching, with two regimes or with three regimes.

Table A.7: Root Mean Squared Errors relative to AR(2) model. Calculations for two-step ahead forecasts (number of lags in the models chosen according to BIC, leads used when possible).

Benchmarks	RMSE										
RW	1.09										
AR(2)	1.00										
Model	Switch	Fac.1	Fac.2	Fac.3	Fac.4	Fac.5	Mean	Med.	S.D.E.	Tr.(1)	Tr.(4)
2 regimes											
MS-U-MIDAS	I,V	1.03	1.15	1.10	1.12	1.02	1.05	1.06	1.06	1.05	1.04
MS-U-MIDAS	I,S,V	1.01	1.16	1.07	1.12	1.02	1.04	1.06	1.05	1.04	1.05
MS-U-MIDAS-AR	I,V	0.97	1.01	1.03	1.02	0.96	0.98	0.99	0.98	0.98	0.97
MS-U-MIDAS-AR	I,S,V	0.94	0.98	1.02	1.03	0.99	0.96	0.96	0.96	0.96	0.96
MS-MIDAS	I,V	1.01	1.12	1.15	1.11	1.02	1.06	1.07	1.06	1.04	1.05
MS-MIDAS	I,S,V	1.00	1.16	1.16	1.11	1.04	1.04	1.07	1.05	1.06	1.07
MS-MIDAS-AR	I,V	0.96	1.00	1.03	0.99	0.96	0.97	0.98	0.97	0.98	0.96
MS-MIDAS-AR	I,S,V	0.92	1.06	1.01	1.03	0.97	0.96	0.98	0.96	0.95	0.95
MS-DL	I,V	1.04	1.09	1.09	1.10	1.03	1.04	1.06	1.05	1.03	1.03
MS-DL	I,S,V	1.05	1.17	1.25	1.10	1.02	1.04	1.05	1.06	1.05	1.05
MS-ADL	I,V	0.98	0.98	0.95	0.99	0.97	0.95	0.97	0.95	0.95	0.95
MS-ADL	I,S,V	0.98	1.02	1.08	1.03	0.96	0.96	0.96	0.97	0.96	0.97
Average		0.99	1.08	1.08	1.06	1.00	1.00	1.02	1.01	1.00	1.01
3 regimes											
MS-U-MIDAS	I,V	1.04	1.03	1.09	1.05	0.98	1.02	1.03	1.02	1.02	1.03
MS-U-MIDAS	I,S,V	0.98	0.98	1.10	1.04	1.00	0.99	1.02	0.99	0.98	0.98
MS-U-MIDAS-AR	I,V	1.00	0.99	1.08	0.99	0.99	0.98	0.99	0.98	0.99	0.97
MS-U-MIDAS-AR	I,S,V	0.97	1.00	1.04	1.03	0.92	0.95	0.95	0.95	0.95	0.96
MS-MIDAS	I,V	0.97	1.04	1.08	1.06	1.00	1.00	1.03	1.00	1.01	1.01
MS-MIDAS	I,S,V	1.00	1.04	1.07	1.02	1.03	1.00	1.02	1.00	1.01	0.99
MS-MIDAS-AR	I,V	0.98	1.04	1.06	1.00	1.00	1.00	1.00	1.00	1.02	1.00
MS-MIDAS-AR	I,S,V	0.97	1.04	1.06	0.95	0.93	0.96	0.93	0.96	0.95	0.95
MS-DL	I,V	0.98	1.01	1.02	1.01	1.02	1.00	1.01	1.00	1.00	1.00
MS-DL	I,S,V	0.96	1.05	1.10	1.08	0.96	0.97	0.97	0.98	0.95	0.95
MS-ADL	I,V	1.00	0.97	0.98	1.06	0.95	0.98	0.98	0.97	0.98	0.97
MS-ADL	I,S,V	0.97	0.96	0.93	1.01	0.98	0.92	0.94	0.92	0.92	0.92
Average		0.98	1.01	1.05	1.03	0.98	0.98	0.99	0.98	0.98	0.98
Models without Markov-switching											
U-MIDAS	—	1.06	1.14	1.12	1.10	1.01	1.03	1.05	1.04	1.05	1.05
U-MIDAS-AR	—	1.00	1.02	1.01	0.99	0.97	0.97	0.97	0.97	0.99	0.97
MIDAS	—	1.04	1.18	1.12	1.16	1.05	1.06	1.05	1.07	1.04	1.05
MIDAS-AR	—	0.98	1.05	1.01	1.02	1.00	0.97	0.98	0.98	0.99	0.98
DL	—	1.16	1.32	1.04	1.13	1.09	0.97	0.98	0.97	0.95	1.00
ADL	—	0.99	1.03	0.98	1.03	0.98	0.98	0.98	0.97	0.97	0.96
Average		1.04	1.12	1.05	1.07	1.02	1.00	1.00	1.00	1.00	1.00

Column "Switch" gives information on the parameters that are allowed to switch in the model: I-intercept β_0 , V-variance of the error term σ^2 , S-slope β_1 .

