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Policy Effects in a Simple Fully Non-Linear New Keynesian Model of the Liquidity Trap

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Abstract

We analyze a simple yet fully non-linear New Keynesian model with a central bank that pursues an inflation targeting strategy. Our analysis shows that expected adverse productivity shocks may drive the economy into a liquidity trap. As our model entails positive or moderately negative inflation in such a situation, it has the potential to explain the so-called “missing disinflation” in the Great Recession. In contrast with some previous papers, we find that the effects of fiscal policy in a liquidity trap are moderate and that reductions in labor income taxes are expansionary. We do not find support for higher inflation targets. Finally, we provide additional support for the view that the common practice of log-linearizing equilibrium relations can be potentially misleading in models with a lower bound on nominal interest rates.

Keywords: Zero lower bound, missing disinflation, fiscal multiplier, liquidity trap, new Keynesian model, multiple equilibria, inflation target.

JEL: E52, E58, E62.

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1 Introduction

Short-term nominal interest rates in Japan dropped to values close to zero in the mid-1990s and have stayed there since. Moreover, in the aftermath of the global financial crisis of 2008 interest rates fell to very low levels in other developed countries as well. These events led to renewed interest in the concept of the liquidity trap and in optimal monetary policy in such a situation (see Eggertsson and Woodford (2003), Adam and Billi (2006, 2007), and Werning (2012)). The fact that conventional interest-rate policy became seriously constrained by the zero lower bound also raised the question about the efficacy of alternative policy tools like fiscal stimuli.¹ This paper aims at contributing to a growing literature on the mechanics behind episodes of very low interest rates and the effects of different policies in these situations.

We analyze a simple New Keynesian model that does not rely on log-linearized equilibrium conditions and consider a central bank that pursues an inflation-targeting strategy.² Our analysis produces the following findings. First, we show that a liquidity trap can be compatible with positive inflation rates that may even be above the central bank's target. While in our model a central bank that is an “inflation nutter,” i.e. exclusively interested in stabilizing inflation, could successfully prevent inflation from being above target, a central bank that also cares about output stabilization might tolerate moderately high inflation rates since lowering inflation may be prohibitively costly in terms of output losses. The possibility of positive inflation rates at the zero lower bound is in line with the so-called “missing disinflation,” i.e. the absence of marked disinflation during the Great Recession, which has proved difficult to understand from the perspective of standard log-linearized New Keynesian models (see Coibion and Gorodnichenko (2015) and Hall (2013)). In particular, our analysis

¹In several countries, nominal interest rates became even slightly negative. However, it is clear that nominal interest rates cannot become significantly smaller than zero, as long as it is possible to hold currency, i.e. banknotes guaranteeing a nominal interest of zero. While, in line with much of the literature, we assume that the minimum level of interest rates is exactly zero, it would be straightforward to extend our framework to allow for a minimum level of interest rates that would be close to zero but negative.

²To be more precise, we use the textbook model from Woodford (2003) with segmented labor markets (see also Eggertsson and Singh (2016)).

has the potential to explain the experience of the United Kingdom in the aftermath of the global financial crisis. CPI inflation was well above the Bank of England’s target for several years while at the same time interest rates were essentially zero.

Second, the ability of our analysis to explain the “missing disinflation” relies on the fact that we do not utilize log-linearized equilibrium conditions. If we considered a Phillips curve that was log-linearized around a zero-inflation steady state, the equilibria at the zero lower bound with mildly negative or positive inflation rates would disappear. Hence our paper demonstrates that the frequent practice of log-linearizing equilibrium relations in New Keynesian models of the liquidity trap can be misleading.³ The rationale behind our finding is that log-linearization around the zero-inflation steady state leads one to exclusively consider a part of the aggregate supply relationship where inflation and output are positively related. This can be seen by looking at the canonical New Keynesian Phillips curve in the absence of shocks, $\pi_t = \beta\pi_{t+1} + \kappa y_t$, where π_t is inflation, y_t the log output gap, and β and κ satisfy $0 < \beta < 1$ and $\kappa > 0$. For constant inflation and output, there is a positive relationship between both variables: $\pi = \frac{\kappa}{1-\beta}y$. However, the log-linear approximation neglects that for inflation rates slightly above zero, output is negatively related to inflation because, under Calvo (1983) price setting without indexation, persistent and sufficiently high inflation rates cause firms to choose higher markups, which lead to lower output.⁴ When neglecting this segment of the aggregate-supply schedule, one may ignore an intersection of the aggregate demand and aggregate supply curves and thus an equilibrium of the economy that may be compatible with positive inflation rates.⁵

³A similar point is made by Boneva et al. (2016) in the context of a model that relies on Rotemberg (1982) pricing. In their framework, log-linearization also eliminates an equilibrium (compare their Figure 3(b) and the discussion thereof).

⁴There are two effects that determine the average markup of firms’ prices over marginal costs as a function of inflation when inflation is positive and persistent. First, higher inflation reduces the markup for firms that have not adjusted their price for some time. This effect tends to lower markups on average, which has a beneficial effect on output. Second, under persistent and positive inflation, a firm takes into account that its markup will decrease over the duration of a price spell. As a consequence, whenever it has the opportunity to reset its price, it will select a particularly high markup. This second effect tends to increase markups on average at higher inflation rates and thereby lowers aggregate output. At inflation rates of exactly zero, the first effect dominates. For even slightly positive inflation, the second effect is stronger, which results in output being a decreasing function of inflation.

⁵For example, Eggertsson (2011) focuses on deflationary equilibria of a log-linearized economy

Third, we examine the effects of different policies in a liquidity trap. In contrast with previous works like Eggertsson (2011), Woodford (2011), or Eggertsson and Singh (2016), we find that current fiscal policy has only moderate expansionary effects in a liquidity trap and that expansionary supply-side policies increase output.⁶ Loosely speaking, the large effects of fiscal policy found in the literature can be traced back to aggregate demand and aggregate supply curves that are almost parallel, which implies that a small shift of the aggregate demand schedule has a sizable impact on output.⁷ By contrast, in the equilibria that we focus on, the slopes of these curves are substantially different, implying a more muted response of output to shifts in aggregate demand.

Fourth, we demonstrate that higher long-term inflation targets, which are proposed by Blanchard et al. (2010), among others, alleviate the zero-lower-bound constraint to a certain degree, as they raise inflation expectations. However, because the liquidity trap is quite persistent according to our calibration, the increase in inflation expectations in the liquidity trap is small and comes at the expense of lower output in all periods. Hence our paper suggests caution against higher inflation targets. In addition, we show that a commitment to reduce government expenditures in the future has very small, contractionary effects at the zero lower bound.

Fifth, to the best of our knowledge we are the first to explore the possibility in the context of a New Keynesian model that the expectation of adverse productivity shocks pushes nominal interest rates towards the zero lower bound.⁸ The mechanism we study is straightforward: The expectation of a severe crisis causes a savings glut, which tends to push down interest rates. There is at least some anecdotal evidence for the view that during the global financial crisis, an even more severe crisis was deemed possible. For example, Barack Obama regarded “saving the economy from a Great Depression” to

where both the AS and the AD curve have positive slopes and the slope of the AD curve is larger than the one for the AS curve. However, as we have argued above, the non-linear AS curve has a negative slope for moderately higher inflation rates, while the AD curve continues to be upward-sloping. As a consequence, there is another intersection of both curves at moderately higher inflation rates that is lost by log-linearization.

⁶A more detailed review of this literature is given in Section 2.

⁷See Eggertsson (2011) for a lucid exposition of this argument.

⁸The main alternative explanations considered in the literature are discount factor shocks and positive technology shocks. See Section 2 for a discussion.

be his presidency’s most important achievement.⁹ Similarly, George W. Bush reported in an interview that “[his] chief economic advisers [had told him] that the situation we were facing could be worse than the Great Depression.”¹⁰ Even currently, there appears to be a non-negligible probability of a resurgence of the Euro crisis with serious repercussions on financial market worldwide, e.g. in case politicians who oppose the Euro are successful in a national election. Understanding the nature of shocks that are responsible for zero-lower-bound episodes is crucial for analyses of welfare. Obviously, shocks to intertemporal preferences, which are often adopted in the literature, have an important influence on the welfare comparisons of different policies if the welfare measure is based on the representative household’s utility.

Our paper is organized as follows. The next section discusses how our paper relates to the literature. Section 3 outlines the model. In Section 4, we analyze a version of our framework where the economy is only subject to preference shocks. This version enables us to highlight the relationship of our findings to those in the literature. The framework with productivity shocks is considered in more detail in Section 5. Section 6 calibrates our model and presents the paper’s results regarding the effects of policies. Multiple equilibria are explored in Section 7. Section 8 concludes.

2 Related Literature

As has been stated before, our paper contributes to the ongoing debate about the effects of fiscal policy when nominal interest rates are stuck at zero. The New Keynesian paradigm has been shown to have potentially intriguing implications in this respect. In particular, several authors find that fiscal policy can be exceptionally powerful in a liquidity trap, as the government spending multiplier may be substantially larger than one (see Eggertsson (2011), Christiano et al. (2011), and Carlstrom et al. (2014)). The underlying mechanism is explained in Woodford (2011) with the help of a simple framework: Expansionary fiscal policy may raise inflation expectations, which in turn

⁹See <http://edition.cnn.com/2016/04/10/politics/obama-libya-biggest-mistake/>.

¹⁰See <http://www.foxnews.com/politics/2009/01/12/raw-data-transcript-bushs-white-house-press-conference.html>.

leads to a decrease in real interest rates, given that the nominal interest rate is stuck at zero.¹¹ While expansionary demand-side policies thus may have very strong desirable effects, positive supply shocks have been shown to be potentially detrimental. For example, according to the paradox of toil (see Eggertsson (2010)), decreases in labor taxes may be contractionary.