The table presents the out-of-sample RMSEs for different models compared in this paper. The original sample is divided into the in-sample period between 1959:Q1 and 1999:Q4 (160 quarters) and the out-of-sample period between 2000:Q1 and 2011:Q3 (47 quarters). For the out-of-sample period two-step ahead forecasts are calculated recursively. In order to evaluate the forecasting performance of the models we calculate the RMSE for each model for the whole out-of-sample period. The RMSEs are calculated for the cases when only a forecast from a single factor is taken into account (columns 3-8), as well as for different forecast combination schemes (in this case the forecasts from single-factor-models are combined). The forecast combination schemes include: simple averaging (Mean), median (Med.), forecasts weighted according to Squared Discounted Mean Forecast Error (S.D.E.) and forecast combinations which trim the recently worst model - when last period is considered (Tr.(1)) and when the last four quarters are considered (Tr.(4)). The row "Average" contains the average RMSE of a particular forecast combination scheme over all considered models. Numbers in italics indicate the best performing model within a class of models without Markov-switching, with two regimes or with three regimes.

Table A.8: Root Mean Squared Errors relative to AR(2) model. Calculations for four-step ahead forecasts (number of lags in the models chosen according to BIC, leads used when possible).

Benchmarks	RMSE										
RW	1.23										
AR(2)	1.00										
Model	Switch	Fac.1	Fac.2	Fac.3	Fac.4	Fac.5	Mean	Med.	S.D.E.	Tr.(1)	Tr.(4)
2 regimes											
MS-U-MIDAS	I,V	1.05	1.03	0.98	1.01	1.01	1.00	1.00	1.00	1.00	1.00
MS-U-MIDAS	I,S,V	1.02	1.04	1.01	1.04	1.01	1.00	1.00	1.00	1.00	0.99
MS-U-MIDAS-AR	I,V	0.99	1.00	0.97	0.99	1.01	0.98	0.98	0.98	0.98	0.97
MS-U-MIDAS-AR	I,S,V	0.98	1.07	0.99	1.04	1.02	0.99	0.99	0.99	1.00	0.97
MS-MIDAS	I,V	1.06	1.02	0.97	0.99	1.01	0.99	1.00	0.99	0.99	0.99
MS-MIDAS	I,S,V	1.02	1.04	1.02	1.01	0.99	0.99	0.99	0.99	1.00	0.99
MS-MIDAS-AR	I,V	1.01	1.00	0.95	0.98	0.99	0.97	0.98	0.97	0.97	0.97
MS-MIDAS-AR	I,S,V	1.03	1.00	1.02	0.97	1.00	0.99	0.99	0.99	0.99	0.99
MS-DL	I,V	1.03	1.01	0.99	1.00	1.00	0.99	1.00	0.99	1.00	0.99
MS-DL	I,S,V	1.01	1.01	1.04	0.95	1.01	0.96	0.99	0.96	0.97	0.98
MS-ADL	I,V	0.99	0.99	0.97	0.96	1.00	0.97	0.99	0.97	0.97	0.97
MS-ADL	I,S,V	0.99	1.00	1.03	0.95	0.99	0.95	0.97	0.95	0.96	0.96
Average		1.01	1.02	1.00	0.99	1.00	0.98	0.99	0.98	0.99	0.98
3 regimes											
MS-U-MIDAS	I,V	0.97	0.97	0.97	0.95	0.96	0.95	0.96	0.95	0.96	0.95
MS-U-MIDAS	I,S,V	0.97	0.97	0.98	1.00	0.97	0.96	0.97	0.96	0.96	0.95
MS-U-MIDAS-AR	I,V	1.05	1.00	0.98	0.95	0.97	0.97	0.98	0.96	0.96	0.97
MS-U-MIDAS-AR	I,S,V	0.97	0.96	0.97	1.00	0.98	0.95	0.96	0.95	0.97	0.96
MS-MIDAS	I,V	0.97	1.00	0.96	0.96	0.97	0.96	0.96	0.96	0.96	0.96
MS-MIDAS	I,S,V	0.96	0.99	0.94	0.90	0.98	0.93	0.94	0.93	0.93	0.94
MS-MIDAS-AR	I,V	0.96	0.97	0.95	1.00	0.94	0.95	0.95	0.95	0.95	0.95
MS-MIDAS-AR	I,S,V	0.94	0.99	0.95	0.92	0.95	0.94	0.95	0.94	0.94	0.94
MS-DL	I,V	0.95	0.95	0.93	0.95	0.96	0.94	0.95	0.94	0.94	0.94
MS-DL	I,S,V	0.98	0.97	0.92	0.93	0.96	0.93	0.96	0.93	0.94	0.93
MS-ADL	I,V	0.98	0.95	0.95	0.98	0.98	0.96	0.96	0.96	0.95	0.96
MS-ADL	I,S,V	0.97	0.98	0.91	0.91	1.01	0.94	0.97	0.94	0.93	0.92
Average		0.97	0.97	0.95	0.95	0.97	0.95	0.96	0.95	0.95	0.95
Models without Markov-switching											
U-MIDAS	—	1.07	1.12	1.03	1.03	1.02	1.02	1.02	1.02	1.01	1.01
U-MIDAS-AR	—	1.00	1.08	1.01	1.01	1.01	0.99	1.00	0.99	0.99	0.99
MIDAS	—	1.07	1.07	1.00	1.02	1.01	1.00	1.02	1.00	1.00	1.01
MIDAS-AR	—	1.04	1.03	0.99	1.04	1.00	0.99	0.99	0.99	0.99	1.00
DL	—	1.22	1.02	1.08	1.14	1.10	1.08	1.06	1.08	1.07	1.07
ADL	—	1.00	1.02	1.01	0.97	1.00	0.98	0.99	0.98	0.98	0.98
Average		1.07	1.06	1.02	1.03	1.02	1.01	1.01	1.01	1.01	1.01

Column "Switch" gives information on the parameters that are allowed to switch in the model: I-intercept β_0 , V-variance of the error term σ^2 , S-slope β_1 .

The table presents the out-of-sample RMSEs for different models compared in this paper. The original sample is divided into the in-sample period between 1959:Q1 and 1999:Q4 (160 quarters) and the out-of-sample period between 2000:Q1 and 2011:Q3 (47 quarters). For the out-of-sample period four-step ahead forecasts are calculated recursively. In order to evaluate the forecasting performance of the models we calculate the RMSE for each model for the whole out-of-sample period. The RMSEs are calculated for the cases when only a forecast from a single factor is taken into account (columns 3-8), as well as for different forecast combination schemes (in this case the forecasts from single-factor-models are combined). The forecast combination schemes include: simple averaging (Mean), median (Med.), forecasts weighted according to Squared Discounted Mean Forecast Error (S.D.E.) and forecast combinations which trim the recently worst model - when last period is considered (Tr.(1)) and when the last four quarters are considered (Tr.(4)). The row "Average" contains the average RMSE of a particular forecast combination scheme over all considered models. Numbers in italics indicate the best performing model within a class of models without Markov-switching, with two regimes or with three regimes.

Table A.9: Root Mean Squared Errors relative to AR(2) model. Calculations for one-step ahead forecasts (number of lags in the models chosen according to AIC, no leads used).

Benchmarks	RMSE										
RW	1.11										
AR(2)	1.00										
Model	Switch	Fac.1	Fac.2	Fac.3	Fac.4	Fac.5	Mean	Med.	S.D.E.	Tr.(1)	Tr.(4)
2 regimes											
MS-U-MIDAS	I,V	0.97	1.24	1.09	1.17	1.07	1.04	1.02	1.03	1.03	1.03
MS-U-MIDAS	I,S,V	0.97	1.32	1.24	1.22	1.01	1.04	1.02	1.02	1.02	1.01
MS-U-MIDAS-AR	I,V	0.88	1.01	0.89	0.99	0.95	0.90	0.90	0.90	0.88	0.91
MS-U-MIDAS-AR	I,S,V	0.95	1.02	1.01	1.03	1.02	0.93	0.95	0.93	0.93	0.92
MS-MIDAS	I,V	0.98	1.19	1.10	1.18	1.05	1.04	1.05	1.03	1.04	1.03
MS-MIDAS	I,S,V	0.99	1.24	1.44	1.21	1.01	1.05	1.06	1.04	1.03	1.03
MS-MIDAS-AR	I,V	0.93	1.06	0.91	1.03	0.99	0.94	0.97	0.93	0.93	0.93
MS-MIDAS-AR	I,S,V	0.97	1.11	0.97	1.11	0.99	0.94	0.91	0.94	0.94	0.93
Average		0.96	1.15	1.08	1.12	1.01	0.98	0.98	0.98	0.98	0.98
3 regimes											
MS-U-MIDAS	I,V	0.94	1.06	1.01	1.12	1.01	0.97	1.00	0.97	0.98	0.97
MS-U-MIDAS	I,S,V	1.10	1.17	1.35	1.16	1.05	0.97	1.06	0.95	0.94	0.94
MS-U-MIDAS-AR	I,V	0.89	1.08	1.04	1.04	0.96	0.94	0.90	0.94	0.97	0.95
MS-U-MIDAS-AR	I,S,V	1.10	1.10	1.56	1.31	1.02	0.95	1.00	0.95	0.98	0.98
MS-MIDAS	I,V	0.94	1.06	0.99	1.09	1.07	0.97	1.02	0.96	0.96	0.97
MS-MIDAS	I,S,V	0.96	1.12	0.94	1.18	1.00	0.96	0.97	0.95	0.94	0.94
MS-MIDAS-AR	I,V	0.95	1.02	0.94	1.04	1.00	0.94	0.98	0.94	0.94	0.94
MS-MIDAS-AR	I,S,V	0.96	1.06	0.93	1.06	0.96	0.92	0.91	0.92	0.93	0.90
Average		0.98	1.08	1.09	1.12	1.01	0.95	0.98	0.95	0.95	0.95
Models without Markov-switching											
U-MIDAS	—	1.02	1.30	1.19	1.18	1.03	1.04	1.05	1.02	1.02	1.02
U-MIDAS-AR	—	0.96	1.07	0.95	1.02	0.97	0.92	0.90	0.91	0.90	0.92
MIDAS	—	1.02	1.23	1.21	1.15	1.04	1.02	1.00	1.01	1.00	1.00
MIDAS-AR	—	1.01	1.09	0.95	1.05	0.98	0.94	0.93	0.93	0.93	0.93
Average		1.00	1.17	1.08	1.10	1.00	0.98	0.97	0.97	0.96	0.97

Column "Switch" gives information on the parameters that are allowed to switch in the model: I-intercept β_0 , V-variance of the error term σ^2 , S-slope β_1 .