These conclusions about the effects of policies have been criticized on both empirical and theoretical grounds. First, Wieland (2016) provides empirical evidence for the effects of supply shocks in Japan and finds that negative supply shocks have conventional, contractionary effects. Second, the theoretical predictions of the New Keynesian model have been criticized as being dependent on the equilibrium selected or the nature of the shock driving the economy to the zero lower bound (see Mertens and Ravn (2014) and Cochrane (2016)).¹² Our paper provides additional support for the view that demand and supply-side policies have conventional effects at the zero bound.

As has been mentioned in the Introduction, we focus on expectations of negative productivity shocks as the source of liquidity-trap episodes. The literature has pursued three main alternative approaches. First, many papers consider shocks to the representative household's discount factor as a reason underlying changes in the natural real rates of interest (see Boneva et al. (2016), Eggertsson and Singh (2016), Gust et al. (2012), Richter and Throckmorton (2015), among others). An obvious disadvantage of this approach is that these shocks represent only a shortcut for other, fundamental shocks. Understanding the nature of these fundamental shocks appears to be important, in particular, for analyses of welfare. Second, the literature has assessed the potential of positive productivity shocks, which lower marginal costs and thereby prices, to explain periods of interest rates at the lower bound.¹³ At least during the Great Recession, productivity declined in the United States (see Fernald (2015)), which does not

¹¹In a recent contribution, Rendahl (2016) proposes an alternative mechanism to explain large effects of fiscal policy in a crisis: When movements in unemployment are persistent, increases in government spending may cause longer-term rises in income, which lead to sizable increases in demand.

¹²The issue of multiple equilibria in New Keynesian models with the zero lower bound have been studied in Benhabib et al. (2001), Aruoba and Schorfheide (2013), Armenter (2017), and Mertens and Ravn (2014).

¹³Gust et al. (2012) and Boneva et al. (2016) find that positive productivity shocks are less important for understanding liquidity-trap episodes than discount factor shocks.

square with the explanation that positive technology shocks were responsible for the low interest rates during that era. Third, some authors have taken sunspot shocks into account (see e.g. Armenter (2017), Boneva et al. (2016), Mertens and Ravn (2014)). Our framework also allows for this possibility as we consider situations in which several Markov-perfect equilibria exist, which opens up the possibility for sunspots to select among the equilibria.

A contribution closely related to this paper is Eggertsson and Singh (2016). They use a canonical New Keynesian model with segmented labor markets and Calvo (1983) pricing (see Woodford (2003) for a textbook exposition), which has the convenient property that the economy does not feature endogenous state variables. In combination with their assumptions that the shocks to the households' intertemporal preferences follow a two state Markov process and that all policies are only a function of this shock realization, this framework allows for a particularly simple exposition of the equilibrium in the non-linear economy.

Our analysis differs from theirs in at least four respects. First, they consider a central bank whose behavior can be described by an exogenously given interest-rate rule. By contrast, the central bank in our model pursues an inflation-targeting strategy and chooses its instrument optimally under discretion. Second, we assume that the central bank may target a positive inflation rate. While such a positive target is typically suboptimal in New Keynesian models, it is arguably realistic as most central banks officially pursue an inflation target of around 2%. Third, while they consider shocks to the representative household's intertemporal preferences, we focus on expected adverse productivity shocks as driving the economy towards the zero lower bound. The fourth point of departure from Eggertsson and Singh (2016) is that we concentrate on another intersection of the AS curve and the AD curve with mildly negative or positive inflation that is not taken into account by Eggertsson and Singh (2016). For the equilibrium that is at the heart of Eggertsson and Singh's analysis, log-linearization only has a negligible effect on the model's quantitative implications. By contrast, as the equilibria that we

focus on in our model disappear under log-linearization, our paper provides support for the view that log-linearization is potentially misleading.¹⁴

The potential pitfalls of using log-linearized New Keynesian models to study liquidity-trap scenarios have recently been studied by Lindé and Trabandt (2014), Mertens and Ravn (2014), Fernández-Villaverde et al. (2015), Christiano et al. (2016), and Eggertsson and Singh (2016). Like our paper, Boneva et al. (2016) stress the importance of nonlinear features different from the zero lower bound constraint and find conventional effects of policies in a nonlinear model with Rotemberg (1982) pricing.¹⁵

Our findings about the effects of fiscal policy are broadly in line with Mertens and Ravn (2014), who find equilibria with moderate effects of fiscal policy as well.¹⁶ Like Mertens and Ravn (2014), we consider a non-linear New Keynesian model with Calvo price-setting. Our model differs from theirs as we consider a central bank that acts under discretion and pursues an inflation targeting strategy. By contrast, they suppose that monetary policy can be described by a Taylor rule. In addition, we utilize a framework with segmented labor markets, whereas Mertens and Ravn (2014) assume a common labor market for all firms. Finally, while they focus on confidence shocks and shocks to intertemporal preferences, we concentrate on the possibility that productivity shocks and expected adverse productivity shocks, in particular, drive the economy into a liquidity trap.

¹⁴For a sufficiently persistent liquidity-trap, bounded log-linear solutions fail to exist in log-linearized models (see Woodford (2011)). By contrast, the equilibrium that is at the heart of our non-linear model exists in this situation. In Section 6.6, we discuss how our results are affected by the size of the parameter governing the persistence of the liquidity trap.

¹⁵Miao and Ngo (2016) compare the dynamics of the New Keynesian model at the zero lower bound for Rotemberg (1982) and Calvo (1983) price-setting.

¹⁶In response to their paper, Christiano et al. (2016) show that these equilibria are not stable under a certain learning procedure. They find unique equilibria that satisfy their learning criterion; these equilibria feature the unconventional policy implications like large government spending multipliers found in earlier analyses of New Keynesian models with the zero lower bound.

3 Model

3.1 Set-up

Like Eggertsson and Singh (2016), we analyze the implications of the zero lower bound in a standard non-linear New Keynesian model with industry-specific labor markets and Calvo (1983) pricing (see Woodford (2003)).¹⁷ The economy is populated by a representative household that supplies a continuum of differentiated types of labor $i \in [0, 1]$ to a continuum of firms that produce differentiated consumption goods. Moreover, there is an independent central bank and a fiscal policy-maker.

The representative household's objective is to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \lambda \frac{\int_0^1 n_t(i)^{1+\omega} di}{1+\omega} \right), \quad (1)$$

where subscripts $t = 0, 1, 2, \dots$ represent the period and the consumption aggregator C_t is given by

$$C_t = \left(\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}. \quad (2)$$

The $c_t(i)$'s with $i \in [0, 1]$ stand for differentiated consumption goods. The elasticity of substitution θ satisfies $\theta > 1$ and the discount factor β satisfies $0 < \beta < 1$; $n_t(i)$ is the labor supplied of type i , ω is the inverse of the Frisch elasticity of labor supply, λ is a positive constant. Variable ξ_t represents a preference shock, which follows a Markov process. While we will abstract from these shocks in our main analysis, we include them at this stage in order to be able to discuss the relation of this paper to the previous literature.

Households trade in one-period risk-free nominal bonds B_t that pay a gross nominal interest I_t . The zero lower bound implies that this rate cannot fall below 1, i.e. $I_t \geq 1$.

¹⁷We do not consider Calvo pricing with indexation, as indexation implies that prices typically change every period, which is at odds with the empirical evidence on individual price-setting (see Nakamura and Steinsson (2008)).

There is a proportional tax on labor income with rate τ_t . Thus the per-period budget constraint of the household is

$$\begin{aligned} & I_{t-1}B_{t-1} + (1 - \tau_t) \int_0^1 w_t(i)n_t(i) di + \text{firms' profits} + \text{lump-sum gov. transfers} \\ & \geq \int_0^1 p_t(i)c_t(i) di + B_t, \end{aligned} \quad (3)$$

where $p_t(i)$ is the price of good i and $w_t(i)$ is the nominal wage for labor of type i .

It is well-known that the household's utility maximization problem results in the following demand for good i as a function of the respective good's price $p_t(i)$:

$$c_t(i) = C_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta}, \quad (4)$$

where the aggregate price level is defined as

$$P_t = \left(\int_0^1 p_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}. \quad (5)$$

The state uses the labor tax and additional lump-sum taxes to finance government spending G_t , which is assumed to be exogenous. Due to Ricardian equivalence, the timing of these lump-sum taxes that are used to balance the government's intertemporal budget constraint does not affect the equilibrium. We assume that, similarly to (2), G_t satisfies $G_t = \left(\int_0^1 g_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$, where $g_t(i)$ is the government's consumption of the good of variety i . Analogously to (4), the government's demand for good i is $g_t(i) = G_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta}$. As a consequence, the total demand for good i , $y_t(i) = c_t(i) + g_t(i)$, can be expressed as

$$y_t(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta}, \quad (6)$$

where $Y_t = C_t + G_t$.

Firm i produces its good with the help of a linear technology, $y_t(i) = A_t n_t(i)$, where A_t stands for the economy-wide productivity. As firms are wage-takers, the wage is determined via the first-order condition from the household's utility maximization

problem: $\frac{w_t(i)}{P_t} = \frac{\lambda n_t(i)^\omega C_t^\sigma}{1-\tau_t}$.¹⁸ Firms face price stickiness à la Calvo (1983). Accordingly, a firm can adjust its price with probability $1 - \alpha$ in each period ($0 < \alpha < 1$); with probability α , the price has to remain fixed.