The table presents the out-of-sample RMSEs for different models compared in this paper. The original sample is divided into the in-sample period between 1959:Q1 and 1999:Q4 (160 quarters) and the out-of-sample period between 2000:Q1 and 2011:Q3 (47 quarters). For the out-of-sample period one-step ahead forecasts are calculated recursively. In order to evaluate the forecasting performance of the models we calculate the RMSE for each model for the whole out-of-sample period. The RMSEs are calculated for the cases when only a forecast from a single factor is taken into account (columns 3-8), as well as for different forecast combination schemes (in this case the forecasts from single-factor-models are combined). The forecast combination schemes include: simple averaging (Mean), median (Med.), forecasts weighted according to Squared Discounted Mean Forecast Error (S.D.E.) and forecast combinations which trim the recently worst model - when last period is considered (Tr.(1)) and when the last four quarters are considered (Tr.(4)). The row "Average" contains the average RMSE of a particular forecast combination scheme over all considered models. Numbers in italics indicate the best performing model within a class of models without Markov-switching, with two regimes or with three regimes.

Table A.10: Root Mean Squared Errors relative to AR(2) model. Calculations for two-step ahead forecasts (number of lags in the models chosen according to AIC, no leads used).

Benchmarks	RMSE										
RW	1.09										
AR(2)	1.00										
Model	Switch	Fac.1	Fac.2	Fac.3	Fac.4	Fac.5	Mean	Med.	S.D.E.	Tr.(1)	Tr.(4)
2 regimes											
MS-U-MIDAS	I,V	1.07	1.10	1.08	1.11	1.06	1.06	1.07	1.06	1.06	1.05
MS-U-MIDAS	I,S,V	1.04	1.16	1.12	1.11	1.02	1.04	1.05	1.05	1.05	1.03
MS-U-MIDAS-AR	I,V	1.00	<i>0.98</i>	<i>0.97</i>	<i>1.00</i>	<i>0.96</i>	<i>0.96</i>	<i>0.97</i>	<i>0.96</i>	0.98	<i>0.97</i>
MS-U-MIDAS-AR	I,S,V	1.00	1.02	1.01	1.01	<i>0.96</i>	0.97	0.98	<i>0.96</i>	<i>0.97</i>	<i>0.97</i>
MS-MIDAS	I,V	1.05	1.10	1.08	1.11	1.06	1.06	1.08	1.07	1.06	1.05
MS-MIDAS	I,S,V	1.03	1.18	1.13	1.12	1.03	1.05	1.07	1.06	1.06	1.04
MS-MIDAS-AR	I,V	<i>0.98</i>	<i>0.98</i>	<i>0.97</i>	1.01	<i>0.96</i>	<i>0.96</i>	<i>0.97</i>	<i>0.96</i>	<i>0.97</i>	<i>0.97</i>
MS-MIDAS-AR	I,S,V	0.99	1.00	0.99	1.01	<i>0.96</i>	0.97	0.98	0.97	<i>0.97</i>	0.98
Average		1.02	1.06	1.04	1.06	1.00	1.01	1.02	1.01	1.01	1.01
3 regimes											
MS-U-MIDAS	I,V	1.04	1.02	1.00	1.02	1.03	1.01	1.02	1.01	1.02	1.01
MS-U-MIDAS	I,S,V	1.02	1.03	1.04	1.05	1.00	0.99	1.00	0.98	0.98	0.98
MS-U-MIDAS-AR	I,V	1.00	0.98	<i>0.98</i>	<i>0.97</i>	0.96	<i>0.95</i>	<i>0.96</i>	<i>0.96</i>	0.96	0.97
MS-U-MIDAS-AR	I,S,V	<i>0.98</i>	1.00	1.01	1.18	<i>0.95</i>	0.97	0.98	<i>0.96</i>	<i>0.95</i>	<i>0.94</i>
MS-MIDAS	I,V	1.00	1.06	1.01	1.00	1.06	1.01	1.02	1.01	1.02	1.00
MS-MIDAS	I,S,V	0.99	1.04	1.06	1.06	1.02	1.00	1.01	1.00	1.00	1.00
MS-MIDAS-AR	I,V	<i>0.98</i>	<i>0.97</i>	1.03	1.02	1.01	0.98	0.99	0.98	0.98	0.98
MS-MIDAS-AR	I,S,V	0.99	1.08	<i>0.98</i>	1.03	0.97	0.99	0.99	0.99	1.00	0.99
Average		1.00	1.02	1.02	1.04	1.00	0.99	1.00	0.99	0.99	0.98
Models without Markov-switching											
U-MIDAS	—	1.09	1.14	1.12	1.10	1.03	1.05	1.06	1.06	1.06	1.06
U-MIDAS-AR	—	1.03	<i>0.99</i>	<i>1.01</i>	<i>0.99</i>	<i>0.97</i>	<i>0.97</i>	<i>0.98</i>	<i>0.97</i>	<i>0.98</i>	<i>0.97</i>
MIDAS	—	1.06	1.15	1.12	1.15	1.07	1.07	1.07	1.07	1.07	1.06
MIDAS-AR	—	<i>1.01</i>	1.03	<i>1.01</i>	1.02	1.00	0.98	1.00	0.99	0.99	0.99
Average		1.05	1.08	1.06	1.07	1.02	1.02	1.03	1.02	1.03	1.02

Column "Switch" gives information on the parameters that are allowed to switch in the model: I-intercept β_0 , V-variance of the error term σ^2 , S-slope β_1 .

The table presents the out-of-sample RMSEs for different models compared in this paper. The original sample is divided into the in-sample period between 1959:Q1 and 1999:Q4 (160 quarters) and the out-of-sample period between 2000:Q1 and 2011:Q3 (47 quarters). For the out-of-sample period two-step ahead forecasts are calculated recursively. In order to evaluate the forecasting performance of the models we calculate the RMSE for each model for the whole out-of-sample period. The RMSEs are calculated for the cases when only a forecast from a single factor is taken into account (columns 3-8), as well as for different forecast combination schemes (in this case the forecasts from single-factor-models are combined). The forecast combination schemes include: simple averaging (Mean), median (Med.), forecasts weighted according to Squared Discounted Mean Forecast Error (S.D.E.) and forecast combinations which trim the recently worst model - when last period is considered (Tr.(1)) and when the last four quarters are considered (Tr.(4)). The row "Average" contains the average RMSE of a particular forecast combination scheme over all considered models. Numbers in italics indicate the best performing model within a class of models without Markov-switching, with two regimes or with three regimes.

Table A.11: Root Mean Squared Errors relative to AR(2) model. Calculations for four-step ahead forecasts (number of lags in the models chosen according to AIC, no leads used).

Benchmarks	RMSE										
RW	1.23										
AR(2)	1.00										
Model	Switch	Fac.1	Fac.2	Fac.3	Fac.4	Fac.5	Mean	Med.	S.D.E.	Tr.(1)	Tr.(4)
2 regimes											
MS-U-MIDAS	I,V	1.04	1.05	0.99	0.99	1.02	1.00	1.02	1.00	1.01	1.00
MS-U-MIDAS	I,S,V	1.00	0.99	1.03	0.98	1.01	0.96	0.99	0.96	0.97	0.97
MS-U-MIDAS-AR	I,V	0.98	1.02	0.97	0.98	1.00	0.98	0.98	0.98	0.98	0.98
MS-U-MIDAS-AR	I,S,V	1.00	1.04	1.03	1.00	1.01	0.98	1.00	0.98	0.99	0.97
MS-MIDAS	I,V	1.06	1.03	0.97	0.99	1.02	1.00	1.01	1.00	1.00	1.00
MS-MIDAS	I,S,V	1.03	1.04	1.06	1.00	1.02	1.00	1.00	1.00	1.00	1.01
MS-MIDAS-AR	I,V	1.01	0.99	0.97	0.97	0.99	0.97	0.98	0.97	0.97	0.98
MS-MIDAS-AR	I,S,V	1.04	1.03	1.00	0.97	0.99	0.99	0.99	0.99	0.99	0.99
Average		1.02	1.02	1.00	0.99	1.01	0.99	1.00	0.98	0.99	0.99
3 regimes											
MS-U-MIDAS	I,V	0.96	0.95	0.94	0.94	0.97	0.94	0.95	0.94	0.95	0.95
MS-U-MIDAS	I,S,V	1.00	1.02	0.97	0.98	0.95	0.95	0.97	0.96	0.97	0.96
MS-U-MIDAS-AR	I,V	0.96	0.97	0.96	0.95	0.98	0.95	0.97	0.95	0.96	0.95
MS-U-MIDAS-AR	I,S,V	1.07	1.02	0.95	1.03	0.99	0.97	0.99	0.98	0.99	0.98
MS-MIDAS	I,V	0.99	0.98	0.96	0.95	0.98	0.96	0.96	0.96	0.96	0.97
MS-MIDAS	I,S,V	0.98	1.00	0.97	0.99	1.02	0.96	0.97	0.96	0.97	0.98
MS-MIDAS-AR	I,V	0.97	0.97	0.95	0.92	0.97	0.95	0.95	0.95	0.95	0.95
MS-MIDAS-AR	I,S,V	0.96	0.99	0.93	0.96	1.00	0.94	0.96	0.94	0.95	0.94
Average		0.99	0.99	0.96	0.96	0.98	0.95	0.97	0.95	0.96	0.96
Models without Markov-switching											
U-MIDAS	—	1.06	1.09	1.04	0.99	1.02	1.01	1.02	1.00	1.01	1.00
U-MIDAS-AR	—	1.07	1.08	1.01	0.98	1.00	1.00	1.02	1.00	1.01	1.00
MIDAS	—	1.07	1.07	1.03	1.00	1.03	1.01	1.03	1.01	1.00	1.01
MIDAS-AR	—	1.04	1.06	1.01	0.98	0.97	0.99	1.00	0.99	0.99	0.98
Average		1.06	1.07	1.02	0.99	1.01	1.00	1.02	1.00	1.00	1.00

Column "Switch" gives information on the parameters that are allowed to switch in the model: I-intercept β_0 , V-variance of the error term σ^2 , S-slope β_1 .