3.2 Private-sector equilibrium

We are now in a position to characterize private-sector equilibria for this economy. For given $\{G_t, \tau_t, \xi_t, A_t, I_t\}_{t=0}^\infty$, a private-sector equilibrium has to satisfy the following equations:

$$1 = \beta I_t \mathbb{E}_t \left[\left(\frac{Y_{t+1} - G_{t+1}}{Y_t - G_t} \right)^{-\sigma} \frac{\xi_{t+1}}{\xi_t} \frac{1}{\Pi_{t+1}} \right] \quad (7)$$

$$K_t = \frac{\theta}{\theta - 1} \frac{\lambda \xi_t}{(1 - \tau_t)} \left(\frac{Y_t}{A_t} \right)^{1+\omega} + \alpha \beta \mathbb{E}_t \left[\Pi_{t+1}^{\theta(1+\omega)} K_{t+1} \right] \quad (8)$$

$$F_t = \xi_t \frac{Y_t}{(Y_t - G_t)^\sigma} + \alpha \beta \mathbb{E}_t \left[\Pi_{t+1}^{\theta-1} F_{t+1} \right] \quad (9)$$

$$\frac{F_t}{K_t} = \left(\frac{1 - \alpha \Pi_t^{\theta-1}}{1 - \alpha} \right)^{\frac{1+\omega\theta}{\theta-1}} \quad (10)$$

The first equation corresponds to the standard consumption Euler equation, where the resource constraint $Y_t = C_t + G_t$ has been used to substitute for C_t . The remaining equations represent the New Keynesian Phillips curve and can be derived from the firms' profit maximization problem (see Appendix A for details).

As stressed by Eggertsson and Singh (2016), this framework has the advantage that there is no endogenous state variable, unless policy-makers respond to such a state variable.¹⁹ This allows for a simple characterization of the equilibria in an economy with Calvo price setting, which is possible otherwise only under Rotemberg (1982) price adjustment costs.

¹⁸As described in more detail in Woodford (2003), the assumption that each firm takes its wage as given can be justified by an economy with infinitely many sectors, where each sector is populated by infinitely many firms. In each period, all firms in a sector are allowed to adjust their prices with probability $1 - \alpha$; with probability α all firms in a sector have to keep their old prices. A certain type of labor can only be employed in one specific industry. As each firm is small in its industry, it takes the wage in its industry as given.

¹⁹A central banker maximizing the representative household's utility would take price dispersion into account. In this case, an appropriate measure of price dispersion, $\Delta_t = \int_0^1 \left(\frac{p_t(i)}{P_t} \right)^{-\theta(1+\omega)} di$, would constitute an endogenous state variable in the framework under consideration.

3.3 Monetary policy

Finally, we close the model by specifying how monetary policy is conducted. We assume for the time being that the central bank's objectives can be described by an instantaneous loss function $L(\Pi_t)$, which is strictly convex and has a unique global minimum at $\Pi_t = \Pi^*$. Hence, the central bank pursues strict inflation targeting, where Π^* has the interpretation of the central bank's inflation target. In the course of our analysis, we will also take more general loss functions into account, where the central bank cares about output stabilization as well.

We focus on Markov-perfect equilibria or discretionary equilibria in the following sense:²⁰ In each period t , the central bank selects Y_t , Π_t , and I_t to minimize the expected value of its loss function, $\mathbb{E}_t \left[\sum_{j=t}^{\infty} \beta^{j-t} L(\Pi_j) \right]$, taking Equations (7)-(10), the zero-lower-bound constraint $I_t \geq 1$, and its own future policies as given. We observe that the absence of an endogenous state variable implies that the central bank's optimization problem effectively amounts to minimizing current losses only at each point in time. Clearly, Π_t will be equal to Π^* , whenever the central bank can achieve this level.

4 Equilibria for Preference Shocks

In order to clarify how our results relate to previous contributions, we study a variant of the economy with preference shocks in this section. In the next section, Section 5, we will concentrate on productivity shocks. In particular, we assume in the present section that A_t is constant and equal to one. By contrast, ξ_t is stochastic and follows a two-state Markov chain with an absorbing state. The two states are $\{\tilde{R}, N\}$, where $\tilde{R}$ is the initial state, which represents a severe recession or a liquidity trap, and N is the “normal state,” which is absorbing.²¹ The probability of the economy remaining

²⁰Markov-perfect equilibria have recently been studied by Armenter (2017) in a log-linearized New Keynesian model with the zero lower bound.

²¹The assumption of a two-state Markov chain with one absorbing state has been frequently employed in the literature since Eggertsson and Woodford (2003).

in state $\tilde{R}$ is denoted by $p_{\tilde{R}\tilde{R}}$ with $0 < p_{\tilde{R}\tilde{R}} < 1$. We normalize the preference shock in the normal state to one, $\xi_N = 1$. The realization of ξ_t in state $\tilde{R}$ satisfies $0 < \xi_{\tilde{R}} < 1$.

We make the assumption that G_t and τ_t are only functions of the state. In a Markov-perfect equilibrium, the same holds true for the policy rate I_t . The respective values of the variables are denoted by I_N , $I_{\tilde{R}}$, G_N , $G_{\tilde{R}}$, τ_N , and $\tau_{\tilde{R}}$. We will construct an equilibrium in which the zero lower bound holds with equality in state $\tilde{R}$, i.e. $I_{\tilde{R}} = 1$, and does not bind in state N . Using (7)-(10), it is straightforward to derive the equilibrium conditions in state N :

$$1 = \frac{\beta I_N}{\Pi_N}, \quad (11)$$

$$K_N = \frac{\theta}{\theta - 1} \frac{\lambda}{(1 - \tau_N)} \frac{Y_N^{1+\omega}}{1 - \alpha\beta(\Pi_N)^{\theta(1+\omega)}}, \quad (12)$$

$$F_N = \frac{1}{1 - \alpha\beta(\Pi_N)^{\theta-1}} \frac{Y_N}{(Y_N - G_N)^\sigma}, \quad (13)$$

$$\frac{F_N}{K_N} = \left(\frac{1 - \alpha(\Pi_N)^{\theta-1}}{1 - \alpha} \right)^{\frac{1+\omega\theta}{\theta-1}}, \quad (14)$$

$$\Pi_N = \Pi^* \quad (15)$$

It is instructive to examine the solution for Y_N more closely, which is implicitly given by

$$(Y_N)^\omega (Y_N - G_N)^\sigma = \frac{1 - \tau_N}{\lambda} \cdot \frac{\theta - 1}{\theta} \cdot \frac{1 - \alpha\beta(\Pi^*)^{\theta(1+\omega)}}{1 - \alpha\beta(\Pi^*)^{\theta-1}} \cdot \left(\frac{1 - \alpha}{1 - \alpha(\Pi^*)^{\theta-1}} \right)^{\frac{1+\omega\theta}{\theta-1}}. \quad (16)$$

Note that the right-hand side of (16) is positive.²² As the left-hand side of (16) is zero for $Y_N = G_N$ and, for larger values of Y_N , increases monotonically with Y_N without bound, Equation (16) therefore specifies a unique solution for Y_N .

As a next step, we examine how Y_N depends on G_N in a comparative-statics sense. Because (16) implies that $(Y_N)^\omega (Y_N - G_N)^\sigma$ is equal to a constant independent of G_N , we conclude that the derivative of $(Y_N)^\omega (Y_N - G_N)^\sigma$ with respect to G_N must be zero, which is equivalent to

$$\omega \frac{dY_N}{dG_N} (Y_N - G_N) + \sigma Y_N \left(\frac{dY_N}{dG_N} - 1 \right) = 0. \quad (17)$$

²²To ensure a well-defined Phillips curve, we have to restrict Π^* to values satisfying $\Pi^* \leq \frac{1}{(\alpha\beta)^{\frac{1}{\theta(1+\omega)}}}$ and $\Pi^* \leq \frac{1}{\alpha^{\frac{1}{\theta-1}}}$.

For $\sigma > 0$, $\omega > 0$, and $Y_N > G_N$, this equation can only be fulfilled if $0 < \frac{dY_N}{dG_N} < 1$. Hence the government spending multiplier is positive and smaller than one. This also implies that private consumption $Y_N - G_N$ is crowded out by increases in government consumption.

Equation (16) also enables us to determine the effects of changes in the tax on labor income, τ_N . As the right-hand side of (16) is a decreasing function of τ_N and the left-hand side is an increasing function of Y_N for $Y_N \geq G_N$, it is clear that an increase in labor taxes always leads to a reduction in output.

We summarize these observations by the following lemma:

Lemma 1. *In state N , a marginal increase in government consumption results in a reduction in private consumption and an increase in output that is positive but smaller than the increase in government consumption. An increase in τ_N lowers output and consumption.*

In state $\tilde{R}$, the zero lower bound imposes a constraint on monetary policy. Hence, the gross interest rate $I_{\tilde{R}}$ equals one and $\Pi_{\tilde{R}}$ is different from Π^* . The equilibrium values of $\Pi_{\tilde{R}}$ and $Y_{\tilde{R}}$ are given by

$$1 = \beta \left[p_{\tilde{R}\tilde{R}} \frac{1}{\Pi_{\tilde{R}}} + (1 - p_{\tilde{R}\tilde{R}}) \left(\frac{Y_N - G_N}{Y_{\tilde{R}} - G_{\tilde{R}}} \right)^{-\sigma} \frac{1}{\xi_{\tilde{R}}} \frac{1}{\Pi_N} \right], \quad (18)$$

$$K_{\tilde{R}} = \frac{\frac{\theta}{\theta-1} \frac{\lambda \xi_{\tilde{R}}}{1-\tau_{\tilde{R}}} \left(\frac{Y_{\tilde{R}}}{A_{\tilde{R}}} \right)^{1+\omega} + \alpha \beta (1 - p_{\tilde{R}\tilde{R}}) \Pi_N^{\theta(1+\omega)} K_N}{1 - \alpha \beta p_{\tilde{R}\tilde{R}} \Pi_{\tilde{R}}^{\theta(1+\omega)}}, \quad (19)$$

$$F_{\tilde{R}} = \frac{\frac{\xi_{\tilde{R}} Y_{\tilde{R}}}{(Y_{\tilde{R}} - G_{\tilde{R}})^\sigma} + \alpha \beta (1 - p_{\tilde{R}\tilde{R}}) \Pi_N^{\theta-1} F_N}{1 - \alpha \beta p_{\tilde{R}\tilde{R}} \Pi_{\tilde{R}}^{\theta-1}}, \quad (20)$$

$$\frac{F_{\tilde{R}}}{K_{\tilde{R}}} = \left(\frac{1 - \alpha \Pi_{\tilde{R}}^{\theta-1}}{1 - \alpha} \right)^{\frac{1+\omega\theta}{\theta-1}}. \quad (21)$$

In line with Eggertsson (2011), we call Equation (18) the AD curve and the relationship between $Y_{\tilde{R}}$ and $\Pi_{\tilde{R}}$ implied by Equations (19)-(21) the AS curve.

In order to verify whether a certain combination of $\Pi_{\tilde{R}}$, $Y_{\tilde{R}}$, I_N , Π_N , and Y_N that satisfies Equations (11)-(15) and (18)-(21) actually is a Markov-perfect equilibrium,

we examine one-period deviations of the central bank in state $\tilde{R}$, which we denote by $\Pi'_{\tilde{R}}$, $Y'_{\tilde{R}}$, and $I'_{\tilde{R}}$, for given future values of inflation and output in states $\tilde{R}$ and N .²³ Intuitively, we have to ensure that the central bank cannot lower its losses by increasing the interest rate above the zero lower bound.

It is straightforward to verify that the constraint $I'_{\tilde{R}} \geq 1$ and the IS curve (7) can be combined to yield an upper bound for the level of output that the central bank can achieve in a certain period.²⁴ Thus every deviation has to satisfy

$$Y'_{\tilde{R}} \leq G_{\tilde{R}} + \beta^{-\frac{1}{\sigma}} \left(\frac{p_{\tilde{R}\tilde{R}}}{\Pi_{\tilde{R}}(Y_{\tilde{R}} - G_{\tilde{R}})^{\sigma}} + \frac{1 - p_{\tilde{R}\tilde{R}}}{\xi_{\tilde{R}}\Pi_N(Y_N - G_N)^{\sigma}} \right)^{-\frac{1}{\sigma}} = Y_{\tilde{R}}. \quad (22)$$

Moreover, every deviation has to be in line with the short-run Phillips curve for given expectations about future levels of output and inflation in states $\tilde{R}$ and N , i.e.

$$K'_{\tilde{R}} = \frac{\theta}{\theta - 1} \frac{\lambda \xi_{\tilde{R}}}{(1 - \tau_{\tilde{R}})} \left(\frac{Y'_{\tilde{R}}}{A_{\tilde{R}}} \right)^{1+\omega} + \alpha \beta \left[p_{\tilde{R}\tilde{R}} \Pi_{\tilde{R}}^{\theta(1+\omega)} K_{\tilde{R}} + (1 - p_{\tilde{R}\tilde{R}}) \Pi_N^{\theta(1+\omega)} K_N \right] \quad (23)$$

$$F'_{\tilde{R}} = \frac{\xi_{\tilde{R}} Y'_{\tilde{R}}}{(Y'_{\tilde{R}} - G_{\tilde{R}})^{\sigma}} + \alpha \beta \left[p_{\tilde{R}\tilde{R}} \Pi_{\tilde{R}}^{\theta-1} F_{\tilde{R}} + (1 - p_{\tilde{R}\tilde{R}}) \Pi_N^{\theta-1} F_N \right], \quad (24)$$

$$\frac{F'_{\tilde{R}}}{K'_{\tilde{R}}} = \left(\frac{1 - \alpha (\Pi'_{\tilde{R}})^{\theta-1}}{1 - \alpha} \right)^{\frac{1+\omega\theta}{\theta-1}}. \quad (25)$$

At this point, it is important to stress the difference between the AS curve, which can be obtained from (19)-(21) by eliminating $K_{\tilde{R}}$ and $F_{\tilde{R}}$, and the short-run Phillips curve, which results from (23)-(25). The AS curve gives all combinations of output and inflation in state $\tilde{R}$ that are compatible with optimal price setting by firms, under the assumptions that output and inflation are constant at these levels for the entire duration of state $\tilde{R}$ and that output and inflation are at their equilibrium levels in state N . By contrast, the short-run Phillips curve specifies the respective combinations of output and inflation for a single period in state $\tilde{R}$, under the assumption that inflation and output in all future periods (including those where state $\tilde{R}$ prevails) are at their equilibrium levels.

²³The central bank cannot profitably deviate in state N , as the inflation rate is already at its optimal level.

²⁴An analogous argument is made by Armenter (2017) in a log-linear framework,

Potential deviations of the central bank have to satisfy the short-run Phillips curve rather than the AS curve because a central bank acting under discretion cannot commit to future policy changes and thus can only affect current output and inflation in a model without endogenous state variables. To examine whether profitable deviations exist for the central bank, it is therefore important to examine the short-run Phillips curve and, in particular, its slope. In Appendix B, we prove the following Lemma:

Lemma 2. *For $\sigma \geq 1$, the short-run Phillips curve in the economy with preference shocks (23)-(25), which gives the value of $\Pi_{\tilde{R}'}$ as a function of $Y_{\tilde{R}'}$, is upward-sloping.*

Henceforth we will assume $\sigma \geq 1$, which is satisfied by the values typically adopted in the literature.

For $\sigma \geq 1$, Lemma 2 implies that the zero lower bound does not only impose an upper bound on $Y'_{\tilde{R}}$, which is given by $Y_{\tilde{R}}$, it also implies an upper bound on $\Pi'_{\tilde{R}}$, namely $\Pi'_{\tilde{R}} \leq \Pi_{\tilde{R}}$. This immediately yields the next lemma:

Lemma 3. *Suppose that $\sigma \geq 1$. Then a triple $(\Pi_{\tilde{R}}, Y_{\tilde{R}}, Y_N)$ that satisfies Equations (11)-(15) and (18)-(21) is a Markov-perfect equilibrium iff $\Pi_{\tilde{R}} \leq \Pi^*$.*

Intuitively, at the zero lower bound all possible deviations of the central bank involve higher interest rates, which entail lower output and, according to Lemma 2, lower inflation as well. As a consequence, deviations can only be profitable if inflation is above its target.

Figure 1 displays the AD curve (18) as a dashed line and the AS curve, which can be obtained by eliminating $F_{\tilde{R}}$ and $K_{\tilde{R}}$ from (19)-(21), as a solid line for the calibration in Eggertsson and Singh (2016), which involves $\Pi^* = 1$.²⁵ The two lines intersect three times. First, there is an equilibrium with substantial deflation and a drop in output of around 70%. As noted by Eggertsson and Singh (2016), this equilibrium is locally indeterminate. Second, there is another point where the two curves intersect. At this point, both curves have a positive slope and the AD curve is steeper than the AS curve. Eggertsson and Singh (2016) show that this point corresponds to a locally unique

²⁵While the results are qualitatively very similar, the numerical values differ. One reason appears to be a small mistake in the calculation of F_N and K_N in Eggertsson and Singh (2016).

Figure 1: The AD curve (dashed line) and the AS curve for the Great Depression scenario in Eggertsson and Singh (2016). Parameters: $\beta = 0.9972$, $\sigma = 1.0238$, $G_N = 0.2$, $G_{\bar{R}} = 0.2$, $\omega = 1.8936$, $\theta = 12.2503$, $\alpha = 0.7273$, $\tau_N = 0.3$, $\tau_{\bar{R}} = 0.3$, $p_{\bar{R}N} = 0.1039$, $\Pi^* = 1$, $\xi_{\bar{R}} = 0.8281$, $\lambda = 0.8079$.

equilibrium. This is the equilibrium that is at the heart of their analysis. Finally, there is also a third intersection, which is not considered by them. At this point, inflation is positive and output is above the level one would obtain in the absence of the zero lower bound.

As inflation is below its target at the first and second intersection, these points correspond to Markov-perfect equilibria. By contrast, the third point does not correspond to such an equilibrium when the central bank is exclusively interested in achieving its inflation target, as will be discussed in more detail now.

Figure 2 displays the short-run Phillips curve, (23)-(25), as a solid line and the maximum level of output that the central bank can achieve in a particular period, which is given in (22), as a dashed line. In line with Lemma 2, the short-run Phillips curve is upward-sloping. Consequently, Lemma 3 implies that profitable deviations exist for the central bank, as inflation is above its target of $\Pi^* = 1$. The central bank could increase the nominal interest rate in a given period in state R , thereby lowering output and inflation.

However, it should be noted that the slope of the short-run Phillips curve is very small. In fact, it is straightforward to compute the slope as 0.004, which is considerably smaller

Figure 2: The maximum output level that the central bank can achieve by deviating in state $\tilde{R}$ (dashed line) and the short-run Phillips curve (solid line) for the Great Depression scenario in Eggertsson and Singh (2016). Parameters: $\beta = 0.9972$, $\sigma = 1.0238$, $G_N = 0.2$, $G_{\tilde{R}} = 0.2$, $\omega = 1.8936$, $\theta = 12.2503$, $\alpha = 0.7273$, $\tau_N = 0.3$, $\tau_{\tilde{R}} = 0.3$, $p_{\tilde{R}N} = 0.1039$, $\Pi^* = 1$, $\xi_{\tilde{R}} = 0.8281$, $\lambda = 0.8079$.

than the slope of the short-run Phillips curve in state N , which can be computed as 0.017.