The table presents the out-of-sample RMSEs for different models compared in this paper. The original sample is divided into the in-sample period between 1959:Q1 and 1999:Q4 (160 quarters) and the out-of-sample period between 2000:Q1 and 2011:Q3 (47 quarters). For the out-of-sample period four-step ahead forecasts are calculated recursively. In order to evaluate the forecasting performance of the models we calculate the RMSE for each model for the whole out-of-sample period. The RMSEs are calculated for the cases when only a forecast from a single factor is taken into account (columns 3-8), as well as for different forecast combination schemes (in this case the forecasts from single-factor-models are combined). The forecast combination schemes include: simple averaging (Mean), median (Med.), forecasts weighted according to Squared Discounted Mean Forecast Error (S.D.E.) and forecast combinations which trim the recently worst model - when last period is considered (Tr.(1)) and when the last four quarters are considered (Tr.(4)). The row "Average" contains the average RMSE of a particular forecast combination scheme over all considered models. Numbers in italics indicate the best performing model within a class of models without Markov-switching, with two regimes or with three regimes.

Table A.12: Root Mean Squared Errors relative to AR(2) model. Calculations for one-step ahead forecasts (number of lags in the models chosen according to BIC, no leads used).

Benchmarks	RMSE										
RW	1.11										
AR(2)	1.00										
Model	Switch	Fac.1	Fac.2	Fac.3	Fac.4	Fac.5	Mean	Med.	S.D.E.	Tr.(1)	Tr.(4)
2 regimes											
MS-U-MIDAS	I,V	0.97	1.20	1.09	1.18	1.06	1.05	1.07	1.04	1.03	1.04
MS-U-MIDAS	I,S,V	0.98	1.23	1.14	1.23	1.01	1.03	1.05	1.02	1.01	1.00
MS-U-MIDAS-AR	I,V	0.90	0.99	0.91	0.99	0.96	0.92	0.89	0.91	0.91	0.90
MS-U-MIDAS-AR	I,S,V	0.95	1.00	1.00	1.03	0.97	0.92	0.92	0.92	0.93	0.93
MS-MIDAS	I,V	0.98	1.19	1.10	1.18	1.05	1.04	1.05	1.03	1.04	1.03
MS-MIDAS	I,S,V	0.99	1.24	1.44	1.21	1.01	1.05	1.06	1.04	1.03	1.03
MS-MIDAS-AR	I,V	0.93	1.06	0.91	1.03	0.99	0.94	0.97	0.93	0.93	0.93
MS-MIDAS-AR	I,S,V	0.97	1.11	0.97	1.11	0.99	0.94	0.91	0.94	0.94	0.93
Average		0.96	1.13	1.07	1.12	1.01	0.99	0.99	0.98	0.98	0.97
3 regimes											
MS-U-MIDAS	I,V	0.94	1.06	1.01	1.06	1.00	0.97	0.98	0.96	0.96	0.96
MS-U-MIDAS	I,S,V	0.95	1.12	1.03	1.16	0.96	0.96	0.99	0.95	0.97	0.97
MS-U-MIDAS-AR	I,V	0.91	1.09	0.97	1.03	0.96	0.93	0.90	0.93	0.96	0.93
MS-U-MIDAS-AR	I,S,V	1.11	1.10	1.04	1.11	1.02	0.95	0.99	0.95	0.91	0.95
MS-MIDAS	I,V	0.94	1.06	0.99	1.09	1.07	0.97	1.02	0.96	0.96	0.97
MS-MIDAS	I,S,V	0.96	1.12	0.94	1.18	1.00	0.96	0.97	0.95	0.94	0.94
MS-MIDAS-AR	I,V	0.95	1.02	0.94	1.04	1.00	0.94	0.98	0.94	0.94	0.94
MS-MIDAS-AR	I,S,V	0.96	1.06	0.93	1.06	0.96	0.92	0.91	0.92	0.93	0.90
Average		0.97	1.08	0.98	1.09	1.00	0.95	0.97	0.94	0.95	0.94
Models without Markov-switching											
U-MIDAS	—	1.04	1.30	1.19	1.18	1.03	1.05	1.04	1.04	1.03	1.03
U-MIDAS-AR	—	0.98	1.12	0.97	1.05	0.97	0.95	0.94	0.94	0.93	0.95
MIDAS	—	1.02	1.23	1.21	1.15	1.04	1.02	1.00	1.01	1.00	1.00
MIDAS-AR	—	1.01	1.09	0.95	1.05	0.98	0.94	0.93	0.93	0.93	0.93
Average		1.01	1.18	1.08	1.11	1.00	0.99	0.98	0.98	0.97	0.98

Column "Switch" gives information on the parameters that are allowed to switch in the model: I-intercept β_0 , V-variance of the error term σ^2 , S-slope β_1 .

The table presents the out-of-sample RMSEs for different models compared in this paper. The original sample is divided into the in-sample period between 1959:Q1 and 1999:Q4 (160 quarters) and the out-of-sample period between 2000:Q1 and 2011:Q3 (47 quarters). For the out-of-sample period one-step ahead forecasts are calculated recursively. In order to evaluate the forecasting performance of the models we calculate the RMSE for each model for the whole out-of-sample period. The RMSEs are calculated for the cases when only a forecast from a single factor is taken into account (columns 3-8), as well as for different forecast combination schemes (in this case the forecasts from single-factor-models are combined). The forecast combination schemes include: simple averaging (Mean), median (Med.), forecasts weighted according to Squared Discounted Mean Forecast Error (S.D.E.) and forecast combinations which trim the recently worst model - when last period is considered (Tr.(1)) and when the last four quarters are considered (Tr.(4)). The row "Average" contains the average RMSE of a particular forecast combination scheme over all considered models. Numbers in italics indicate the best performing model within a class of models without Markov-switching, with two regimes or with three regimes.