The low value of the slope of the short-run Phillips curve in state R implies that a central bank that is not an “inflation nutter” but also aims at stabilizing output might not be willing to incur the large output losses necessary to lower inflation. This point is strengthened by the observation that the central bank would have to choose contractionary policy in a situation where output is already rather low.

Suppose, for example, that the objectives of the central bank were adequately described by the standard loss function $(\Pi_t - \Pi^*)^2 + a(Y_t - Y^*)^2$, where Y^* would be the level of output compatible with $\Pi_t = \Pi^*$ in state N and a is a positive parameter that measures the importance the central bank attaches to output stabilization. Unless a was very small, the third point where the AS curve and the AD curve intersect would correspond to a Markov-perfect equilibrium with positive inflation in this case.

We now turn to a version of the model where the economy may be pushed into a liquidity trap not by shocks to the representative household’s preferences but by aggregate

productivity shocks. In particular, we will see that the expectation of a very low future realization of productivity will tend to drive nominal interest rates downwards. We will demonstrate that the third point where the AS curve and the AD curve intersect, which does not represent a Markov-perfect equilibrium under strict inflation targeting in the scenario considered in this section, will also occur in the modified framework of the next section and will correspond to a meaningful Markov-perfect equilibrium for a plausible calibration of our model.

5 The Economy with Productivity Shocks

As we examine the possibility that expectations about a catastrophic event drive interest rates towards the zero lower bound, we introduce a third state, D , which represents a severe depression, in addition to the normal state N and the recession state, which we call R in this variant of our framework. While ξ_t is constant across the three states, we assume that $0 < A_D < A_R \leq A_N$ and normalize A_N to $A_N = 1$. In line with a large literature following Eggertsson and Woodford (2003), we make the assumption that N is an absorbing state and that the economy is initially stuck in a severe recession R . Differently from this literature, we consider the possibility that the economy in state R may move to state D with probability p_{RD} and to state N with probability p_{RN} . For simplicity, we look at the case where an economy in a depression does not move back to state R . In each period, the economy remains mired in state D with probability p_{DD} ; with probability $p_{DN} = 1 - p_{DD}$, the economy escapes to state N .

In order to compute the equilibria, it will be useful to start with the analysis of the absorbing state N . As a next step, we will focus on state D because this state can lead to N but not to R . State R , which may be followed by both D and N , will be examined last. We notice that the analysis of state N has already been completed: The equilibrium levels of output and inflation in state N are identical to those computed in Section 4.

We proceed by considering the economy in state D . It is instructive to examine the household's Euler equation first. In line with (7), the nominal interest rate in D satisfies

$$I_D = \beta^{-1} \frac{1}{\frac{p_{DD}}{\Pi_D} + \frac{p_{DN}}{\Pi_N} \left(\frac{C_D}{C_N} \right)^\sigma} \quad (26)$$

Conjecture, for the moment that the zero lower bound does not bind in state D , which implies that the central bank can achieve its inflation target Π^* not only in state N but also in state D . In this case, we can conclude

$$I_D = \beta^{-1} \frac{\Pi^*}{p_{DD} + p_{DN} \left(\frac{C_D}{C_N} \right)^\sigma}. \quad (27)$$

As state D corresponds to a depression, it appears plausible that consumption in D will be smaller than in the normal state N . For $p_{DD} < 1$ and therefore $p_{DN} > 0$, this implies that $I_D > I_N = \beta^{-1}\Pi^*$.²⁶ Hence interest rates in the depression are pushed away from the zero lower bound, which confirms our initial conjecture that the zero lower bound represents no constraint on monetary policy in state D .²⁷ The conclusion that interest rates are comparably high in a depression should be taken with a pinch of salt. It is an artifact of our assumption that the economy cannot deteriorate further in a depression, which we made for analytical convenience.²⁸

With the help of the Phillips curve (8)-(10), the conditions $\Pi_D = \Pi^*$ and $\Pi_N = \Pi^*$ as well as the solutions for Y_N , F_N , and K_N derived in Section 4, it is now straightforward to compute Y_D from

$$K_D = \frac{\frac{\theta}{\theta-1} \frac{\lambda}{(1-\tau_D)} \left(\frac{Y_D}{A_D} \right)^{1+\omega} + \alpha\beta p_{DN} (\Pi^*)^{\theta(1+\omega)} K_N}{1 - p_{DD} \alpha\beta (\Pi^*)^{\theta(1+\omega)}}, \quad (28)$$

$$F_D = \frac{\frac{Y_D}{(Y_D - G_D)^\sigma} + \alpha\beta p_{DN} (\Pi^*)^{\theta-1} F_N}{1 - p_{DD} \alpha\beta (\Pi^*)^{\theta-1}}, \quad (29)$$

$$\frac{F_D}{K_D} = \left(\frac{1 - \alpha (\Pi^*)^{\theta-1}}{1 - \alpha} \right)^{\frac{1+\omega\theta}{\theta-1}}. \quad (30)$$

²⁶For $p_{DD} = 1$ and $p_{DN} = 0$, i.e. in the case where D is an absorbing state as well, $I_D = I_N$ would hold.

²⁷It is nevertheless conceivable that equilibria exist where the zero lower bound binds in states D or N . Multiple equilibria are analyzed in more detail in Section 7.

²⁸This assumption does not affect the equilibrium in state R , which takes center stage in our analysis, to a significant extent.

It will be instructive to consider the special case where the central bank targets zero net inflation, i.e. $\Pi^* = 1$. In this case, $F_D = K_D$ and $F_N = K_N$ hold, which results in the following simple equation:

$$(Y_D)^\omega (Y_D - G_D)^\sigma = (A_D)^{1+\omega} \frac{1 - \tau_D}{\lambda} \frac{\theta - 1}{\theta}. \quad (31)$$

A few comments are in order. First, we note that, just like (16) implies a unique value of Y_N , Equation (31) implies a unique solution for Y_D . Second, we note that, unsurprisingly, output is lower in the depression than in state N , provided that $\tau_D = \tau_N$ and $G_D = G_N$. This can be easily seen by observing that, in line with $A_D < A_N$, the right-hand side of (31) is smaller than the right-hand side of (16) and that the left-hand sides of both equations are increasing functions of output, which are identical for identical levels of output and government expenditures. Third, and as a consequence of the second point, consumption in state D , which is given by $C_D = Y_D - G_D$, is lower than in state N for identical fiscal policies, $\tau_D = \tau_N$ and $G_D = G_N$, which confirms our previous conjecture. Fourth, arguments completely analogous to those that led to Lemma 1 imply the following Lemma that describes the effects of government expenditures and changes in labor income taxes on output and consumption in a depression:

Lemma 4. *Suppose that $\Pi^* = 1$. Then a marginal increase in government consumption in state D leads to (i) a reduction in private consumption and (ii) an increase in output in state D that is positive but smaller than the increase in government consumption. An increase in τ_D leads to a decrease in output and consumption in state D .*

It is worth noting that, due to the continuity of the expressions in (11)-(15) and (28)-(30), the statement of the Lemma also holds for values of Π^* that are different but sufficiently close to one.

Finally, we examine the initial state R . The observation made in our analysis of state D that an expected increase in consumption tends to increase interest rates suggests that the possibility of a severe drop in consumption may have the opposite effect. Hence, we will look at constellations where the zero lower bound is binding in

state R . Equations (7)-(10) and $I_R = 1$ entail that

$$1 = \beta \left[\frac{p_{RR}}{\Pi_R} + \frac{(Y_R - G_R)^\sigma}{\Pi^*} \left(\frac{p_{RD}}{(Y_D - G_D)^\sigma} + \frac{p_{RN}}{(Y_N - G_N)^\sigma} \right) \right], \quad (32)$$

$$K_R = \frac{\frac{\theta}{\theta-1} \frac{\lambda}{(1-\tau_R)} \left(\frac{Y_R}{A_R} \right)^{1+\omega} + \alpha \beta (\Pi^*)^{\theta(1+\omega)} (p_{RD} K_D + p_{RN} K_N)}{1 - \alpha \beta p_{RR} \Pi_R^{\theta(1+\omega)}}, \quad (33)$$

$$F_R = \frac{\frac{Y_R}{(Y_R - G_R)^\sigma} + \alpha \beta (\Pi^*)^{\theta-1} (p_{RD} F_D + p_{RN} F_N)}{1 - \alpha \beta p_{RR} (\Pi_R)^{\theta-1}}, \quad (34)$$

$$\frac{F_R}{K_R} = \left(\frac{1 - \alpha \Pi_R^{\theta-1}}{1 - \alpha} \right)^{\frac{1+\omega\theta}{\theta-1}}. \quad (35)$$

In our discussions of the economy with preference shocks ξ_t in Section 4, we found that the slope of the short-run Phillips curve matters for whether a solution lying on the AS curve and the AD curve represents a Markov-perfect equilibrium. In fact, the proof of Lemma 2 in Appendix B can be directly applied also to the case under consideration. Hence we obtain

Lemma 5. *For $\sigma \geq 1$, the short-run Phillips curve in state R of the economy with productivity shocks is upward-sloping.*²⁹

As a result, we conclude that a Markov-perfect equilibrium for a central bank that pursues a strict inflation-targeting strategy, i.e. a central bank whose loss function depends only on inflation, has to satisfy $\Pi_R \leq \Pi^*$ because otherwise the central bank could profitably deviate by raising interest rates above the zero lower bound, thereby lowering output and inflation.

We summarize these findings in the following lemma:

Lemma 6. *A tuple (Y_N, Y_D, Y_R, Π_R) that satisfies (12)-(14), (28)-(30), and (32)-(35), is a Markov-perfect equilibrium of the economy with productivity shocks iff $\Pi_R \leq \Pi^*$.*

²⁹Analogously to the short-run Phillips curve (23)-(25), the short-run Phillips curve in state R for a particular period t can be readily obtained from (8)-(10) by taking all future values of F_j and K_j , $j \geq t+1$, as given. Depending on the state they are equal to F_N , K_N , F_D , K_D , F_R , or K_R respectively.

6 Numerical findings

6.1 Calibration

Finally, we calibrate our model to be able to derive quantitative predictions. We select standard values $\beta = 0.99$ and $\sigma = 1$. Moreover, we set $\alpha = 0.5$, which corresponds to an expected price duration of two quarters, which is the median duration of regular prices in the United States for the time period 1998-2005 (see Nakamura and Steinsson (2008)).³⁰ Parameter ω , the inverse of the Frisch elasticity of the labor supply, is set to $\omega = 1/0.75$, which is the value chosen by Mertens and Ravn (2014). We select $\theta = 11$ for the elasticity of substitution, which implies a markup of 10% in the long-run state N . For the levels of taxes and government expenditures, we follow Eggertsson and Singh (2016) and pick $G_N = G_R = G_D = 0.2$ and $\tau_N = \tau_R = \tau_D = 0.3$.

We restrict the values of p_{RD} and p_{RN} by imposing $p_{RD} = p_{RN}$. Moreover, we choose $p_{RR} = 0.95$. This value entails that the expected duration of the liquidity trap is five years, which appears to be a reasonable magnitude, given that many economies had essentially zero nominal interest rates for several years following the global financial crisis of 2008. While the analyses in Gust et al. (2012) and Fernández-Villaverde et al. (2015) involve zero-lower-bound events that last for one year in expectations or even shorter, the corresponding values of p_{RR} would imply that the zero-lower-bound episodes observed in reality with durations of e.g. seven years in the United States are highly unlikely events (see Section 3.3 in Boneva et al. (2016)). We will later discuss in more detail how our results depend on the choice of p_{RR} . We set $p_{DD} = 0.95$, which has the implication that the expected duration of a depression is five years as well.

Many central banks like the ECB or the Fed pursue an inflation target of approximately two percent. Hence $\Pi^* = 1.02^{1/4}$ is a plausible choice. Finally, we select $A_R = 0.93$, which causes a decline of output by roughly 7%, which is targeted by Boneva et al.

³⁰Taking into account product substitutions, Nakamura and Steinsson (2008) find median durations for regular prices of seven to nine months. In the literature, larger values are sometimes employed. These do not affect our results qualitatively.

Figure 3: Left panel: The AD curves (dashed lines) and the AS curves (solid lines) for the parameter constellation given in the text with $p_{RR} = 0.95$ (black lines) and the same parameter constellation except for $p_{RR} = 0.6$ (gray lines). Right panel: the maximum level of output attainable through a one-period deviation of the central bank (dashed line) and the short-run Phillips curve (black line) for the first parameter constellation.

(2016) for the Great Recession.³¹ For A_D , we pick a value of 0.7, which causes a drop in output of approximately 30%, like in the Great Depression scenario of Eggertsson and Singh (2016).

6.2 Benchmark equilibrium

The left panel of Figure 3 displays the AD curve and the AS curve for this calibration as a dashed and a solid black line. We observe that the lines intersect only once and that this point corresponds to the third point discussed in Section 4. The corresponding inflation rate is below the target rate of Π^* . According to Lemma 3 and the right panel of the Figure, which plots the short-run Phillips curve in state R as a solid line, no profitable deviation exists for the central bank in each period where the economy is in state R . By raising interest rates above the zero lower bound in a given period, the central bank could lower output in this period. However, this would move inflation further away from the target $\Pi^* = 1.02^{1/4}$ and thus would not be beneficial. Hence, the intersection of the AD curve and the AS curve represents a Markov-perfect equilibrium.

³¹Fernald (2015) uses growth accounting to show that TFP growth fell markedly during the Great Recession.

We would like to highlight that this is true not only for a central bank that is solely interested in stabilizing inflation at Π^* but would continue to hold for a central bank that also cares about output stabilization.

The values of the relevant economic variables in this equilibrium are reported in Table 1. In state R , the economy is stuck at the zero lower bound with moderate deflation, i.e. a net annual inflation rate of -1% . It is noteworthy that the interest rate is rather high in the depression state D . As has been discussed before, this is due to the fact that consumption is expected to increase strongly once the economy recovers to the normal state N . This tends to push nominal interest rates up, just like the possibility of a deterioration of the economy pushes interest rates down in a recession.³²

state	N	R	D
output	1.00	0.93	0.73
gross annual inflation rate	1.02	0.99	1.02
gross annual nominal interest rate	1.06	1.00	1.14

Table 1: Benchmark equilibrium where the ZLB binds in state R .

6.3 Relationship to Section 4

One might ask why there is only one intersection of the AS curve and the AD curve for our model with the benchmark calibration as opposed to the three intersections discussed for the economy with preference shocks in Section 4. The main reason is that three intersections occur only for sufficiently small values of p_{RR} . To illustrate this, Figure 3 also shows the AD curve and the AS curve for $p_{RR} = 0.6$, which corresponds to an expected duration of state R of 7.5 months, as gray lines. It is clear that the graphs look qualitatively similar to the ones in Figure 1.

In particular, for $p_{RR} = 0.6$ both curves intersect three times like in the scenario with preference shocks considered in Section 4. The two points with lower output levels

³²We have evaluated whether a log-linear approximation of our model around the equilibrium considered in this section is locally determinate. For this purpose, we have log-linearized (7) for $I_t = 1$ and (8)-(10) around the equilibrium values in state R under the assumption that the economy will be in the equilibrium under consideration in states D and N . This exercise yields three stable roots for four forward-looking variables and hence local indeterminacy. However, it is not clear whether this analysis should lead one to exclude the equilibrium under consideration.

correspond to the equilibria discussed in detail by Eggertsson and Singh (2016). It is worth noting that the third point does not represent a Markov-perfect equilibrium for a central bank only interested in stabilizing inflation at $\Pi^* = (1.02)^{1/4}$ because it implies an inflation rate that is higher than this target. Accordingly, this point would only correspond to an equilibrium if the central bank cared sufficiently strongly about the difference between output and the long-run level of output. We will discuss the role of p_{RR} in more detail in Section 6.6.

6.4 Policy effects

In this section, we analyze the effectiveness of fiscal policy at the zero lower bound as well as the consequences of changes in labor taxes and increases in the inflation target for the equilibrium described in Section 6.2. As has been discussed before, several studies find large government spending multipliers and harmful effects of positive supply stimuli like reductions in labor income taxes. Interestingly, our numerical simulations reveal that the government multiplier is quite small in our model: An increase of government spending by one small unit of goods leads to an output gain of 0.42 units. The effects of tax reductions are also conventional and thereby different from those in Eggertsson (2011) and other papers that find harmful effects of supply-side stimulus. A reduction of the tax rate by one percentage point leads to a rise in output of 0.006 units, which corresponds to an increase of approximately 0.6%.

Additionally, one might ask how long-term changes in government expenditures affect the economy in a recession. To be more precise, we examine how a commitment of the government to raise government expenditures in state N influences output in a liquidity trap.³³ One might expect that increases in government expenditures lower consumption in state N (see Lemma 1) and thereby exacerbate the liquidity trap in state R , in line with our discussion that expected low levels of future consumption tend to drive interest rates downwards. However, our numerical simulations reveal that an increase in G_N by one unit has a positive, albeit almost negligible effect on output Y_R ,

³³Eggertsson (2011) discusses a similar experiment.

as Y_R increases by only 0.02 units. By comparison, output in state N increases by 0.48, which is in line with Lemma 1.³⁴

Several macroeconomists have argued recently that inflation targets should be revised upwards in light of the zero lower bound. For example, Blanchard et al. (2010) proposed to consider an inflation target of 4%. One rationale for this proposal is that higher inflation targets increase inflation expectations, thereby lowering real interest rates in situations where the central bank cannot lower nominal interest rates because of the zero lower bound.³⁵

Our model can be used to estimate the consequences of such a strategy change. The respective results are presented in Table 2. The increase in the target leads to corresponding increases in inflation in states N and D , where monetary policy is unconstrained by the zero lower bound. However, it is only moderately effective in raising inflation in state R . This is due to the fact that state R is rather persistent and therefore higher expected inflation in states N and D does not have a substantial effect on inflation expectations in state R . While the higher inflation target is moderately successful in increasing inflation in state R , Table 2 also reveals that it has adverse consequences for output in all states. Although an analysis of welfare is beyond the scope of this paper, our model therefore suggests some caution towards higher inflation targets.

state	N	R	D
relative change in output (in percent)	-1.6	-0.5	-1.5
change in the annual inflation rate (in percentage points)	2.0	0.2	2.0

Table 2: The consequences of raising the inflation target from 2% to 4%.

6.5 Equilibria without a binding ZLB

While it is clear from Figure 3 that the equilibrium for the benchmark calibration is the only equilibrium where the zero lower bound binds in state R but not in the other

³⁴Hence the government spending multiplier is even larger in state N (0.48) than in state R (0.42).

³⁵The argument that central banks should attempt to raise inflation expectations in a liquidity trap is put forth in Eggertsson and Woodford (2003).

states, one might ask whether additional equilibria exist where the zero lower bound is always slack. This is indeed the case for the calibration specified in Section 6.1. The economic variables for this equilibrium are stated in Table 3.

state	N	R	D
output	1.00	0.94	0.73
gross annual inflation rate	1.02	1.02	1.02
gross annual nominal interest rate	1.06	1.03	1.14

Table 3: Equilibrium where the ZLB does not bind in state R .