Table A.13: Root Mean Squared Errors relative to AR(2) model. Calculations for two-step ahead forecasts (number of lags in the models chosen according to BIC, no leads used).

Benchmarks	RMSE										
RW	1.09										
AR(2)	1.00										
Model	Switch	Fac.1	Fac.2	Fac.3	Fac.4	Fac.5	Mean	Med.	S.D.E.	Tr.(1)	Tr.(4)
2 regimes											
MS-U-MIDAS	I,V	1.07	1.11	1.08	1.12	1.06	1.07	1.09	1.07	1.07	1.06
MS-U-MIDAS	I,S,V	1.05	1.16	1.09	1.10	1.02	1.06	1.09	1.06	1.08	1.06
MS-U-MIDAS-AR	I,V	0.99	0.99	<i>0.97</i>	<i>1.00</i>	<i>0.96</i>	0.97	0.98	0.97	0.98	<i>0.97</i>
MS-U-MIDAS-AR	I,S,V	0.99	0.99	<i>0.97</i>	1.01	<i>0.96</i>	<i>0.96</i>	<i>0.97</i>	<i>0.96</i>	<i>0.97</i>	0.98
MS-MIDAS	I,V	1.05	1.10	1.08	1.11	1.06	1.06	1.08	1.07	1.06	1.05
MS-MIDAS	I,S,V	1.03	1.18	1.13	1.12	1.03	1.05	1.07	1.06	1.06	1.04
MS-MIDAS-AR	I,V	<i>0.98</i>	<i>0.98</i>	<i>0.97</i>	1.01	<i>0.96</i>	<i>0.96</i>	<i>0.97</i>	<i>0.96</i>	<i>0.97</i>	<i>0.97</i>
MS-MIDAS-AR	I,S,V	0.99	1.00	0.99	1.01	<i>0.96</i>	0.97	0.98	0.97	<i>0.97</i>	0.98
Average		1.02	1.06	1.04	1.06	1.00	1.01	1.03	1.02	1.02	1.01
3 regimes											
MS-U-MIDAS	I,V	1.04	1.02	1.00	1.02	1.01	1.00	1.01	1.01	1.00	1.01
MS-U-MIDAS	I,S,V	<i>0.96</i>	1.03	1.01	1.05	0.99	1.00	1.00	1.00	0.99	0.99
MS-U-MIDAS-AR	I,V	1.00	0.98	1.00	<i>0.97</i>	0.96	0.96	0.97	0.96	<i>0.96</i>	<i>0.96</i>
MS-U-MIDAS-AR	I,S,V	0.97	1.00	<i>0.97</i>	0.99	<i>0.95</i>	<i>0.95</i>	<i>0.94</i>	<i>0.95</i>	<i>0.96</i>	<i>0.96</i>
MS-MIDAS	I,V	1.00	1.06	1.01	1.00	1.06	1.01	1.02	1.01	1.02	1.00
MS-MIDAS	I,S,V	0.99	1.04	1.06	1.06	1.02	1.00	1.01	1.00	1.00	1.00
MS-MIDAS-AR	I,V	0.98	<i>0.97</i>	1.03	1.02	1.01	0.98	0.99	0.98	0.98	0.98
MS-MIDAS-AR	I,S,V	0.99	1.08	0.98	1.03	0.97	0.99	0.99	0.99	1.00	0.99
Average		0.99	1.02	1.01	1.02	1.00	0.99	0.99	0.99	0.99	0.99
Models without Markov-switching											
U-MIDAS	—	1.06	1.14	1.12	1.10	1.03	1.04	1.06	1.05	1.06	1.04
U-MIDAS-AR	—	<i>1.01</i>	<i>1.01</i>	<i>1.00</i>	<i>0.99</i>	<i>0.97</i>	<i>0.97</i>	<i>0.99</i>	<i>0.97</i>	<i>0.99</i>	<i>0.97</i>
MIDAS	—	1.06	1.15	1.12	1.15	1.07	1.07	1.07	1.07	1.07	1.06
MIDAS-AR	—	<i>1.01</i>	1.03	1.01	1.02	1.00	0.98	1.00	0.99	<i>0.99</i>	0.99
Average		1.04	1.08	1.06	1.07	1.02	1.02	1.03	1.02	1.03	1.02

Column "Switch" gives information on the parameters that are allowed to switch in the model: I-intercept β_0 , V-variance of the error term σ^2 , S-slope β_1 .

The table presents the out-of-sample RMSEs for different models compared in this paper. The original sample is divided into the in-sample period between 1959:Q1 and 1999:Q4 (160 quarters) and the out-of-sample period between 2000:Q1 and 2011:Q3 (47 quarters). For the out-of-sample period two-step ahead forecasts are calculated recursively. In order to evaluate the forecasting performance of the models we calculate the RMSE for each model for the whole out-of-sample period. The RMSEs are calculated for the cases when only a forecast from a single factor is taken into account (columns 3-8), as well as for different forecast combination schemes (in this case the forecasts from single-factor-models are combined). The forecast combination schemes include: simple averaging (Mean), median (Med.), forecasts weighted according to Squared Discounted Mean Forecast Error (S.D.E.) and forecast combinations which trim the recently worst model - when last period is considered (Tr.(1)) and when the last four quarters are considered (Tr.(4)). The row "Average" contains the average RMSE of a particular forecast combination scheme over all considered models. Numbers in italics indicate the best performing model within a class of models without Markov-switching, with two regimes or with three regimes.