First, it is worth noting that the equilibrium values in states N and D are identical to those shown in Table 1. This is a consequence of the assumption that the economy cannot move back to state R once it is in states N or D . Therefore changes in state R leave the equilibrium values in the other states unaffected. Second, we would like to point out that the nominal interest rate in state R is lower than in state N , which is in line with our previous argument that an expected decrease in consumption tends to lower nominal interest rates. The following section will discuss under which circumstances equilibria occur where the zero lower bound never binds.

6.6 Persistence of state R

It is known from the literature (see Mertens and Ravn (2014) and Carlstrom et al. (2014), among others) that the probability with which the economy remains in the liquidity trap affects the behavior of the economy and the efficacy of different policies strongly. We therefore analyze how the equilibria considered in Sections 6.2 and 6.5 are affected by changes in p_{RR} , while maintaining our assumption that $p_{RD} = p_{RN}$.³⁶

In order to show how the liquidity-trap equilibrium considered in Section 6.2 is influenced by the persistence of state R , Figure 4 displays the values of output and inflation that solve Conditions (32)-(35) as a function of the expected duration of state R in years, i.e. $\frac{1}{4} \cdot \frac{1}{1-p_{RR}}$. Interestingly, the left panel of the figure reveals that inflation would be above the target when the expected duration of state R is smaller than 2.5 years.

³⁶As $p_{RN} + p_{RR} + p_{RD} = 1$, this implies $p_{RD} = p_{RN} = \frac{1}{2}(1 - p_{RR})$.

Figure 4: Left panel: deviation of the inflation rate from the target as a function of the expected duration of state R in years, $\frac{1}{1-p_{RR}} \cdot \frac{1}{4}$, for the potential equilibrium where the ZLB binds in state R . Right panel: output as a function of the expected duration. Other parameters as introduced in Section 6.1.

Lemma 5 implies that a central bank that focuses exclusively on attaining its inflation target could profitably deviate by increasing the interest rate in this case. Hence, for durations shorter than 2.5 years, the equilibrium analyzed in Section 6.2 fails to exist for such a central bank. For durations longer than 2.5 years, the equilibrium exists and inflation is a decreasing function of the expected duration of state R .

One might also wonder how the additional equilibrium examined in Section 6.5, where interest rates are always above the zero lower bound, is affected by changes in the persistence of state R . Figure 5 plots output and the nominal interest rate that solve the IS curve (7) in state R and the AS curve (33)-(35) under the assumption that the central bank can always achieve its target Π^* . Importantly, the figure shows that these equilibria fail to exist for durations below 2.5 years as nominal interest rates would violate the zero lower bound constraint.

To sum up, both types of equilibria exist if state R is sufficiently persistent.³⁷ Otherwise, both equilibria fail to exist. This raises the question which equilibria exist for shorter durations. First, as we have shown, for sufficiently short expected durations

³⁷Note that the minimum duration of state R that guarantees the respective equilibrium to exist is identical for both equilibria. At this duration, both equilibria coincide and $\Pi_R = \Pi^*$ and $I_R = 1$ hold at the same time.

Figure 5: Left panel: annual net nominal interest rate as a function of the expected duration of state R in years, $\frac{1}{1-p_{RR}} \cdot \frac{1}{4}$, for a potential equilibrium where the ZLB never binds in state R . Right panel: output as a function of the expected duration. All other parameters are taken from Section 6.1.

of state R , deflationary equilibria similar to those examined by Eggertsson and Singh (2016) exist (see the gray lines in Figure 3). Second, for intermediate durations, one can show that no Markov equilibrium exists, unless the central bank is also concerned about output stabilization, in which case the liquidity-trap equilibria studied in Section 6.2 may exist.

To examine the case of an intermediate duration of state R more closely, we consider a calibration of our model with $p_{RR} = 0.856$, which is the value considered in Denes et al. (2013). In this case, the AS curve and the AD curve intersect only once and the corresponding values of the economic variables are displayed in Table 4. In line with our previous discussion, we observe that inflation is above target (but at a level that is in line with the experience of the United Kingdom at zero interest rates after the global financial crisis). Hence this constellation would not correspond to an equilibrium if the central bank was only interested in achieving its inflation target. However, as we explain now, a central bank attempting to bring inflation to its target level would have to cause sizable losses in output. The target for quarterly inflation is approximately 1.005 and the value of quarterly inflation is 1.012 according to Table 4. As the slope of the short-run Phillips curve is 0.048, bringing inflation to its target would approximately involve a reduction of output by $\frac{1.012-1.005}{0.048} \approx 0.15$. Taking output in the normal state $Y_N = 1$

as a reference point, this would involve a 15% drop in output, in addition to the 7% output drop that already occurs when gross annual inflation is at 1.050. As it appears unlikely that a central bank would be willing to incur such significant output losses, one can conclude that the constellation described in Table 4 represents an equilibrium for a central bank that puts a plausible emphasis on output stabilization. At such an equilibrium, the government spending multiplier is positive but clearly below one.

Our finding that the equilibrium discussed in Section 6.2 tends to exist only for a sufficiently persistent state R may be reminiscent of Mertens and Ravn (2014), who demonstrate that liquidity-trap equilibria driven by confidence shocks have long expected durations as opposed to liquidity-trap equilibria driven by fundamental shocks, which are more temporary phenomena. In our model, the liquidity-trap equilibria when state R is rather persistent can be interpreted as being driven by confidence shocks to some extent because they coexist with equilibria for which interest rates are positive. However, while in Mertens and Ravn (2014) expansionary fiscal policy is deflationary in equilibria driven by confidence shocks, expansionary fiscal policy can easily be shown to be inflationary in the liquidity-trap equilibrium considered in Section 6.2.

economic variable in state R	value
output	0.932
gross annual inflation rate	1.050
gross quarterly inflation rate	1.012
government spending multiplier	0.696
slope of the short-run Phillips curve	0.048

Table 4: Economic variables in state R for the point where the AS curve and the AD curve intersect under the assumption that $p_{RR} = 0.856$.

7 Permanent liquidity traps

In a recent contribution, Armenter (2017) analyzes log-linearized monetary economies with the zero lower bound and central banks that pursue nominal targets. He shows that, in addition to the Markov-perfect equilibrium where the zero bound does not bind, there is typically a deflationary equilibrium where self-fulfilling expectations keep

nominal interest rates at a zero level. It may therefore be interesting to ask whether this finding also holds in our fully non-linear economy.

To address this question, we consider the simplest possible case and assume that the economy is permanently in state N . In this case, Equations (11)-(15) describe a Markov-perfect equilibrium with positive nominal interest rates. As a next step, we attempt to construct an additional equilibrium with zero nominal interest rates. This is easy, as together with the condition $\Pi_N = \beta$, an equilibrium is given by (12)-(14). For an inflation target $\Pi^* \geq \beta$ and $\sigma \geq 1$, no profitable deviation exists for the central bank, as the short-run Phillips curve is upward sloping, which is a consequence of arguments similar to those leading to Lemma 2.

Hence, Armenter's finding extends to our economy:

Lemma 7. *Suppose that $\sigma \geq 1$ and $\Pi^* \geq \beta$ and consider an economy whose initial state is N . Then an additional Markov-perfect equilibrium exists with permanent deflation $\Pi_N = \beta$. If $\Pi^* = 1$ and*

$$\frac{1 - \alpha\beta^{1+\theta(1+\omega)}}{1 - \alpha\beta^\theta} \cdot \left(\frac{1 - \alpha}{1 - \alpha\beta^{\theta-1}} \right)^{\frac{1+\omega\theta}{\theta-1}} < 1, \quad (36)$$

then output in the additional equilibrium is lower than in the equilibrium with $\Pi_N = \Pi^$.*

In Appendix C, we provide a proof for the claim that output is lower in the deflationary equilibrium if $\Pi^* = 1$ and (36) are satisfied. We have verified that Condition (36) is fulfilled for a large set of empirically plausible values of α , β , and ω . For example, for $\beta = 0.99$, $\theta = 11$, $\alpha = 0.5$, and $\omega = 1/0.75$, which are the values selected in Section 6.1, the left-hand side of Condition (36) is approximately 0.97. Hence, we can conclude that the additional deflationary equilibrium typically involves lower output.³⁸

Armenter (2017) proposes that central banks should pursue long-term interest rate targets to eliminate the additional equilibria appearing in the presence of the zero

³⁸Lemma 7 is also closely related to the finding in Mertens and Ravn (2014) that a permanent liquidity trap may arise in an economy with Calvo price setting and a central bank whose behavior can be described by a Taylor rule. From this perspective, Lemma 7 extends their finding of permanent liquidity traps to an economy with segmented labor markets and a central bank that pursues an inflation targeting strategy and chooses monetary policy under discretion.

lower bound. In the following, we reassess his proposal in our fully non-linear model. While we could readily introduce bonds with longer maturities into our framework in order to be able to examine the consequences of a stabilization objective for long-term interest rates, we focus on a short-term interest rate stabilization objective for simplicity. Importantly, this simplification does not affect our results qualitatively.

Suppose that the central bank's loss function was given by

$$L^I(\Pi_t, I_t) = (\Pi_t - \Pi^*)^2 + b(I_t - I^*)^2, \quad (37)$$

where b is a positive parameter and I^* is the central bank's interest rate target, which is given by the equilibrium interest rate in state N when interest rates are positive, i.e. $I^* = \frac{1}{\beta} \cdot \Pi^*$. For sufficiently large values of b , the equilibrium described in Lemma 7 ceases to exist because the central bank can profitably deviate by raising interest rates in a given period in state R . While this would come at the expense of lower inflation in this period, a sufficiently large value of b guarantees that the central bank is willing to incur these costs in order to bring interest rates closer to their target level. To sum up, the proposal made by Armenter (2017) is also effective in our framework. An interest rate target makes it credible that the central bank would raise interest rates in a non-fundamental liquidity trap and thereby invalidate the self-fulfilling beliefs that would cause such an equilibrium.