Table A.14: Root Mean Squared Errors relative to AR(2) model. Calculations for four-step ahead forecasts (number of lags in the models chosen according to BIC, no leads used).

Benchmarks	RMSE										
RW	1.23										
AR(2)	1.00										
Model	Switch	Fac.1	Fac.2	Fac.3	Fac.4	Fac.5	Mean	Med.	S.D.E.	Tr.(1)	Tr.(4)
2 regimes											
MS-U-MIDAS	I,V	1.00	1.01	0.99	0.99	1.01	0.99	1.00	0.99	0.99	0.99
MS-U-MIDAS	I,S,V	1.02	1.01	1.03	1.01	1.01	1.00	1.01	1.00	1.00	0.99
MS-U-MIDAS-AR	I,V	0.99	0.99	0.97	0.98	1.00	0.98	0.99	0.98	0.98	0.98
MS-U-MIDAS-AR	I,S,V	0.99	1.00	1.01	1.00	1.01	0.98	1.00	0.98	0.99	0.99
MS-MIDAS	I,V	1.06	1.03	0.97	0.99	1.02	1.00	1.01	1.00	1.00	1.00
MS-MIDAS	I,S,V	1.03	1.04	1.06	1.00	1.02	1.00	1.00	1.00	1.00	1.01
MS-MIDAS-AR	I,V	1.01	0.99	0.97	0.97	0.99	0.97	0.98	0.97	0.97	0.98
MS-MIDAS-AR	I,S,V	1.04	1.03	1.00	0.97	0.99	0.99	0.99	0.99	0.99	0.99
Average		1.02	1.01	1.00	0.99	1.00	0.99	1.00	0.99	0.99	0.99
3 regimes											
MS-U-MIDAS	I,V	0.96	0.95	0.94	0.92	0.95	0.93	0.94	0.93	0.94	0.94
MS-U-MIDAS	I,S,V	0.99	0.95	0.97	0.98	0.98	0.96	0.96	0.96	0.97	0.96
MS-U-MIDAS-AR	I,V	0.96	0.97	0.96	0.95	0.98	0.95	0.96	0.95	0.96	0.95
MS-U-MIDAS-AR	I,S,V	1.08	1.02	0.96	0.99	0.99	0.98	0.98	0.98	0.98	0.98
MS-MIDAS	I,V	0.99	0.98	0.96	0.95	0.98	0.96	0.96	0.96	0.96	0.97
MS-MIDAS	I,S,V	0.98	1.00	0.97	0.99	1.02	0.96	0.97	0.96	0.97	0.98
MS-MIDAS-AR	I,V	0.97	0.97	0.95	0.92	0.97	0.95	0.95	0.95	0.95	0.95
MS-MIDAS-AR	I,S,V	0.96	0.99	0.93	0.96	1.00	0.94	0.96	0.94	0.95	0.94
Average		0.98	0.98	0.96	0.96	0.98	0.96	0.96	0.95	0.96	0.96
Models without Markov-switching											
U-MIDAS	—	1.02	1.09	1.04	0.99	1.02	1.01	1.02	1.00	1.00	1.00
U-MIDAS-AR	—	0.99	1.06	1.01	0.97	1.00	0.99	1.01	0.98	0.99	0.99
MIDAS	—	1.07	1.07	1.03	1.00	1.03	1.01	1.03	1.01	1.00	1.01
MIDAS-AR	—	1.04	1.06	1.01	0.98	0.97	0.99	1.00	0.99	0.99	0.98
Average		1.03	1.07	1.02	0.99	1.01	1.00	1.01	0.99	1.00	1.00

Column "Switch" gives information on the parameters that are allowed to switch in the model: I-intercept β_0 , V-variance of the error term σ^2 , S-slope β_1 .

The table presents the out-of-sample RMSEs for different models compared in this paper. The original sample is divided into the in-sample period between 1959:Q1 and 1999:Q4 (160 quarters) and the out-of-sample period between 2000:Q1 and 2011:Q3 (47 quarters). For the out-of-sample period four-step ahead forecasts are calculated recursively. In order to evaluate the forecasting performance of the models we calculate the RMSE for each model for the whole out-of-sample period. The RMSEs are calculated for the cases when only a forecast from a single factor is taken into account (columns 3-8), as well as for different forecast combination schemes (in this case the forecasts from single-factor-models are combined). The forecast combination schemes include: simple averaging (Mean), median (Med.), forecasts weighted according to Squared Discounted Mean Forecast Error (S.D.E.) and forecast combinations which trim the recently worst model - when last period is considered (Tr.(1)) and when the last four quarters are considered (Tr.(4)). The row "Average" contains the average RMSE of a particular forecast combination scheme over all considered models. Numbers in italics indicate the best performing model within a class of models without Markov-switching, with two regimes or with three regimes.