8 Discussion and Conclusions

In this paper, we have examined a simple yet fully non-linear New Keynesian model with a zero-lower-bound constraint. We have shown that, for a plausible calibration of our model, fiscal policy has only moderate effects on output in a liquidity trap. Moreover, we have found that the effects of changes in labor taxes in a liquidity trap are not qualitatively different from the respective effects in the absence of a liquidity trap.

Our paper has concentrated on expected adverse productivity shocks as opposed to shocks to the representative household's discount factor as a source of liquidity-trap

episodes. We have already stressed that a disadvantage of the conventional approach that uses preference shocks to model liquidity-trap scenarios may be that it is not clear whether it leads to a reliable analysis of welfare, given that the shocks imply that some periods receive a lower weight in the intertemporal social welfare function than others. Consequently, it would be interesting to examine socially optimal policies in our framework and to compare them with the corresponding results for preference shocks.³⁹ There are other potentially interesting issues that could be addressed in future research. For example, one could examine whether the equilibria examined in this paper are stable under learning (see Christiano et al. (2016)).

³⁹This would necessitate an explicit modeling of price dispersion.

A Derivation of the New Keynesian Phillips Curve (8)-(10)

For completeness, we derive the New Keynesian Phillips curve with stochastic shocks to preferences ξ_t and aggregate productivity shocks A_t . Taking the wage $w_t(i)$ in its sector as given, a firm i that has the opportunity to adjust its price chooses the price $p_t(i)$ to maximize the following sum of discounted profits, where profits in period j ($j \geq t$) are weighted by the factor $\xi_j \alpha^{j-t} \beta^{j-t} C_j^{-\sigma}$:

$$\sum_{j=t}^{\infty} \xi_j (\alpha \beta)^{j-t} C_j^{-\sigma} \left[\frac{p_t(i)}{P_j} \left(\frac{p_t(i)}{P_j} \right)^{-\theta} Y_j - \frac{w_j(i)}{P_j} \left(\frac{p_t(i)}{P_j} \right)^{-\theta} \frac{Y_j}{A_j} \right] \quad (38)$$

Equation (38) uses that the demand in period j for a good with price $p_t(i)$ is given by $y_j(i) = \left(\frac{p_t(i)}{P_j} \right)^{-\theta} Y_j$ (compare (6)) and that the labor the firm has to employ to satisfy demand $y_j(i)$ is $y_j(i)/A_j = \left(\frac{p_t(i)}{P_j} \right)^{-\theta} \frac{Y_j}{A_j}$.

Computing the derivative with respect to $p_t(i)$ results in the following first-order condition for the optimal price p_t^* of firms that can adjust their prices $p_t(i)$ in period t :

$$\sum_{j=t}^{\infty} \xi_j (\alpha \beta)^{j-t} C_j^{-\sigma} \left[(1 - \theta) \frac{(p_t^*)^{-\theta}}{P_j^{1-\theta}} Y_j + \theta \frac{w_j(i)}{P_j} \frac{(p_t^*)^{-1-\theta}}{P_j^{-\theta}} \frac{Y_j}{A_j} \right] = 0 \quad (39)$$

With the help of the household's first-order condition for an optimal choice of labor, which can be combined with demand (6) and the production function $y_t(i) = A_t n_t(i)$,

$$\begin{aligned} \frac{w_j(i)}{P_j} &= \frac{\lambda n_j(i)^\omega C_j^\sigma}{1 - \tau_j} \\ &= \frac{\lambda \left(\frac{y_j(i)}{A_j} \right)^\omega C_j^\sigma}{1 - \tau_j} \\ &= \frac{\lambda \left(\frac{p_t^*}{P_j} \right)^{-\theta \omega} (Y_j)^\omega C_j^\sigma}{(1 - \tau_j)(A_j)^\omega}, \end{aligned} \quad (40)$$

Equation (39) can be stated as

$$(\theta - 1) \sum_{j=t}^{\infty} \xi_j (\alpha \beta)^{j-t} C_j^{-\sigma} \frac{(p_t^*)^{-\theta}}{P_j^{1-\theta}} Y_j = \theta \sum_{j=t}^{\infty} \xi_j (\alpha \beta)^{j-t} \frac{\lambda \left(\frac{p_t^*}{P_j} \right)^{-\theta \omega} (Y_j)^\omega}{(1 - \tau_j)(A_j)^\omega} \frac{(p_t^*)^{-1-\theta}}{P_j^{-\theta}} \frac{Y_j}{A_j}. \quad (41)$$

This expression can be re-arranged in the following way:

$$\begin{aligned}
& (\theta - 1) \left(\frac{p_t^*}{P_t} \right)^{1+\theta\omega} \sum_{j=t}^{\infty} \xi_j (\alpha\beta)^{j-t} C_j^{-\sigma} \left(\frac{P_j}{P_t} \right)^{\theta-1} Y_j \\
& = \theta \sum_{j=t}^{\infty} \xi_j (\alpha\beta)^{j-t} \frac{\lambda}{1 - \tau_j} \left(\frac{P_j}{P_t} \right)^{\theta(1+\omega)} \left(\frac{Y_j}{A_j} \right)^{1+\omega}.
\end{aligned} \tag{42}$$

With the definitions of K_t and F_t given in the main text, the price of a firm that can re-optimize its price, p_t^* , can be written as:

$$\frac{p_t^*}{P_t} = \left(\frac{K_t}{F_t} \right)^{\frac{1}{1+\omega\theta}} \tag{43}$$

With the help of the well-known equation describing the evolution of the price level under Calvo price-setting,

$$P_t = ((1 - \alpha)(p_t^*)^{1-\theta} + \alpha P_{t-1}^{1-\theta})^{\frac{1}{1-\theta}}, \tag{44}$$

we obtain the following relationship between p_t^*/P_t and the gross rate of inflation Π_t :

$$1 = (1 - \alpha) \left(\frac{p_t^*}{P_t} \right)^{1-\theta} + \alpha \Pi_t^{\theta-1}. \tag{45}$$

Using (45) to substitute for p_t^*/P_t in (43) yields (10). $\square$

B Proof of Lemma 2

We can rewrite (23)-(25) as

$$K'_{\tilde{R}} = \phi(Y'_{\tilde{R}})^{1+\omega} + C, \tag{46}$$

$$F'_{\tilde{R}} = \xi_{\tilde{R}} \frac{Y'_{\tilde{R}}}{(Y'_{\tilde{R}} - G_{\tilde{R}})^{\sigma}} + D, \tag{47}$$

$$\frac{F'_{\tilde{R}}}{K'_{\tilde{R}}} = \left(\frac{1 - \alpha(\Pi'_{\tilde{R}})^{\theta-1}}{1 - \alpha} \right)^{\frac{1+\omega\theta}{\theta-1}}, \tag{48}$$

where ϕ , C , and D are positive terms that are constant for short-term deviations in state $\tilde{R}$. As a next step, we observe that $K'_{\tilde{R}}$ is an increasing function of $Y'_{\tilde{R}}$. Moreover, $F'_{\tilde{R}}$ decreases with $Y'_{\tilde{R}}$ for all $Y'_{\tilde{R}} > G_{\tilde{R}}$ provided that $\sigma \geq 1$. This has the implication

that the left-hand side of (48) is a decreasing function of $Y'_{\tilde{R}}$. Together with the observation that the right-hand side of (48) decreases with $\Pi'_{\tilde{R}}$ for $\Pi'_{\tilde{R}} \in \left[0, \frac{1}{\alpha^{\frac{1}{\theta-1}}}\right]$ and the implicit function theorem, this proves the claim of the lemma.⁴⁰ $\square$

C Proof of Lemma 7

In this appendix, we prove that output in the additional equilibrium is lower than in the equilibrium with $\Pi_N = P^* = 1$. For this purpose, we note that output in both equilibria is given by (12)-(14) and the respective level for Π_N , i.e. $\Pi_N = 1$ or $\Pi_N = \beta$. Equations (12)-(14) can be re-arranged as

$$(Y_N)^\omega (Y_N - G_N)^\sigma = C \frac{1 - \alpha\beta(\Pi_N)^{\theta(1+\omega)}}{1 - \alpha\beta(\Pi_N)^{\theta-1}} \cdot \left(\frac{1 - \alpha}{1 - \alpha(\Pi_N)^{\theta-1}} \right)^{\frac{1+\omega\theta}{\theta-1}}, \quad (49)$$

where C is a positive constant. We make the following observations: First, the left-hand side of (49) is a monotonically increasing function of Y_N . Second, the right-hand side equals C for the standard equilibrium with $\Pi_N = 1$. Third, for $\Pi_N = \beta$ the right-hand side of (49) is strictly smaller than C , provided that

$$\frac{1 - \alpha\beta^{1+\theta(1+\omega)}}{1 - \alpha\beta^\theta} \cdot \left(\frac{1 - \alpha}{1 - \alpha\beta^{\theta-1}} \right)^{\frac{1+\omega\theta}{\theta-1}} < 1. \quad (50)$$

Taken together, these observations imply that output is smaller in the equilibrium where the zero lower bound holds with equality if condition (50) is fulfilled. $\square$

⁴⁰The New Keynesian Phillips curve does not admit values of inflation that are larger than $\frac{1}{\alpha^{\frac{1}{\theta-1}}}$.

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